# Power Plants (part 3)

## Electro-Thermal Technology

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**Contents**

**A list of the most important quantities of electro-thermal technology**

| Abbreviation | Meaning | Unit |
|---|---|---|
| $M$ | irradiance | W·m$^{-2}$ |
| $C$ | heat capacity | J·K$^{-1}$ |
| $J$ | current density | A·m$^{-2}$ |
| $P$ | Power | W |
| $Q$ | heat | J |
| S | area | m$^{2}$ |
| $T$ | thermodynamic temperature | K |
| $V$ | volume | m$^{3}$ |
| $W$ | energy | J |
| $C$ | specific heat capacity | J·kg$^{-1}$·K$^{-1}$ |
| $p_{skut}$ | actual surface loading | W·m$^{-2}$ |
| $a$ | thermal conductivity coefficient | m$^{2}$·s$^{-1}$ |
| $q$ | heat flux density | W·m$^{-2}$ |
| $h$ | Planck constant | J·s |
| $t$ | time | s |
| $f$ | frequency | Hz |
| α | heat transfer coefficient | W·m$^{-2}$·K$^{-1}$ |
| $\gamma$ | conductivity | S·m$^{-1}$ |
| $\varepsilon$ | emissivity | - |
| $\varepsilon$ | permittivity | F·m$^{-1}$ |
| $\sigma$ | Stefan-Boltzmann constant | W·m$^{-2}$·K$^{-4}$ |
| Λ | wave length | $M$ |
| Λ | thermal conductivity | W·m$^{-1}$·K$^{-1}$ |
| $\mu$ | permeability | H·m$^{-1}$ |
| $\rho$ | density | kg·m$^{-3}$ |
| $\rho$ | resistivity | Ω·m |
| tg$\delta$ | dissipation factor | - |
| cos $\varphi$ | power factor | - |
| $\Phi$ | heat flux, heat losses | W |
| $\vartheta$ | temperature | °C |

Other symbols and abbreviations used in the text are explained upon their occurrence.

# 1. Introduction

The presented book is intended mainly for students of electro-technical study fields, but also for students of other related fields of engineering, civil engineering, metallurgy and others, where they deal with the issues of the electro-thermal technology. Some parts of the book can be used by workers in practice. The electro-thermal technology deals with the transformation of electric energy into heat in various kinds of electro-thermal devices described herein. A significant part of this book is devoted to the energy issues of electro-thermal facilities in the industry as well as to heating, air-conditioning and electric energy savings. The theoretical parts of the book can be a valuable aid for study, especially in doctoral study programs.

# 2. Basics of heat transfer

## 2.1. Terms, symbols, quantities and units

- A **thermodynamic (TD) system** is a defined volume of space with its substance filling. Outside there are its surroundings. The system is separated from its surroundings with boundaries, be they real, or imagined. The properties of this boundary determine its characters in terms of:
  - substance permeability:
    - open boundary - both energy and substance can pass through it,
    - closed boundary - only the energy, not the substance, can pass through it in both the directions,
  - energy permeability:
    - a non-insulated border allows the transfer of heat as well as mechanical work,
    - an insulated border allows the transfer of heat and mechanical work between the system and its surroundings,
  - uniformities:
    - homogeneous - properties of the system are the same in all parts,
    - non-homogeneous - properties of the system change in steps in some parts.
- **Thermodynamic variables** are appropriately selected functions (quantities) which express properties of the TD system and its interaction with the surroundings.
- A **thermodynamic process** expresses changes which occur in the system or on its border with the surroundings. It is a sequence of states of the system when the TD variables change over space and time.
- **Thermodynamic equilibrium** is a condition of the system when the TD variables are independent of the place in the system or time. This condition occurs in the insulated and closed system after elapsing of specific time. The full thermodynamic equilibrium is heat, mechanical and chemical equilibrium.
- **Energy** is a TD variable expressing the ability of the system to perform work. On the physical basis, external and internal energies need to be distinguished.
- **The external energy** is related to the movement and position of the

system in the field of external forces. This is kinetic energy.

- **The internal energy** is related to the internal state of the system and to the micro-physical movements in it. The internal energy of the system equals to the total energy which need to be supplied in order to transfer from one state into the other.
- **Heat** is a TD variable which expresses an increment or loss of internal energy of the system, provided that this energy does not perform work and if no chemical reactions or changes to the form happen in the system. Thus, it is a part of the internal energy of the system, which can be exchanged with the surroundings by means of micro-physical interactions. Heat as a form of energy transfer is an expression of the action of non-directed microscopic forces. Therefore, it is not a state quantity as it depends on the way of mutual interaction of the system with its surroundings.
- **Work** is also a form of energy transfer and one of the forms by which the TD system is in relationship with the surroundings. Unlike heat, work is related to the action of macroscopic directed forces (e.g. pressure). Similarly to heat, it does not depend on the state of the system, but on the mutual interaction with the surroundings. Therefore, it is not a state quantity, either. Work is description of coordinated reversible transfer, whereas heat is description of perfectly chaotic transfer.
- **Thermodynamic temperature** is a status variable expressing the thermal state of the system and it is a measure of its total internal energy. This is a quantitative value. It can be measured by means of the change to other physical properties of the system, such as volume and pressure.
- **Laws of thermodynamics**
    - **Zeroth law of thermodynamics -** defines the law of energy conservation. Non-insulated TD systems which are both in mutual interaction and in the TD equilibrium, show the same temperature.
    - **First law of thermodynamics** defines a general principle of energy conservation in TD systems. It defines a status function - internal energy which is a feature of the specific condition of the closed system dependent on the physical quantities of work and heat. Internal energy change is a sum of heat which has been supplied to the system, and work which has been performed on the system.
    - **Second law of thermodynamics** expresses the fact that a cooler body does not pass heat to a warmer body. Therefore, it determines the direction of the thermal energy transfer. It is based on the definition of entropy, which is an extensive function defining the condition of the system. Entropy defines disorderliness of the system. The total change to entropy in the closed system is a sum of

a change to entropy inside the system and entropy which is transferred to the system from its surroundings.

- **Third law of thermodynamics** - at the absolute zero temperature, the entropy of the substance equals to zero.

- **Temperature difference** - If there is a difference in temperatures in the TD system, then energy exchange and their mutual balancing in the form of heat transfer occur. Each element of the system has its own internal energy, i.e. temperature, and thus it forms a scalar field of temperature.
- **Thermal power** *P* is heat per a unit of time ($J \cdot s^{-1}$ = W). It equals to the heat flux *Φ*. This is a scalar quantity.

$$P = \frac{Q}{t} \tag{2.1}$$

- **Heat flux density** *q* has a direction given by a normal and it expresses the amount of energy passing through the specific cross-section for a certain period of time. Its unit is $W \cdot m^2$.

$$q = \frac{\mathrm{d}P}{\mathrm{d}S} \tag{2.2}$$

- **Heat capacity** *C* ($J \cdot K^{-1}$) is a physical quantity expressing the amount of heat by which the body is heated up by 1 K.
- **Specific heat capacity** *c* ($J \cdot kg^{-1} \cdot K^{-1}$) is heat capacity of one kilogram of a substance.

## 2.2. Calorimetric calculations

Calorimetric calculations are among the basic tasks of thermal technology. The material type determines which amount of heat needs to be supplied to 1 kg of a substance in order to be heated up by 1 thermal degree. The ability of the substance to accumulate heat is specified by the specific heat capacity *c*. Generally it is a constant in a specific thermal interval. Tab. 2.1 gives values of this quantity for the most common substances and materials [2].

| **Substance** | ***c*** **($J \cdot kg^{-1} \cdot K^{-1}$)** | **Substance** | ***c*** **($J \cdot kg^{-1} \cdot K^{-1}$)** |
|---|---|---|---|
| water | 4187 | iron | 450 |
| air (°C) | 1003 | copper | 383 |
| ethanol | 2460 | zinc | 385 |

| | | | |
|---|---|---|---|
| ice | 2090 | aluminum | 896 |
| oil | 2000 | tin | 227 |
| dry wood (°C) | 1450 | lead | 129 |
| oxygen | 917 | gold | 129 |
| silicon | 703 | platinum | 133 |

**Tab. 2.1: Specific heat capacity of substances and materials**

The basic issue of the calorimetric calculation is the amount of heat which needs to be supplied for heating up (cooling) of the substance with a weight *m* at the thermal difference *ΔT*. The calorimetric equation then takes the form:

$$Q = m \cdot c \cdot \Delta T \tag{2.3}$$

The conversion of electric energy to heat energy is conducted with efficiency *η*. The total supplied energy then depends on the electric power input of the equipment and on time. The following relation applies to the electricity:

$$Q_e = P \cdot t \tag{2.4}$$

*P* is power input (W), *t* is time (s).

Upon combining of both the equations and including of the efficiency, one can get the resulting relation for the equilibrium of heat and electrical energy.

$$m \cdot c \cdot \Delta T = P \cdot \eta \cdot t \tag{2.5}$$

The relations between the most commonly used units are stated in the following table.

| Unit | J | Wh | cal |
|---|---|---|---|
| J | 1 | $2.778 \cdot 10^{-4}$ | 0.239 |
| Wh | 3600 | 1 | 860 |
| cal | 4.186 | $1.163 \cdot 10^{-3}$ | 1 |

**Tab. 2.2: Relations between units**

## 2.3. Energy transfer and heat transfer

When heat is used for both industrial applications and heating applications, its distribution is controlled according to the general principle of energy distribution; from the place with higher energy density to the place with lower energy density. Energy volumetric mass density ($J \cdot m^{-3}$) is the amount of energy *W* belonging to the volumetric unit of the environment, i.e.

$$w = \frac{\mathrm{d}W}{\mathrm{d}V} \tag{2.6}$$

Heat transfer intensity is then expressed by heat flux (power)

$$P = \frac{\mathrm{d}Q}{\mathrm{d}t} \tag{2.7}$$

Heat flux density (W·m$^{-2}$) is considered to be the passage of the specific thermal power through the area $S$ = 1 m$^2$.

$$\boldsymbol{q} = \frac{\mathrm{d}P}{\mathrm{d}\boldsymbol{S}} = \frac{\mathrm{d}^2 Q}{\mathrm{d}\boldsymbol{S} \cdot \mathrm{d}t} \tag{2.8}$$

Energy transfer (heat) through the environment is executed by energy carriers. These are particles which occur in the respective environment, but which feature higher energy than the particles in their vicinity or which get into the specific environment from the surroundings. As a result, the kind, speed and way of particle movement are different and they depend on the type of the environment. They can be elementary particles (electrons, atoms), but also electromagnetic waves which transmit energy by photons.

There are two particular ways of movement of particles (energy carriers) and they depend on the concentration of substance particles in the respective environment.

The first method is applied mainly in the environments with high concentration of particles (solid or liquid state). Then the particles which move against the energy density gradient are in a constant contact with other particles of the environment. The constant interaction between these particles is called diffusion. It is natural transfer of heat from the places with higher thermal concentration to the places with lower concentration. Energy is transmitted through continuous collisions between particles.

The other method of heat transfer occurs in flowing liquids, where the energy transfer by means of diffusion is supported by the energy transfer of environment masses. It is, therefore, a diffusion-convection way of transfer and it strongly depends on the speed of fluid flow.

Another and different way of energy transfer occurs in the environment with a low content of particles. The transfer of energy carriers is carried out by radiation. The radiation can be in the form of light.

These principles describe the heat transfer through the environment, and three ways of heat transfer are derived for them [1]:

- heat conduction,
- heat flow (convection),

- thermal radiation.

## 2.4. Thermodynamic processes

Heat transfer by conduction, convection and radiation and their possible combination are called thermodynamic processes.

### 2.4.1. Heat conduction, energy transfer by diffusion

Energy diffusion transfer (W·m$^{-2}$) is conditioned by the movement of environmental particles against the energy density gradient.

$$\varphi = -\frac{1}{2} \cdot v_{\mathrm{av}} \cdot l_{\mathrm{av}} \cdot \mathrm{grad}\, w \qquad (2.9)$$

where the expression $\frac{1}{2} \cdot v_{\mathrm{av}} \cdot l_{\mathrm{av}} = a_{\mathrm{D}}$ is also called the coefficient of diffusion heat transfer.

Energy in the solid environment close to the thermodynamic equilibrium is represented only by the internal energy, its density and ambient temperature. Then change in the internal energy d*w* and temperature d*T* equals to the heat capacity in the volumetric unit of the environment. Therefore the analogy between d*w* = d*u* can be applied.

$$\frac{\mathrm{d}w}{\mathrm{d}T} \equiv \frac{\mathrm{d}u}{\mathrm{d}T} = \rho \cdot c \qquad (2.10)$$

$$\mathrm{d}w \equiv \mathrm{d}u = \rho \cdot c \cdot \mathrm{d}T \qquad (2.11)$$

If we substitute the equations (2.10) and (2.11) to (2.9), we obtain the density of the heat flux by heat conduction.

$$\mathbf{q}_{\mathrm{v}} = -a \cdot \mathrm{grad}\, u = -a \cdot \mathrm{grad}\,(\rho \cdot c \cdot T) \qquad (2.12)$$

If only the temperature gradient changes, then

$$\boldsymbol{q}_{\mathrm{v}} = -a \cdot \rho \cdot c \cdot \mathrm{grad}\, T = -\lambda \cdot \mathrm{grad}\, T \qquad (2.13)$$

The equation (2.13) expresses conduction heat flux density and this equation is also the mathematical expression of the Fourier law of heat conduction. The $\lambda$ (W·m$^{-1}$·K$^{-1}$) coefficient expresses a thermophysical property of the environment and it is called **thermal conductivity**.

The set of physical parameters

$$a = \frac{\lambda}{\rho \cdot c} \qquad (2.14)$$

is called **the heat conductivity coefficient** ($m^2 \cdot s^{-1}$); it is an equivalent to the $a_D$ coefficient in the equation (2.9).

As to space,

- the temperature field can be
    - one-dimensional $\vartheta = \vartheta(x,t)$,
    - two-dimensional $\vartheta = \vartheta(x,y,t)$,
    - three-dimensional $\vartheta = \vartheta(x,y,z,t)$,
- and as to time
    - stationary, for instance $\vartheta = \vartheta(x,y,z)$,
    - non-stationary $\vartheta = \vartheta(x,y,z,t)$.

For the calculation of heat losses and thermal comfort of the environment, the steady state is assumed; the case of stationary temperature field is dealt with. The actual temperature fluctuations in time are taken into account in complementary coefficients used in the applied relations.

The connecting lines of places with the same level of heat energy are called isotherms (Fig. 2.1), to be specific isothermal surfaces, and they are also places with the same temperature.

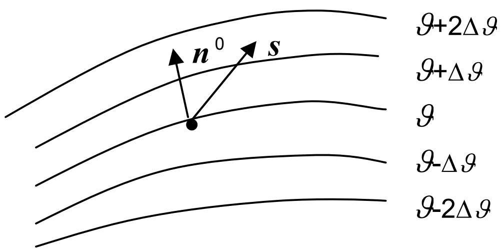



**Fig. 2.1: Isotherms**

The greatest changes to temperatures occur in the direction of the normal line towards the isothermal surfaces. The limit value of the thermal gradient is the temperature gradient.

$$\text{grad}\,\vartheta = \lim_{n\to 0}\frac{\Delta\vartheta}{\Delta n}\boldsymbol{n}^0 \qquad (2.15)$$

It is a vector perpendicular to the isothermal surface. The set of thermal gradients forms a vector field. The existence of the field (provided that it is non-zero) means that heat transmission in the space occurs.

**Heat conduction through a plane wall.**

Heat flux $\Phi$ (W) passing through a homogeneous plane wall with a thickness of $l$, surface of $S$ and difference of surface temperatures $\vartheta_1$-$\vartheta_2$ (Fig. 2.2a) is

$$\Phi = \frac{\lambda}{l}\cdot S\cdot(\vartheta_1 - \vartheta_2) \qquad (2.16)$$

The temperature $\vartheta$ decreases linearly with the distance $x$ from the value $\vartheta_1$ on the left interface to the temperature $\vartheta_2$ on the right interface (equation 2.17). The actual course for ceramic materials is indicated by the dashed line in Fig. 2.2 and above the linear course. Under the linear course, there is a representation for pure metals. In Fig. 2.2b, the isothermal surfaces are indicated by the dashed line.

$$\vartheta = \frac{\vartheta_1 - \vartheta_2}{l}\cdot x + \vartheta_1 \qquad (2.17)$$

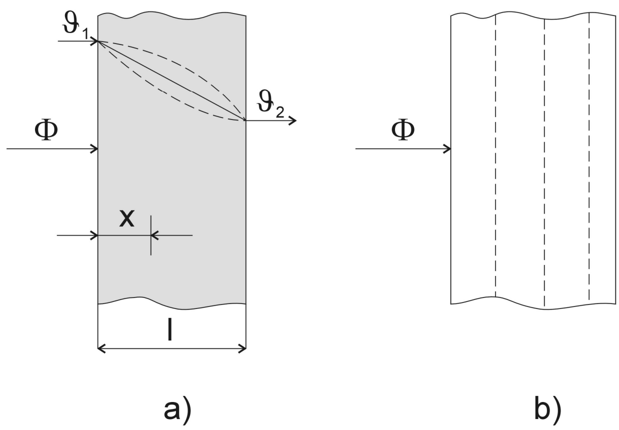



**Fig. 2.2: Heat conduction through a simple plane wall**

If the wall is composed of several differently thick layers of materials featuring various thermal conductivity (Fig. 2.3), then the heat flux through this structure is

$$\Phi = \frac{S \cdot (\vartheta_1 - \vartheta_2)}{\frac{l_1}{\lambda_1} + \frac{l_2}{\lambda_2} + \ldots + \frac{l_n}{\lambda_n}} \tag{2.18}$$

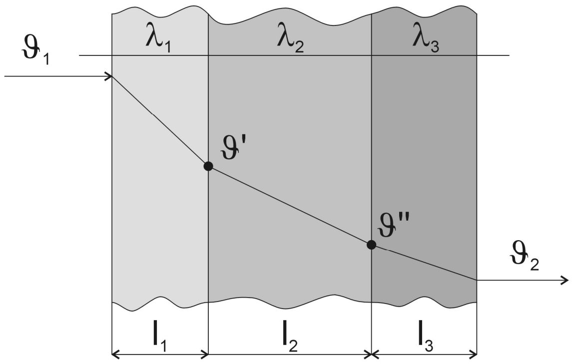



**Fig. 2.3: Heat conduction through a composite plane wall**

With the composite plane wall, the following relations apply to the temperature on the layer interface.

$$\vartheta' = \vartheta_1 - \frac{\Phi \cdot l_1}{\lambda_1 \cdot S} \tag{2.19}$$

$$\vartheta'' = \vartheta_2 + \frac{\Phi \cdot l_2}{\lambda_2 \cdot S} \tag{2.20}$$

**Heat conduction through a cylindrical wall**

During heat conduction from the inner surface of a thick-walled cylindrical tube towards the outer surface (Fig. 2.4), the surface through which the heat passes increases with increasing diameter. The course of temperatures depending on the radius, therefore, has the shape of a logarithmic curve. The following relation then applies to the heat flux

$$\Phi = \frac{2 \cdot \pi}{\frac{1}{\lambda} \cdot \ln \frac{r_2}{r_1}} \cdot l \cdot (\vartheta_1 - \vartheta_2) \tag{2.21}$$

Similarly, the following relation then applies to the composite cylindrical wall.

$$\Phi = \frac{2 \cdot \pi \cdot l \cdot (\vartheta_1 - \vartheta_2)}{\frac{1}{\lambda_1} \cdot \ln \frac{r_2}{r_1} + \frac{1}{\lambda_2} \cdot \ln \frac{r_3}{r_2} + ... + \frac{1}{\lambda_n} \cdot \ln \frac{r_{n+1}}{r_n}} \tag{2.22}$$

The temperatures on the layer interfaces can then be calculated according to

$$\vartheta' = \vartheta_1 - \frac{\Phi}{\pi \cdot l} \cdot \frac{1}{2 \cdot \lambda_1} \cdot \ln \frac{d'}{d_1}$$
$$\vartheta'' = \vartheta_2 - \frac{\Phi}{\pi \cdot l} \cdot \frac{1}{2 \cdot \lambda_3} \cdot \ln \frac{d_2}{d''} \tag{2.23}$$

The following table shows thermal conductivity values for selected materials.

| **The type of material (substance)** | **Thermal conductivity** ($W \cdot m^{-1} \cdot K^{-1}$) |
|---|---|
| air | 0.025 (at 20°C) |
| water | 0.6 (at 20°C) |
| ice | 2.2 |
| thermal insulators | 0.03 – 0.1 |
| wood | 0.1 - 0.5 |
| construction materials | 0.2 – 1.2 |
| stone | 15 – 3.5 |
| pure metals | 50 – 400 |
| alloys | 10 – 200 |

**Tab. 2.3: Thermal conductivity values for selected materials**

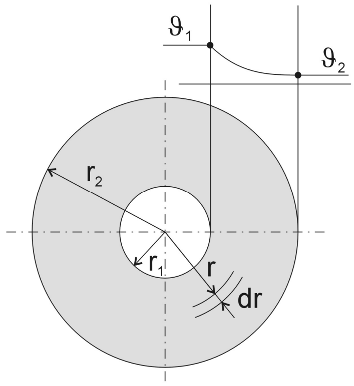



**Fig. 2.4: Heat conduction through a cylindrical wall**

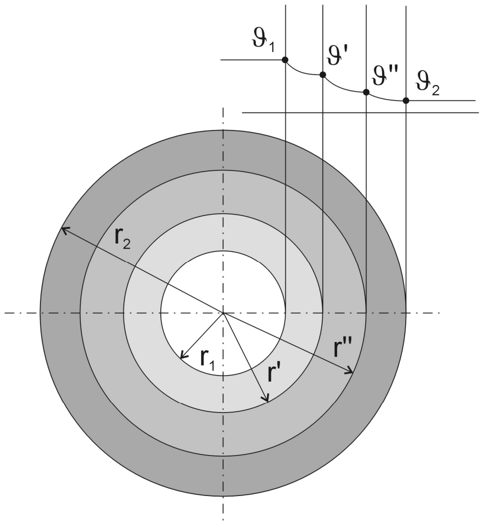



**Fig. 2.5: Heat conduction through a composite cylindrical wall**

## 2.4.2. Heat flux, diffusion-convection energy transmission

Convection is such a way of heat transfer which is bound to the transfer of mass with a specific internal energy from one place to another. From the macroscopic point of view, it is not heat transfer, but transfer of mass, to which heat is bound. This movement is initiated by either the gradient of temperature, or also external effects (e.g. by a fan). Two options are therefore distinguished:

- natural convection,
- forced convection.

In both the cases, transfer of the weight of the environment is expressed using a vector of mass flux ($kg \cdot m^{-2} \cdot s^{-1}$)

$$\boldsymbol{m}_\mathrm{k} = \rho \cdot v \tag{2.24}$$

The vector of heat flux density can be obtained by multiplication of the mass flux $\boldsymbol{m}_\mathrm{k}$ transferred by internal energy ( $m \cdot c \cdot \vartheta$ ) per mass unit, i.e.

$$\boldsymbol{q}_\mathrm{k} = \boldsymbol{m}_\mathrm{k} \cdot c \cdot T = v \cdot \rho \cdot c \cdot T = v \cdot u \tag{2.25}$$

which is in accordance with the first law of thermodynamics and it expresses the heat transfer only by flow. In the real environment, diffusion also participates in heat transfer in the environment (diffusion heat transfer). The overall heat transfer is then diffusion-convection.

$$\boldsymbol{q}_{\mathrm{v,k}} = \boldsymbol{q}_{\mathrm{v}} + \boldsymbol{q}_{\mathrm{k}} = -\lambda \cdot \mathrm{grad}\, T + v \cdot \rho \cdot c \cdot T \qquad (2.26)$$

The contribution of each heat fluxes in the equation (2.26) is dependent on the type, velocity, thermodynamic and hydrodynamic conditions of the environment. In practice, this assessment is very complex and demanding; therefore, a mathematical-experimental model is used for common needs. For bypassing of the surface of the hard environment with a temperature of $T_p$ and by flowing liquid with a temperature of $T_i$, the following equation applies:

$$\mathbf{q}_k = \alpha \cdot (T_p - T_i) = \alpha \cdot \Delta T \qquad (2.27)$$

This means that the density of heat flux by convection is directly determined by the temperature difference between the surface and the fluid. The density is affected by the heat transfer coefficient $\alpha$ ($W \cdot m^{-2} \cdot K^{-1}$).

The convective heat transfer coefficient $\alpha$ depends on pressure, temperature and velocity of fluid flow as well as on the kind of flow (laminar or turbulent) and on the physical properties of the liquid (its density, specific heat capacity, thermal conductivity and viscosity) and also on the shape, dimensions and roughness of the bypassed body. The following table shows the values of this coefficient for some well-known cases.

| | $\alpha_{min}$ ($W \cdot m^{-2} \cdot K^{-1}$) | $\alpha_{max}$ ($W \cdot m^{-2} \cdot K^{-1}$) |
|---|---|---|
| calm air | 12.5 | 125 |
| flowing air | 40 | 2100 |
| flowing liquid | 8400 | 21000 |
| boiling liquid | 16800 | 25100 |
| condensing vapors | 29000 | 50000 |

**Tab. 2.4: The values of heat transfer coefficient [2]**

Convective heat transfer is one of the most difficult calculation issues in the thermal technology. It is dealt with in numerous scientific books. For important cases, it is best to determine the heat transfer coefficient $\alpha$ by our own measuring using a model which best corresponds to our case using relations, in which $\alpha$ occurs. The Newton law applies to convective heat transfer (Fig. 2.6):

$$\Phi = \alpha_1 \cdot (\vartheta_{p1} - \vartheta_1) \cdot S \qquad (2.28)$$

$$\Phi = \alpha_2 \cdot (\vartheta_2 - \vartheta_{p2}) \cdot S \qquad (2.29)$$

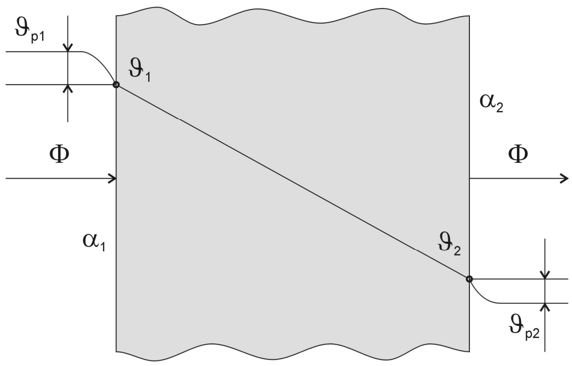



**Fig. 2.6: Convective heat transfer**

There is a difference in temperature between the ambient temperature and the surface temperature, which is a result of the fact that there is always a thin layer of gas or liquid on the wall surface; the layer does not participate in the flowing. The heat flux passes through this layer only by conduction. Since the thermal conductivity of gases and liquids is small, a thermal shock occurs here.

## 2.4.3. Radiation heat transfer

Radiation heat transfer differs from conductive heat transfer and flux by a different transfer mechanism. Energy transfer, or to be specific, heat transfer, happens by means of electromagnetic waves in the entire range of wavelengths. Electromagnetic waves are generated by each non-transparent body with a temperature exceeding 0 K and at the same tame absorbing surrounding radiation.

Radiation can be decomposed into individual components corresponding to each wavelength; thus the radiation spectrum can be obtained.

The body is also subject to heat flux from other bodies in space. Heating of the body occurs, of course, only if it receives more energy from its surroundings than it emits, and vice versa. The amount of emitted energy is proportional to the area of the active surface of the body and the fourth root of its thermodynamic temperature. It is also dependent on the characteristics of the body surface. The energy flow incident onto the body can be divided into three parts:

- *A* - absorbed flux (A is variable absorption),
- *B* - reflected flux (*B* is variable reflection),
- *C* - permeation flux (*C* is variable permeability).

The following must apply:

$$A + B + C = 1 \qquad (2.30)$$

The following extremes can be defined:

$A$ = 1 ..... absolutely black surface (all of the energy of heat flux is absorbed by the body),

$B$ = 1 ..... absolutely white surface (all of the energy is reflected by the body),

$C$ = 1 ..... transparent (diathermanous) environment - two-atomic gases and the air,

$C$ = 0 ..... thermally non-transparent environment - e.g. metals.

These coefficients can be dependent on the frequency of the electromagnetic wave; therefore it is possible to define its spectral values and they must apply to all wavelengths.

$$A_\lambda + B_\lambda + C_\lambda = 1 \tag{2.31}$$

**Laws of radiation**

Radiation heat transfer is governed by physical laws. The surface of a heated absolutely black body emits a continuous spectrum of radiation with various wavelengths (Fig. 2.7).

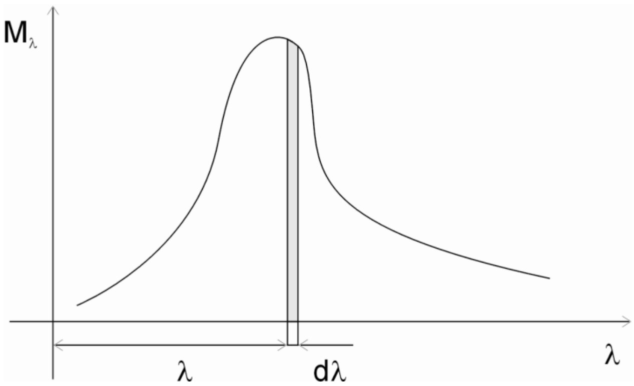



**Fig. 2.7: The course of spectral radiance dependent on the wave length**

**Snell's law**

Snell's law is used to describe the character of radiation spread upon the transition from one environment to the other (Fig. 2.8). The following relation applies to the spreading direction

$$\frac{\sin\alpha}{\sin\beta} = \frac{v}{v'} = n \tag{2.32}$$

where $n$ is the refractive index, $v$ and $v'$ are velocities of propagation in the respective environments.

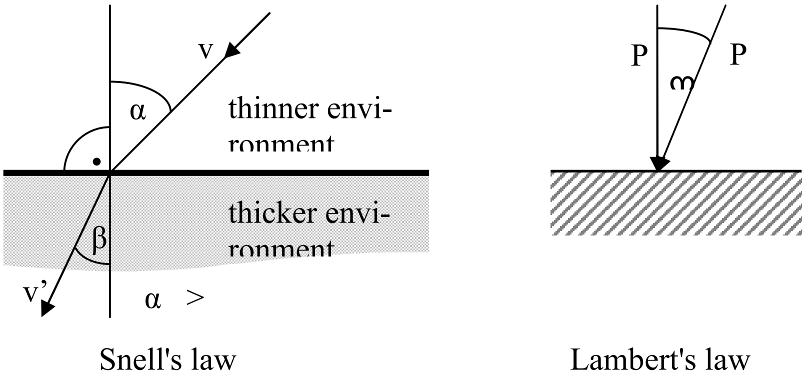



**Fig. 2.8: Spread of radiation on the transition of two environments [4]**

**Lambert's law**

Lambert's law defines that only the perpendicular component of radiation applies in terms of power.

$$P = P_{\varphi} \cdot \cos\varphi \tag{2.33}$$

where $\varphi$ is the incident angle of radiation, $P_{\varphi}$ is the energy in the direction of the $\varphi$ angle.

**Stefan-Boltzmann law**

The Stefan-Boltzmann law describes the total intensity of radiation of the absolutely black body. This law says that the radiation intensity $M$ ($W \cdot m^{-2}$) grows with the fourth power of the thermodynamic temperature of the radiating body.

$$M = \sigma \cdot T^4 \tag{2.34}$$

where σ is a Stefan-Boltzmann constant, $\sigma$ = 5,67.10$^{-8}$ $W \cdot m^{-2} \cdot K^{-4}$, $T$ is thermodynamic temperature (K).

**Planck's law**

Planck's law defines the dependence of radiation spectral intensity $M_{\lambda}$ ($W \cdot m^{-3}$) of the absolutely black body on its surface temperature

$$M_{\lambda} = f(T,\lambda) = \frac{C_1}{\lambda^5 \left( e^{\frac{C_2}{\lambda \cdot T}} - 1 \right)} \tag{2.35}$$

where $C_1 = 3.74 \cdot 10^{-16}$ W·m$^{-2}$, $C_2 = 1.44 \cdot 10^{-2}$ m·K.

The equation (2.35) indicates the radiated power from 1 m$^2$ of the area only for 1 wavelength $\lambda$. The total radiated power will be a sum for all wavelengths, i.e. for $\lambda = 0$ do $\lambda = \infty$.

$$M(T) = \int_0^\infty M(T,\lambda) \cdot \mathrm{d}\lambda = \int_0^\infty \frac{C_1}{\lambda^5 \left( e^{\frac{C_2}{\lambda \cdot T}} - 1 \right)} \cdot \mathrm{d}\lambda \qquad (2.36)$$

Through integration and substituting of the constants, the following relation will be obtained:

$$M(T) = \sigma \cdot T^4 \qquad (2.37)$$

which is a relation expressing the Stefan-Boltzmann law - see the equation.

**Wien's law**

The spectral intensity of radiation $M_\lambda$ is the most intensive at the respective temperature for the wavelength λm, which is inversely proportional to the temperature *T.* As a result, the body emits only long-wave (infrared) radiation by its surface at a low temperature. With rising temperature, the body radiance grows and the maximum of the emitted spectrum also moves towards shorter wavelengths - Wien's displacement law

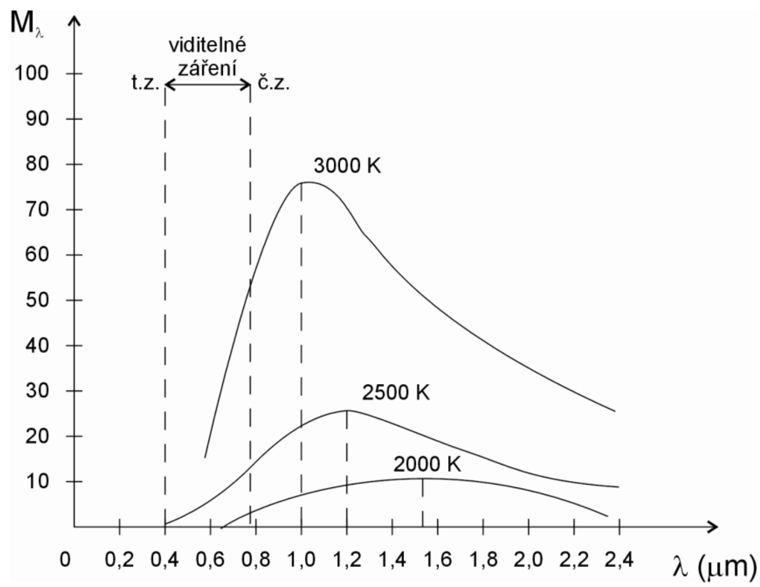



**Fig. 2.9: Wien's displacement law**

$$\lambda_{\mathrm{m}} = \frac{2892}{T} \quad (2.38)$$

The human eye is adapted for the sun radiation, whose maximum lies in the area of the yellow-green color, which corresponds to the wavelength $\lambda$ = 500 nm. By substituting this value to Wien's law, one can obtain the temperature of the Sun surface

$$T = \frac{2892}{0{,}5} = 5784\,\mathrm{K} \quad (2.39)$$

**Kirchhoff's law**

Kirchhoff's law applies to both spectral and total radiance of gray bodies in relation to bodies with absolutely black surfaces.

The ratio of the total and relative absorption of the gray body is dependent only on the absolute temperature of the body *T* and is independent of the surface color. Therefore, Kirchhoff's law of radiation can be also described as follows - for spectral radiance:

$$\frac{M_{\check{\mathrm{S}}}}{A_{\check{\mathrm{S}}}} = f(T) = \frac{M_{\check{\mathrm{C}}}}{A_{\check{\mathrm{C}}}} = M_{\check{\mathrm{C}}} \quad (2.40)$$

$$\frac{M_{\lambda\check{\mathrm{S}}}}{A_{\lambda\check{\mathrm{S}}}} = f(T,\lambda) = M_{\lambda\check{\mathrm{C}}} \quad (2.41)$$

where $M_{\check{s}}$, $M_{\check{c}}$ ($M_{\lambda\check{s}}$, $M_{\lambda\check{c}}$) are total (spectral) radiances of gray and black surfaces, and $A_{\check{s}}$, $A_{\check{c}}$ ($A_{\lambda\check{s}}$, $A_{\lambda\check{c}}$) are relative (spectral) absorptions of gray and black surfaces.

It follows from the curves in Fig. 2.9 that the absolute black surface emits a continuous spectrum of radiance which is not interrupted by gaps. For most real, i.e. physically gray surfaces, the spectral radiance curve is also uninterrupted and similar to the curve of the black body (Fig. 2.10). So, it can be said that

$$\frac{M_{\lambda\check{\mathrm{S}}}}{M_{\lambda\check{\mathrm{S}}}} = \mathrm{konst.} = \varepsilon \quad (2.42)$$

or in other words:

$$A_{Š} = \frac{M_{Š}}{M_{Č}} = \frac{\varepsilon \cdot \sigma_{č} \cdot T^4}{\sigma_{č} \cdot T^4} = \varepsilon \qquad (2.43)$$

Therefore, the relative absorption $A_Š$ is numerically equal to the degree of blackness ε of the surface subject to consideration.

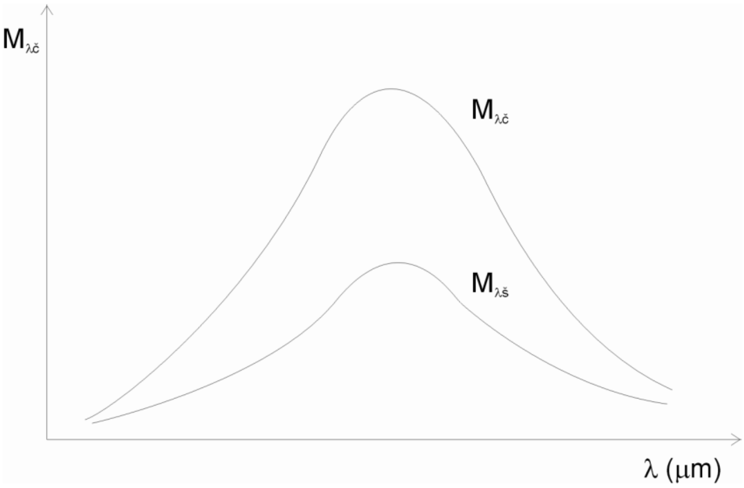



**Fig. 2.10: The spectral radiance for black and gray surfaces**

Informative values of emissivity are given in the following table.

| **Material** | **Emissivity ε (-)** |
|---|---|
| absolutely black body | 1 |
| carbon black, graphite | 0.95 |
| oxidized steel | 0.85-0.95 |
| oxidize copper | 0.7 |
| burnt brick | 0.9 |
| fireclay brick | 0.8 |
| oxidized aluminium | 0.3 |
| glossy aluminium | 0.1 |
| polished steel | 0.29 |
| polished nickel | 0.07 |
| polished silver | 0.02 |
| water, ice (smooth surface) | 0.96 |
| glass | 0.94 |

**Tab. 2.5: Emissivity values [2]**

**Mutual radiation of body surfaces**

The body with the area of *S* emits the radiant flux:

$$\Phi = M \cdot S = \sigma \cdot T^4 \cdot S \qquad (2.44)$$

We will consider two bodies with areas of S1, S2,thermodynamic temperatures of surfaces T1, T2 and emissivities $\varepsilon_1$ and $\varepsilon_2$. Then, the following applies to the radiant heat flux in a steady state

- in the case of two surfaces which form a closed envelope

$$\Phi = \frac{\sigma_{č} \cdot \left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1 \cdot S_1} + \frac{1}{F_{12} \cdot S_1} + \frac{1-\varepsilon_2}{\varepsilon_2 \cdot S_2}} \qquad (2.45)$$

where $F_{12}$ is a radiation factor (how the surfaces see each other).

- the case of two parallel areas with the same size, when $S_1 = S_2 = S$, $F_{12} = 1$

$$\Phi = \frac{S \cdot \sigma_{č} \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \qquad (2.46)$$

- the case of two bodies, when one of the bodies spatially surrounds the other, i.e. $S_1 << S_2$

$$\Phi = \frac{S_1 \cdot \sigma_{č} \cdot \left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{S_1}{S_2} \cdot \left(\frac{1}{\varepsilon_2} - 1\right)} \qquad (2.47)$$

## 2.5. General equation of energy propagation

The nature of energy propagation can be applied to the design of a mathematical model - the equation of energy propagation in the environment The default assumption consists in a theory that the environment has the character of a closed, but non-insulated TD system. This system can exchange energy only with its surroundings.

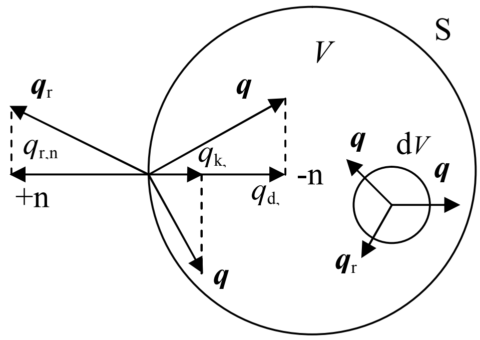



**Fig. 2.11: Energy propagation in a non-insulated TD system [1]**

The environment is in an unsteady state and it does not have an internal power source. If you apply the law of conservation of energy, it is possible to express the energetic balance of such an environment as follows: the change to the total energy of the environment with the volume V at the elementary time d*t* equals to the sum of all possible energy fluxes into and out of the environment through a boundary surface *S* at the same time d*t*.

The change to all possible components situated in the environment can be considered the change to the total energy of the environment [1]:

a change to internal energy d*U*

$$\mathrm{d}U = \int_V \mathrm{d}u \cdot \mathrm{d}V = \int_V \mathrm{d}(\rho \cdot c \cdot \vartheta) \cdot \mathrm{d}V \tag{2.48}$$

a change to kinetic energy $\mathrm{d}W_\mathrm{k}$

$$\mathrm{d}W_\mathrm{k} = \int_V \mathrm{d}w_\mathrm{k} \cdot \mathrm{d}V = \int_V \mathrm{d}(\frac{1}{2} \cdot \rho \cdot v^2) \cdot \mathrm{d}V \tag{2.49}$$

a change to potential energy $\mathrm{d}W_\mathrm{p}$

$$\mathrm{d}W_\mathrm{p} = \int_V \mathrm{d}w_\mathrm{p} \cdot \mathrm{d}V = \int_V \mathrm{d}(\sum_{i=1}^{n} \rho_i \cdot e_i) \cdot \mathrm{d}V \tag{2.50}$$

a change to radiation energy $\mathrm{d}W_\mathrm{r}$

$$\mathrm{d}W_\mathrm{r} = \int_V \mathrm{d}w_\mathrm{r} \cdot \mathrm{d}V \tag{2.51}$$

The sum of the individual energies gives us the total change to the energy of the thermodynamic system subject to the examination.

$$\mathrm{d}W = \mathrm{d}U + \mathrm{d}W_{\mathrm{k}} + \mathrm{d}W_{\mathrm{p}} + \mathrm{d}W_{\mathrm{r}} \tag{2.52}$$

The cause of the above-mentioned changes is the existence of the energy source and the subsequent interaction of the TD system with the surroundings by means of individual energy fluxes.

The internal source of energy can be defined using volumetric density $w_z$

$$W_{\mathrm{z}} = \int_V \mathrm{d}w_{\mathrm{z}} \cdot \mathrm{d}V \tag{2.53}$$

whose time change expresses the volumetric density of the internal source energy.

$$q_{\mathrm{z}} = \frac{\mathrm{d}w_{\mathrm{z}}}{\mathrm{d}t} \tag{2.54}$$

If the density vectors of individual fluxes are expressed using components normal to the surface $S$ of the system (Fig. 2.11), then the change is given by the following equation

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\int_S \left(\boldsymbol{q}_{\mathrm{d}} + \boldsymbol{q}_{\mathrm{k}} + \boldsymbol{q}_{\mathrm{r}}\right) \cdot \mathrm{d}\boldsymbol{S} \tag{2.55}$$

corresponding to the identical equation

$$\frac{\mathrm{d}W}{\mathrm{d}t} = -\int_V \mathrm{div}\left(\boldsymbol{q}_{\mathrm{d}} + \boldsymbol{q}_{\mathrm{k}} + \boldsymbol{q}_{\mathrm{r}}\right) \cdot \mathrm{d}V \tag{2.56}$$

The total energy balance of the environment under consideration - the TD system without any internal sources - is expressed by the following equation

$$\int_V \left[\mathrm{d}u + \mathrm{d}w_{\mathrm{k}} + \mathrm{d}w_{\mathrm{p}} + \mathrm{d}w_{\mathrm{r}}\right] \cdot \mathrm{d}V = \left[-\int_V \mathrm{div}\left(\mathbf{q}_{\mathrm{d}} + \mathbf{q}_{\mathrm{k}} + \mathbf{q}_{\mathrm{r}}\right) \cdot \mathrm{d}V\right] \cdot \mathrm{d}t + \left[\int_V \mathrm{div}\,\mathbf{q}_{\mathrm{z}} \cdot \mathrm{d}V\right] \cdot \mathrm{d}t \tag{2.57}$$

This equation is called the general equation of energy propagation. The changes to individual energy forms included in the volumetric unit of the system per a time unit are always a result of action of the internal source of energy (if any) and the energy transfer through a surface unit of the system expressed by vectors of possible energy fluxes.

The equation (2.57) does not have a direct practical application as there is not a general environment, in which all contained energy forms would change at the same time and simultaneously all energy fluxes would be applied. The particular solution to this equation respecting conditions of energy transfer and character of the environment can be found in [1].

## 2.6. Basic criteria for the similarity of temperature fields in the solid

**Fourier's law**

$$\mathrm{Fo} = \frac{a}{l_{\mathrm{ch}}^2} \cdot \tau = \frac{\lambda \cdot l_{\mathrm{ch}}}{\rho \cdot c \cdot l_{\mathrm{ch}}^3} \cdot \tau \qquad (2.58)$$

where $l_{ch}$ is a typical dimension of the environment.

The Fourier number Fo is the dimensionless time of a thermal action, and it expresses a ratio of the time of the ongoing thermal process to the time of molecular diffusion of heat. The relationship between the velocity of thermal field change, physical parameters and dimensions of the thermal system reflects itself in the dimensionlessly expressed time. In the other form, it expresses the ratio of the heat transferred in the system by conduction to the heat accumulated in the system.

**Biot number**

$$\mathrm{Bi} = \frac{\alpha \cdot l_{\mathrm{ch}}}{\lambda} = \frac{s / \lambda}{1 / \alpha} = \frac{R_\lambda}{R_\alpha} \qquad (2.59)$$

The Biot number Bi expresses the ratio of the heat flux transferred by the surface of charge (burden) by flowing towards the heat flux transferred inside the charge by conduction. Actually, it is a ratio of the thermal resistance of the charge (burden) $R\lambda$ to the thermal resistance of the interface - the charge surface - $R_\alpha$ surroundings.

In other words, in the first form, it expresses the ratio of the characteristic length of the body to the equivalent thickness of the environment adjacent to the surface. At Bi > 100 the thermal resistance at the heat transfer on the surface is negligible compared to the conductive heat transfer. The temperature field features significant irregularity. On the contrary, at Bi < 0.1, the temperature gradient on the body surface is high, inside the body negligible and the temperature field is regular.

**The criterion for temperature field irregularity**

$$\Psi = \frac{\vartheta_o - \vartheta_p}{\vartheta_o - \vartheta_{ob}} \quad (2.60)$$

The temperature field irregularity $\Psi$ expresses the ratio of the difference in the ambient temperature and surface temperature of the body ($\vartheta_o - \vartheta_p$) to the difference in the temperature of the environment and the medium integral volumetric temperature ($\vartheta_o - \vartheta_{ob}$). It characterizes the irregularity of the temperature field in the body.

## 2.7. The analogy between the temperature and the electric fields.

The analogy greatly simplifies calculations of heat propagation in simpler systems and in the steady state. The stationary current field and the stationary temperature field are curl-free, divergent fields; therefore, LaPlace's law applies. In Fig. 2.6, there are some examples of the analogy between the electric and the temperature fields.

| Electric field | Temperature field |
|---|---|
| **Potential**<br>The zero potential is at infinity<br>scalar quantity, unit (V) | **Thermodynamic temperature**<br>Absolute zero = -273.15 °C<br>scalar quantity, unit (K) |
| **Voltage**<br>$U = V_1 - V_2$ (V) | **Temperature difference**<br>$\Delta T = T_1 - T_2$ (K) |
| **Conductivity**<br>γ ($S \cdot m^{-1}$) | **Thermal conductivity**<br>λ ($W \cdot m^{-1} \cdot K^{-1}$) |
| **Resistivity**<br>$\rho = \frac{1}{\gamma}$ (Ω·m) | **Specific thermal resistance**<br>$\frac{1}{\lambda}$ ($m \cdot K \cdot W^{-1}$) |
| **Electrical conductivity**<br>$G = \frac{\gamma \cdot S}{l}$ (S) | **Thermal conductivity**<br>$G = \frac{\lambda \cdot S}{l}$ ($W \cdot K^{-1}$) |
| **Electric resistance**<br>$R = \frac{l}{\gamma \cdot S} = \frac{\rho \cdot l}{S}$ (Ω) | **Thermal resistance**<br>$R = \frac{l}{\lambda \cdot S}$ ($K \cdot W^{-1}$) |
| **Electric current**<br>$I = \int_S \boldsymbol{J} \cdot d\boldsymbol{S}$ (A) | **Heat flux**<br>$\Phi = \int_S \boldsymbol{q} \cdot \mathrm{d}\boldsymbol{S}$ (W) |

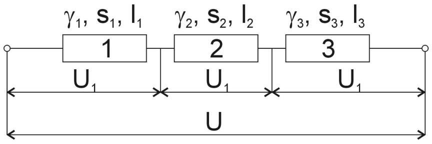
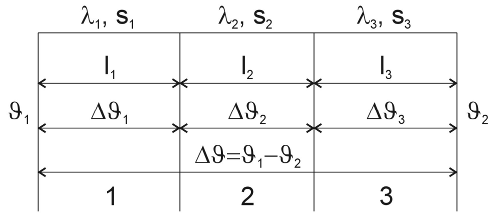
| **Resistors in series** | **Heat conduction through a composite wall** |
|---|---|
|   |   |
| $R = R_1 + R_2 + R_3$ | $R = R_1 + R_2 + R_3$ |

**Fig. 2.6: Examples of electro-thermal analogy [3]**

## 2.8. References: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# 3. Resistance electro-thermal equipment

## 3.1. Direct resistance heating

In the equipment for direct resistance heating, heat emerges by direct passage of current through electrically conductive solid charge (burden) or electrically conductive fluid - the electrolyte surrounding the charge. Theoretical basics of the direct resistance heating are simple (Joule's law) If current *I* passes through a conductor with resistance *R* for the period of time *t*, heat Q generates in the conductor.

$$Q = R \cdot I^2 \cdot t = P \cdot t \qquad (3.1)$$

The resistance of the conductor with a length of l (m) and cross section ($mm^2$) is

$$R = \frac{\rho \cdot l}{s} \qquad (3.2)$$

where $\rho$ is material resistivity. For most materials, it depends on the temperature. Upon warming by $\Delta T$, it is

$$R_\vartheta = R \cdot (1 + \alpha \cdot \Delta T) \qquad (3.3)$$

where α is a temperature coefficient of resistance, which is positive for most metals, negative for ceramic materials, and it is strongly dependent on the temperature.

Calculation and designing of these equipments are demanding. The issues related to the non-linear dependence of physical properties of the charge (burden) or electrolyte on the temperature manifest themselves here. They include mainly resistivity, specific heat capacity and also thermal conductivity. These quantities affect thermal balance of heating, which can be expressed by the relation:

$$Q = Q_u + Q_z \qquad (3.4)$$

where *Q* is the heat generated by the passage of current, $Q_u$ is useful heat necessary for heating of the charge (burden) and $Q_z$ represents heat losses. The mentioned thermal balance is fundamental for the specification of the required power:

$$P \approx \frac{dQ}{dt} \qquad (3.5)$$

It depends on the time course during charge heating. The total demand $P_c$ of electro-thermal equipment increases by heat losses caused by charging equipment, cooling of the charge, by the power required for driving mechanisms and by losses of the relevant transformer.

According to heat formation, the equipment for direct resistance heating can be divided into two basic types:

- equipment for heating of solid charge,
- equipment for heating of liquid charge.

### 3.1.1. Heating of long metal rods, wires, belts and others.

The principle of resistance heating is schematically shown in Fig. Fig. 3.1. The length of the rod subject to heating must be at least 10 times longer than its diameter, so that the heating is sufficiently even along its entire length. Heavy current is loaded to the rod 1 with a constant cross-section from the regulating transformer 2 through contacts 3. Heating is very quick and efficient.

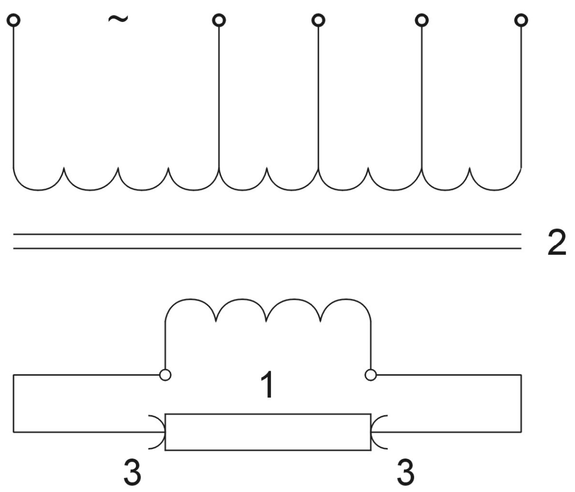


**Fig. 3.1: Heating of long metal rods, wires and belts**

Optimum conditions occur if the active resistance of the rod equals to the impedance of the entire supply power line. A cold copper rod approaches these conditions. Steel resistance, however, grows up to 7 times if heated from 20 °C to 1 200 °C. For acceptable impedance matching, the voltage in the rod is increased during heating simultaneously with the growth of resistance by switching of the branches on the input winding of the transformer. The rod ends must be, if possible, clean, contacts are usually made of copper, cooled by water and they are pressed to the rod subject to heating pneumatically or hydraulically.

Heating by direct current passage is conveniently applied at power values up to 100 kW and where exceptionally appropriate. For a single-phase equipment over

500 kW, a bazooka must be used.

The courses of power input, temperature and losses at direct resistive heating of a steel rod without voltage switching can be seen in Fig. 3.2. Power input drops, according to the rod resistance, with temperature, heat losses grow. If the power input equals to losses, the temperature has reached its limit value. The heating temperature must be lower than the limit temperature.

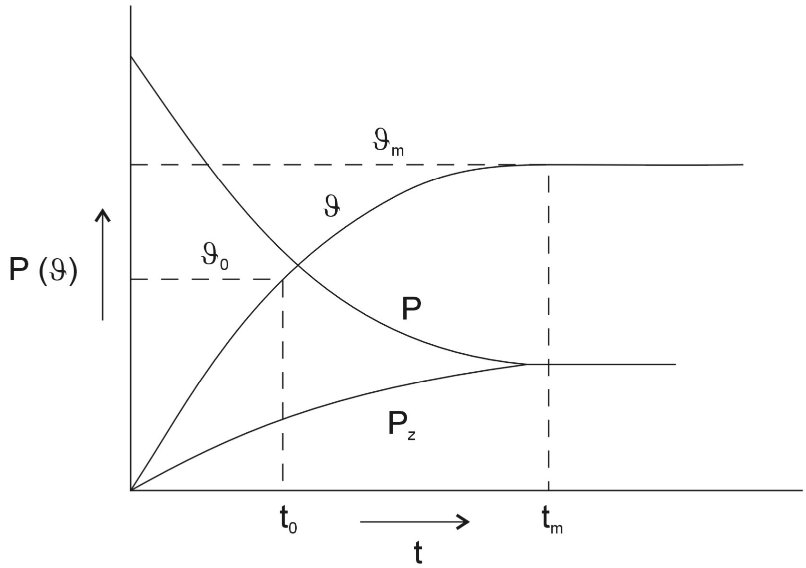


**Fig. 3.2: The courses of power input, temperature and losses**

The power factor *cosφ* is low at direct resistance heating; this is due to the fact that reactance of intakes to contacts with high currents play its role here. Switching the transformer on and off during heating causes voltage fluctuations in the network. For a single-phase equipment over 500 kW, a bazooka must be used.

At direct heating of steel ferromagnetic rods by alternating current, the surface phenomenon acts significantly. The greatest heat (86.4 %) is generated approximately in the penetration depth *a*. It can be determined from the following relation:

$$a = \sqrt{\frac{2 \cdot \rho}{\omega \cdot \mu_0 \cdot \mu_r}} \qquad (3.6)$$

where $\rho$ is the resistivity of the rod, $\omega$ is angular velocity, $\mu_o$ vacuum permeability and $\mu_r$ is relative permeability.

For common structural steel (magnetic) up to the temperature of 768 °C (Curie point - ferromagnetic loss), the penetration depth *a* at a frequency of 50 Hz is in the order of mm units. $\rho$ is approximately 7times higher above the temperature of 768

°C and the penetration depth is approx. 70 mm.

The equipment for continuous heating of wire or belt is shown in the diagram Fig. 3.3. The wire or belt 1 is connected to the current circuit of the output side of the transformer 2 by rollers 3 (or by graphite blocks).

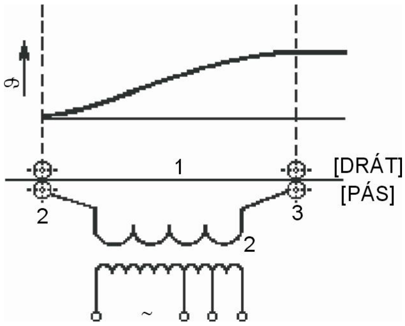


**Fig. 3.3: Equipment for continuous heating**

The final temperature of heating can be achieved by changing voltage between rollers, changing their mutual distance or changing the drawing speed of the wire. The described heating is used e.g. for soft annealing of copper wires and belts prior to sheathing, for heating of steel rods for forging, for bending, wire hardening into prestressed concrete etc.

### 3.1.2. Furnaces for production of graphite and silicon carbide

Graphite and silicon carbide (carborundum) are produced in Acheson furnaces (Fig. 3.4). Graphite is made from carbon (graphitization - a chemical process at a temperature of about 2 500 °C, during which amorphous carbon changes itself structurally into graphite with excellent physical, chemical as well as mechanical properties).

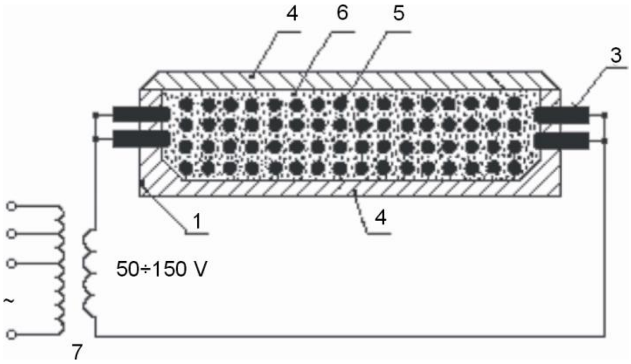


**Fig. 3.4: Acheson graphitization furnace**

In Fig. 3.4, there are 1 furnace bottom, 2 front wall, 3 graphite blocks, 4 furnace cover, 5 charge, 6 filling mixture and 7 power transformer.

Furnaces are up to 20 m long with weight of charge of 50 t and over. Power input of transformers up to 10 MVA. Supply voltage is controlled within the range of 50 V and 150 V. These furnaces feature low cos$\varphi$ (about 0.5) and they load the power network unevenly. Therefore, a bazooka is used for furnaces with high power values. If supplied by direct current, there is no need for compensation or symmetrization, and power regulation is easier.

Electric energy consumption for production of 1 kg of graphite based on the size of the furnace and the kind of products varies from 4 kWh to 6 kWh; for 1 kg silicon carbine about 8 kWh. Heating lasts for 2 to 4 days. After switch off, such a furnace cools down for 10 to 14 days. Then the charge can be taken out of the furnace.

### 3.1.3. High-temperature electrolysis

Electrolyte is heated up by direct passage of direct current at simultaneous electrolysis or refining. The most widely used high-temperature electrolysis is electrolysis production of aluminium. It is also used for production of sodium and magnesium.

Aluminum is made from bauxite ($Al_2O_3$), whose melting temperature is about 2050 °C. With the dissolution of bauxite in molten cryolite (sodium hexafluoroaluminate), aluminium can be obtained even at 950 °C, which is much more convenient technically.

Electricity consumption for production of 1 kg varies from 16 kWh to 22 kWh according to the size and technical condition of the equipment.

### 3.1.4. Electrode salt baths

Salt baths are used mainly for heating of steel components for hardening, e.g. balls or rings for ball bearings. They are also used for heat processing of non-ferrous metals or alloys at temperatures up to 1 400 °C. Two basic types are distinguished:

- Current passes not only through electrolyte, but also through the charge submerged in the electrolyte; the power input is dependent on the charge. In Fig. 3.5 one can see 1 electrodes, 2 ceramic crucible and 3 its heat insulation.
- The charge is inserted into the salt at a place where there is no electric field, and electric current does not pass through the charge; the power input does not depend on the charge.

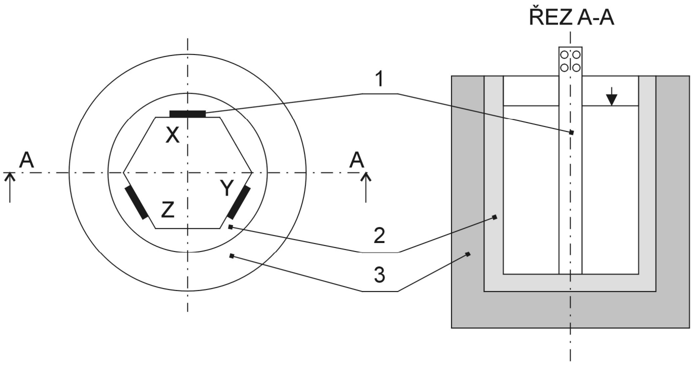


**Fig. 3.5: Salt bath - type 1**

Electrodes feature large contact surfaces in order to prevent excessive local overheating. Heat is generated by the passage of electric current through molten salt. The applied salts are non-conductive in the solid state. Therefore, an additional resistive heating element, which is able to melt the thin layer of salt and then it disconnects itself, must be used. Further heating occurs upon the passage of current through this layer. With the heating of the salt, its resistance decreases, therefore a transformer with a possibility of controlling of secondary voltage within the range of 4 to 24 V is needed for power supply. The most suitable salt mixture is chosen according to the working temperature of the salt bath.

| Composition of the mixture (%) | Working temperature (°C) |
|---|---|
| 55 KNO + 45 $NaNO_3$ | 230 ÷ 480 |
| 28 NaCl + 72 CaCl | 550 ÷ 870 |
| 50 $Na_2CO_3$ + 50 KCl | 600 ÷ 820 |
| 65 $Na_2CO_3$ + 35 NaCl | 650 ÷ 880 |
| 20 KCl + 80 $BaCl_2$ | 850 ÷ 1350 |

**Tab. 3.1 The chemical composition of some used salts and the scope of their application**

The main advantage of salt baths consists in quick, accurate and uniform heating of charge without the access of the air. The velocity of heating is determined by the great value of the convective heat transfer coefficient between the electrolyte and the solid charge. The salt baths must comply with strict safety operation rules, for example, for exhausting and cleaning of fumes released from the molten salts.

### 3.1.5. Electrode water heating

For preparation of hot water and steam production, heat generated by the direct passage of current through heated water. Current is conducted by graphite (for low power inputs) or metal (for high power inputs) electrodes. Alternating current is used to prevent formation of explosive gases and corrosion. Current density through the electrode surface should not exceed 1.5 $A \cdot cm^{-2}$; usually the value about 0.5 $A \cdot cm^{-2}$ is selected. Electrical conductivity of water depends on its composition and mainly on its temperature.

A lot of designs of electrode boilers are being used. They are designed not only for voltage of 231 V and 400 V, but also for high voltage up to 30 kV. Flow-through boilers for water heating are designed from power of several kilowatts up to several megawatts. Water is often heated by the current with a special night rate as an accumulation medium for heating of technologies in paper, textile, food and other industries. For example, electric boilers for steam production are usually used for cooking (Fig.3.6); the steam is distributed to cooking boilers and it returns back in the form of condensate. For industrial production of hot water and steam, boilers with power of up to 60 MW with an operating voltage of 30 kW and overpressure of 4 MPa are designed.

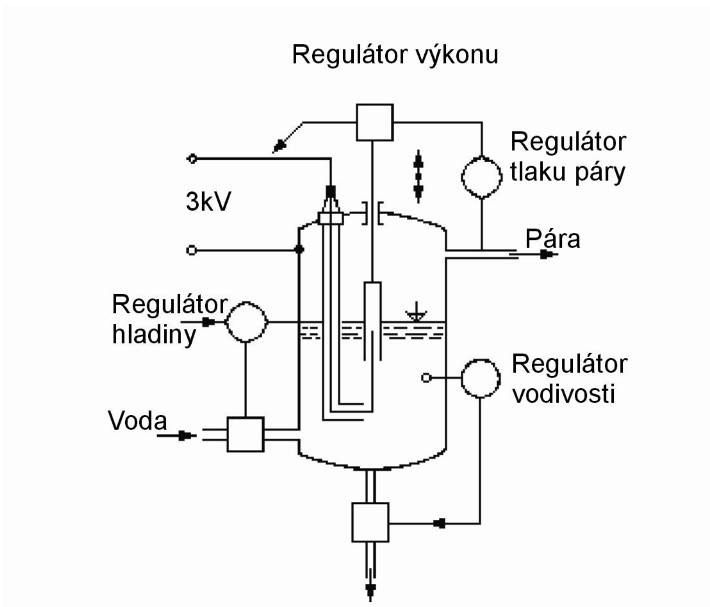


**Fig.3.6: Single-phase electrode boiler**

The power of electrode equipment at the respective voltage can be controlled as follows:

1) modification to water conductivity,
2) electrode surfaces - covering with insulation sleeves (porcelain or quartz tubes), shortening, extending,

3) approximation and retreating of electrodes,
4) dividing of electrodes into groups which switches and bridges in various ways,
5) mechanical immersing into and emerging of electrodes from water,
6) a change to water level height in the boiler using a pump with fixed electrodes suspended from above,
7) a change to the number of jets with the jet electrode high-voltage boilers.

## 3.2. Indirect resistance heating

In equipment with indirect resistance heating, heat is generated in heating elements placed directly in the area of the furnace. Heat is then transferred into the charge by radiation of heating elements and fettling (lining), by flowing of the atmosphere in the space of the furnace and/or by conduction. Electric resistive equipments with indirect heating, i.e. resistance furnaces, can be classified according to several criteria (e.g. as per ČSN IEC 60050-841).

Based on temperature, one can distinguish the following furnaces:

- low-temperature furnaces up to 600 °C,
- medium-temperature furnaces from 600 °C to 1100 °C,
- high-temperature furnaces over 1100 °C.

Based on the atmosphere in the furnace area, one can distinguish the following furnaces:

- with the normal atmosphere (air),
- with the controlled atmosphere (e.g. for carburizing, nitriding and to prevent oxidation),
- working with a vacuum - vacuum furnaces.

Based on application in process, there are furnaces:

- for heat processing of metals,
- for melting of metals,
- for glass melting,
- for cooling of glass,
- for laboratories, for households,
- with infrared heating and others.

Based on the fact whether the charge moves or does not move during heating, there are furnaces:

- with stable - non-moving charge, with intermittent operation,
- with charge passing through the furnace - continuous furnaces with mobile bottoms, with continuous operation.

The last mentioned criterion is decisive for the furnace classification; therefore, we will follow this classification hereunder.

### 3.2.1. Resistance furnaces with stable charge

The most common furnaces, in which the charge does not move during heating, are:
chamber, car-hearth, shaft, hood (bell), elevator and crucible melting furnaces.

**Chamber furnaces**

Chamber furnaces are among the oldest types of electric resistance furnaces. They are very universal, therefore widely used. Heating elements are usually placed on the sides, sometimes on the floor, in the rear wall and in the door, or also on the ceiling. Chamber furnaces are designed for temperatures of up to 1100 °C with metal heating elements, up to 1400 °C with silicon-carbide (SiC) elements or from other materials (cements).

**Car-hearth furnaces**

Car-hearth furnaces are larger chamber furnaces which feature stable sides with a ceiling and rear front. The bottom with the front face is formed by a car which goes out of the furnace. The charge is loaded onto the car outside the furnace and enters the heated furnace. Car wheels are controlled precisely. The car-hearth furnaces are economical only for large-sized charges weighing 100 tons and over and with a power of 3 000 to 5 000 kW.

**Shaft (pit) furnaces**

Shaft furnaces feature a vertical axis and round or square cross-section. The furnaces are sometimes 10 to 20 m deep and they are called “pit furnaces”. Usually they are recessed under the floor. In order to achieve faster heating and uniform distribution of temperature in the furnace, fans for atmosphere circulation in the furnace are sometimes installed into the bottom or the cover.

Shaft furnaces are not as universal as chamber or car-hearth ones, but they can be easily sealed and insulated against heat losses.

**Hood (bell) furnaces**

A hood furnace (Fig. 3.7) features a well-heat-insulated heating cover 1 (bell) with a round or square cross-section and it has heating winding 6 on its inner surface. The cover is fitted with a crane onto the working platform 4, on which charge has been put. The charge is covered by a heat-resistant muffle 2 (hood) against direct radiation of heating elements, thus also against local overheating. Under the charge, there is a fan 5. The furnace features several working platforms and muffles. Hood furnaces are designed for power of several hundreds of kilowatts.

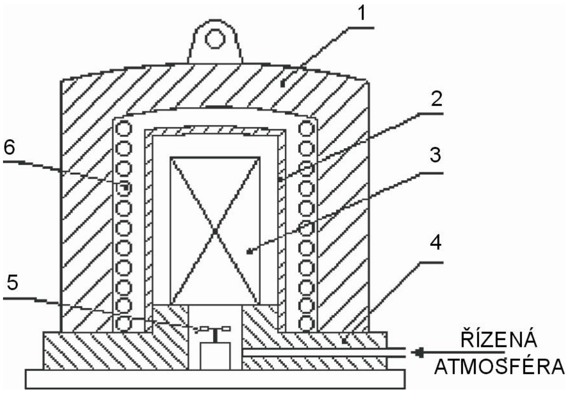


**Fig. 3.7 Hood furnace**

**Elevator furnaces**

Elevator furnaces are among the largest resistance furnaces with stable charge. The furnaces work with good efficiency and they are designed for temperatures of up to 1000 to 1200 °C with a power of 500 kW up to 2 000 kW. Elevator furnaces are suitable for large-sized charges with a high weight of tens of tons. Their great advantage consists in the fact that they can be integrated into a continuous production line as the car with the charge goes on in the direction of arrival at the furnace after heating.

**Melting crucible furnaces and melting tanks**

They are designed for melting of metals or alloys with a lower melting point (Sn, Pb, Al, Zn and others). There is heating winding around the metal or ceramic crucible. Inside the heating, there are heat insulation and furnace framework. These furnaces are usually tilting, so that the melted metal can be poured out.

Melting tanks feature various designs, for instance, they have heat insulated heating winding on the outer side of the tank, they are not tilting (for galvanizing, tinning, alitizing etc.). For aluminium remelting (equalizing), they have heating in the ceiling and during casting they tilt on rotating rollers.

## 3.2.2. Resistance continuous furnaces

They are used where heat processing for multiple products is required. Various types of continuous furnaces are designed; they are usually rated for lower temperatures. The prescribed heating, dwell and cooling can be performed according to a technological process in these furnaces. Generally, these furnaces feature multiple temperature zones, which are supplied and controlled separately. In the case of slow cooling of charged parts, a cooling chamber is connected to the furnace; the chamber is equipped either by heat insulation, or water cooling, according to the required velocity of cooling.

Several continuous furnaces can form one entirely mechanized and automated unit. Mostly hardening and tempering (drawing) furnaces are connected with hardening tanks as well as cleaning and drying devices.

The furnaces are designed for continuous operation. According to the kind of mechanism for charge transportation, the following furnaces are used most often: conveyor and chain, roller-hearth, pusher-type, jolt ramming, walking beam, drawing, drum and rotary-hearth furnaces.

**Conveyer furnaces**

Charge to a conveyor furnace (Fig. 3.8) is introduced manually or using special automation through a feed table 5 onto a conveyor belt 3 passing through the furnace. Belts for light components are formed by metal mesh, belts for heavy components are formed by embossed boards which are interconnected using pins and connectors. It is tensioned by means of a device 7, 4 is the belt drive. At the end of the furnace, material is removed from the area 6.

Heating elements 2 NiCr are usually placed on the ceiling and the bottom, under the upper level of the belt. The working area of the furnace is lined with heat-proof fettling (lining) 1. Conveyer furnaces are intended for heat processing of smaller components up to a temperature of 900 °C.

**Roller-hearth furnaces**

They are designed for temperatures not exceeding 900 °C. A roller track passes through the entire furnace and it is composed of heat-proof rollers whose axis is perpendicular to the direction of movement. Its bearings are inside the furnace on both sides. Charge is put directly on the rollers or pads, so that it is carried away well.

Designing must respect thermal expansion of the rollers and the fettling (lining) of the furnace.

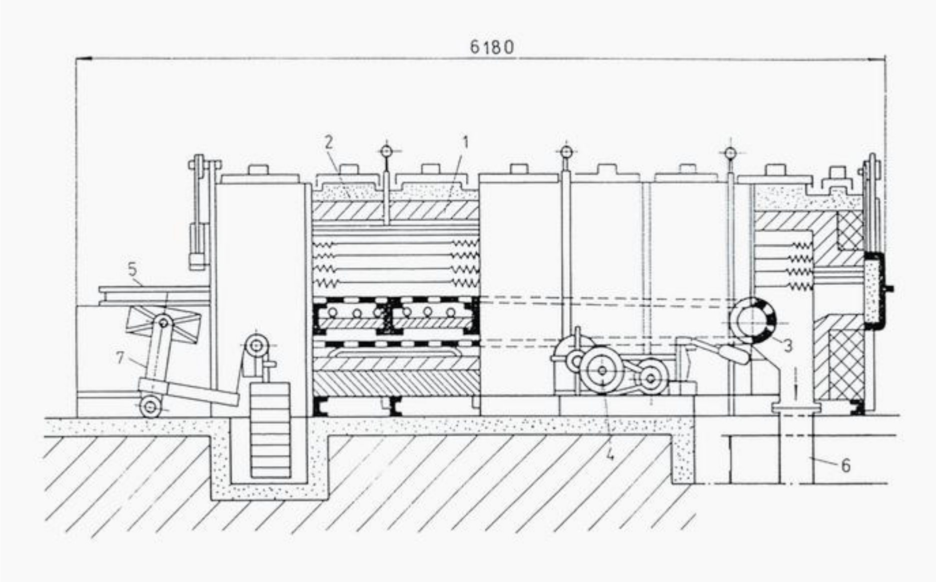


**Fig. 3.8: Conveyer furnaces**

**Pusher-type (push) furnaces**

Working temperature of pusher-type furnaces does not exceed 1000 °C.

**Jolt ramming furnaces**

Jolt ramming furnaces are designed for heating of small unit charge up to the temperature not exceeding 900 °C.

**Walking beam furnaces**

Walking beam furnaces are designed for heating of large forged pieces and castings to medium and high temperatures. The walking beam mechanism is outside the working area of the furnace. While moving forward, the charge lifts and moves. While moving backwards, the mechanisms drops and prepares for another step ahead. It is driven hydraulically or by an electric motor.

**Drawing furnaces**

Drawing furnaces are designed for heating of wires and belts, mainly of non-ferrous metals (as well as steel), which are drawn through the furnace. Uniform heating is thus achieved. For great power, furnaces with vertical movement of wires are in several loops (production of enameled wires).

**Drum furnaces**

The drum of a drum furnace is made from heat-resistant material. It is stressed thermally as well as mechanically. Its service life is limited. Accurate and uniform temperatures are achieved in the furnace.

These furnaces are suitable for heat processing (hardening, annealing, etc.) of washers, bolts, smaller bearing rings and balls and also for mica cleavage.

**Rotary-hearth furnaces**

Rotary-hearth furnaces are designed for the highest temperatures because furnace mechanisms are entirely outside the area with the working temperature. The furnace cross-section can be seen in Fig. 3.9. The actual furnace body 1 is rotary with heating elements 2. The furnace floor 3 is rotary and charge is put onto it. Rotation is ensured by an electric motor 5. The charge is inserted into the furnace through the opening with a door 6 and after one revolution, it is removed from the discharge opening with a door next to the entrance. The door are closed by a device 7. Charge heating time differs on the basis of change to floor revolving velocity.

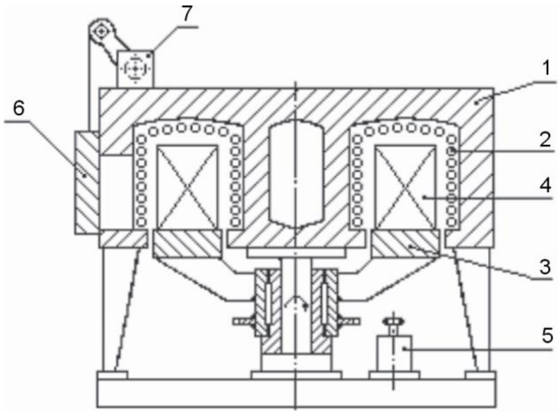


**Fig. 3.9: Rotary-hearth furnace**

### 3.2.3. Materials and components of electric resistance furnaces with indirect heating

The typical resistance furnace with indirect heating is composed of the following fundamental parts:

1) refractory fettling (lining),
2) heat insulation,
3) furnace enclosure,
4) heating elements,
5) feeding mechanisms and their drives.

In addition, the furnaces can be equipped with devices for production of the controlled (protective) atmosphere or vacuum. All furnaces are equipped with apparatuses for measuring and controlling of temperature.

Refractory fettling (lining) surrounds the working area inside the furnace. At the working temperature, it has to withstand heat sufficiently, it has to be strong enough and chemically stable. In resistance furnaces, fireclay (chamotte) components made from 38% to 44% aluminium oxide $Al_2O_3$, the rest is silicon dioxide $SiO_2$.

Heat-insulating materials feature natural or artificial porosity (magnesite, slag, aluminium oxide, glass wool).

Enclosure and structure are usually made from sheet steel and steel profiles. Some components are made from alloy and cast steel. These parts work at normal temperature and they do not have any special requirements.

Materials for heating elements are subject to high demands. They should have the following properties:

- resistance to heat at the working temperature of the element,
- great mechanical heat strength,
- resistance to chemical effects of atmospheres in furnaces and ceramics they are in contact with,
- great resistance for the possibility of application of larger cross-sections and appropriate length of conductors and to allow direct connection to the network,
- small temperature coefficient of resistance providing small difference between cold and hot resistance of the element,
- stable resistivity throughout the service life of the element,
- small thermal expansion,
- good workability to various shapes.

The above-mentioned requirements are highly demanding. In practice, only some of the requirements can be fulfilled at the same time; sometimes a compromise solution is selected for achieving of maximum service life.

Materials for heating elements can be classified into two fundamental groups:

1) metal materials,
2) non-metal materials.

### 3.2.4. Materials for metal heating elements

Metal materials include alloys of Ni, Cr, Fe, Al, non-magnetic and magnetic pure metals, steel and special alloys.

**Austenitic alloys**

Austenitic alloys are non-magnetic, i.e. chromium-nickel. The alloy of Ni+Cr and the alloy of Ni+Cr+Fe are used for heating elements. These alloys feature top quality, they have lower refractoriness and they resist to frequent switching on and off.

They are easily welded and formed. They feature high resistivity and a low temperature coefficient of resistance. They do not age and they are stable.

**Ferritic alloys**

Ferritic alloys are magnetic alloys of Cr+Al+Fe without nickel, highly heat-proof and with higher resistivity than the previous group. They include alloys with trade names: Kanthal, Alsichrom, Alkrothal, Chromal, Aluchrom, Thermal and others. Resistivity values of these wires are about 1.4 μΩ·m at 20 °C and they change with temperature very little. These materials are applicable for working temperatures up to 1375 °C.

**Pure metals**

Pure metals are precious, hardly meltable, e.g. platinum, tungsten and molybdenum. They are used for heating elements of laboratory or other special furnaces, where very high temperature is required.

Platinum does not oxidise, but it is carburized intensively. It cannot be used in the reducing atmosphere. Since its resistance changes significantly with temperature, its cold start causes a big current surge.

Tungsten is very brittle. Heating elements are usually designed as tubes whose internal area is directly the working area. Current intakes are cooled down with water. Tungsten heating elements work in a vacuum or in controlled (protective) atmosphere up to 2600 °C.

Molybdenum is used for temperatures from 1400 to 2000 °C. It requires the controlled (protective) atmosphere (e.g. alcohol vapors or hydrogen). It is sprayed in a vacuum at a temperature of 1650 °C.

**Steel and special resistance alloys.**

Steel wire can be used at temperature of up to 900 °C, but only in the hydrogen atmosphere. In the normal atmosphere, it can be used only at the temperature not exceeding 400 °C. It is cheap, and used in drying kilns.

Constantan (56% Cu + 44% Ni) and Nikelin (65% Cu + 34% Ni + 1% Fe) are special alloys whose resistance does not change with temperature. They are mainly used in measuring and controlling technologies. However, they can be used as heating elements for small appliances and for low temperatures.

### 3.2.5. Materials for non-metal heating elements.

The working temperatures of metal heating elements do not reach more than 1 375 °C; therefore, the materials which can work at higher temperatures in the normal atmosphere while preserving the same basic properties as metal were searched.

**Silicon carbide (SiC)**

Silicon carbide is the most commonly used non-metal material for heating elements with trade names Silit, Globar, Crusilir, Cesiwid and others. The resistivity is significantly higher than the resistivity of metal materials (0.6-3.0 mΩ·m), which allows making heating elements in the form of rods with reinforced ends. Rod diameters are from 1.2 to 5 cm, their length from 8 to 200 cm.

The SiC elements can be used up to the temperature of 1500 °C. The temperature coefficient of resistance is negative up to approx. 800 °C, and positive over 800 °C. The lifetime of heating elements ranges from 3000 to 10000 working hours.

**Cermet elements**

Cermet elements are produced by powder metallurgy. The basic material is a mixture of molybdenum disilicide ($MoSiO_2$) with silicon dioxide ($SiO_2$). The most common shape of the elements is a U shape (hairpins). They can also be designed as rods or tubes.

The working temperature of the elements ranges between 1600 to 1700 °C. Refractoriness is the result of a protective layer of $SiO_2$ occurring on the surface of the element in operation. Cermet elements are brittle, they do not tolerate shocks. They are resistant to oxidizing, nitrogen, argon and carbon monoxide atmospheres. Sulfur and chlorine do harm to them. Resistivity varies according to the temperature (at 20 °C $\rho$=0,25 μΩ·m, at 1600 °C $\rho$ = 3,5 μΩ·m), therefore they are connected through regulating transformers.

**Carbon and graphite heating elements**

The basic raw materials and graphite production were described in the article 3.1.2. Heating elements are made in the form of rods, tubes, etc.

Working temperatures reach up to 2000 °C in the vacuum or controlled atmosphere preventing oxidation. In the normal atmosphere, oxidation of carbon elements occurs from the temperature of about 400 °C; with graphite elements it is from about 600 °C. Carbon resistivity decreases with increasing temperature; e.g. at 1400 °C to about 67 % of the full value at 0 °C. With graphite, resistivity decreases firstly with increasing temperature, at the temperature of about 400 °C it is about 77 % and then it grows again. At 1400 °C, resistivity is about 96 % of the original value.

### 3.2.6. The basic application of resistance furnaces in the industry and mechanical engineering

The basic area of the application of electric resistance furnaces in the industry is heat processing. These are processes, in which metal objects in the solid state are subject to certain temperature changes to achieve the required properties of the material. In particular, the processes include increasing of strength and stress limits while maintaining the shape of the item subject to heat treatment. If this process is influenced by chemical effects of the environment, it is chemical-thermal processing.

The following processes of ČSN EN 10052 are used in resistance furnaces for heat processing, mainly of engineering components made of steel or non-ferrous metals and their alloys:

1) **Annealing** reduces hardness, improves machinability, reduces internal stress and encourages the required micro-structure. Steel components are heated up to the temperature of 700 °C (up to 800 °C with brass, 960 °C to 1200 °C with nickel), they dwell for 2 hours at this temperature and then they cool down slowly. The types of annealing include, for example: bright annealing, normalizing, isothermal, recrystallization annealing, etc.
2) **Hardening** increases the hardness of steel components, which are heated up to over recrystallization temperature and then they are quickly cooled down by immersion into water or oil. The types of hardening include: marquenching, austempering, broken, surface hardening etc.
3) **Tempering** usually follows hardening. Hardened steel objects are very hard, but also very brittle, therefore they are further heat processed by tempering. They are heated up to the temperature ranging from 150 °C to 600 °C. After a delay at tempering temperature, they cool down slowly.
4) **Case hardening** causes great hardness of the surface layer of steel components, while maintaining toughness of the core. Case hardening is surface saturation of a steel object with carbon in the solid, liquid and mainly gaseous environments at about 900 °C.
5) **Nitriding** increases resistance of the surface layer of steel components to abrasive wear. Nitriding is surface saturation of steel components with nitrogen in the gaseous or liquid environments at 470 °C to 580 °C.
6) **Hardening treatment** of steel products. Hardening treatment encourages great strength, hardness and toughness Prior to the last mechanical machining, annealing is carried out; it is followed by oil or water hardening and then by tempering with a delay and subsequent controlled cooling.

Electric resistance furnaces are used for heat processing of glass products. Automatic lines of heat processing of semi-finished products and products are expanding to various areas of production, including the textile and food industries.

### 3.2.7. The actual calculation of the heat element

For designing of heating elements with round or rectangular cross-sections of a resistance conductor, the following relations are used:

**an element with a round cross-section**

The diameter of the resistance wire can be calculated as follows:

$$d = \sqrt[3]{\frac{4 \cdot \rho \cdot P^2}{\pi^2 \cdot U^2 \cdot p_{\mathrm{skut}}}} \qquad (3.7)$$

**an element with a rectangular cross-section**

$$b = \sqrt[3]{\frac{P^2 \cdot \rho}{2 \cdot \beta \cdot (\beta + 1) \cdot U^2 \cdot p_{\text{skut}}}} \tag{3.8}$$

$$a = b \cdot \beta \tag{3.9}$$

The required length can be calculated as follows:

$$l = \frac{U^2 \cdot S_{\text{p}}}{P \cdot \rho} \tag{3.10}$$

or

$$l = \frac{P}{o \cdot p_{\text{skut}}} \tag{3.11}$$

where

- $P$ (W)..... power input to one heating element
- $U$ (V)...... voltage on the heating element (it can be star (phase) voltage or delta voltage.
- $R$ (Ω) ..... resistance of the element under consideration at the working temperature
- $\rho$ (Ω·m).. resistivity of the heating material at the working temperature
- $l$ (m)....... length of the conductor of one heating element
- $d$ (m)...... diameter of the conductor with a round cross-section
- $o$ (m)...... circumference of the conductor
- $S_P$ ($m^2$) .. the cross-section of the conductor of one heating element
- $p_{skut}$ ($W.m^{-2}$)the actual surface load of the heating element
- $\beta$............ the length-width ratio of the rectangle

### 3.2.8. Connection and controlling of electric resistance furnaces

The most common way of electrical connection of a resistance furnace is shown in Fig. 3.10.

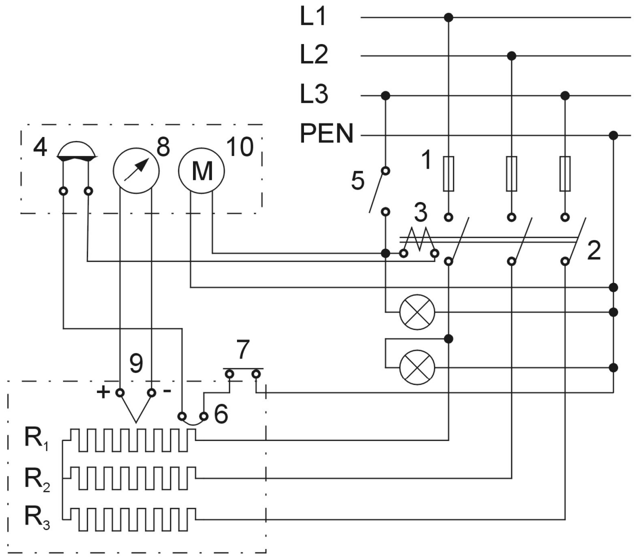


**Fig. 3.10: Connection of the resistance furnace**

The electric resistance furnace connected according to Fig. 3.10 is linked to a three-phase network with a neutral conductor by means of a contactor 2. Protection is ensured by fuses 1. The heating winding of furnaces present resistances with the same value $R_1$, $R_2$, $R_3$. The furnace shall be started with the switch 5. Upon enabling of this switch, we close the auxiliary switching circuit, which is composed of the following elements: contactor coil 3, mercury switch of the automatic temperature controller 4, fuse heating wire in the furnace 6, door contact 7. The operation of the furnace is indicated by the illuminated red light. At the same time, the motor of the automatic temperature controller 10 starts. After energizing of the switching coil 3, the contactor 2 switches and it connects the heating elements to the network (mains). The thermocouple 9 supplies voltage for the apparatus showing the temperature in the furnace. Upon reaching the required temperature, the mercury switch of the automatic temperature controller 4 flips, current in the control circuit is interrupted and the contactor 2 releases the heating elements from the network. In the diagram Fig. 3.10., furnace temperature controlling is possible only by switching off and on of the entire power input.

### 3.2.9. Automatic temperature control in resistance furnaces

The task of the electric furnace temperature control consists in maintaining the required temperature of the working area continuously and precisely, or to respond to changes according to the pre-set program. In the industry, step or continuous controlling is used.

The easiest way to make use of the step controlling is to switch off and on the whole power input of the furnace. Furthermore, we can make use of the switching of the resistance sections of star-delta, or the switching of the groups of resistance elements. The course of temperature and power input of a resistance furnace with single-pole or dipole step controlling is shown in Fig. 3.11.

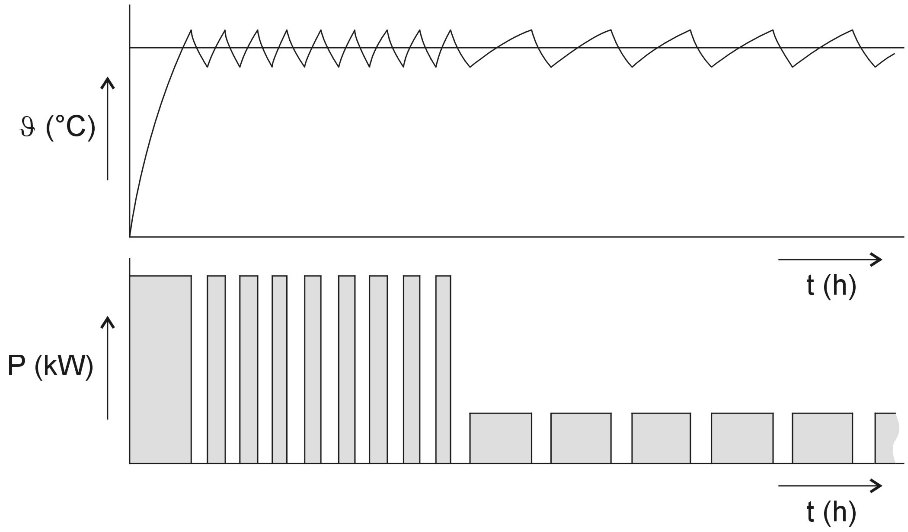


**Fig. 3.11: The course of temperature and power input of a resistance furnace with single-pole or dipole step controlling**

In the first part Fig. 3.11, the single-pole controlling on, off is shown. In the other part of the figure, the dipole control of temperature and power input while using the switching of resistive sections star-delta is shown.

Semiconductor elements connected to the supply to the furnace are used for continuous regulation of power input of the electric resistance furnace (Fig. 3.12). Controlling with a final control element with controllable output alternating voltage and constant frequency is applied here. The basic element is anti-parallel connection of thyristors. The output voltage of thyristor converters can be controlled by an ignition angle of thyristors. To do this, it is necessary to supply the control electrode with controlling signals of suitable properties at the appropriate moment. This function is provided by circuits, which can be classified based on their purposes as switching and control circuits. The switching circuits provide only switching of thyristors always at the beginning of a positive voltage half-wave. In addition, the control circuits provide continuous control of the moment of thyristor switching during the positive voltage half-wave.

The summary of requirements for controlling can be summed up as follows:

1) The control signal must not overload the thyristor control electrode in order to prevent loss of blocking and closing capabilities of the thyristor.

2) The control signal must feature the necessary width. Its value is influenced mainly by the type of load. Shorter impulses must feature a higher amplitude.

The control signal must feature necessary increase slope as the low slope decreases also the slope of current increase.

Using this continuous controlling, significant unification of temperature courses of the resistance furnace occurs as shown in Fig. 3.13.

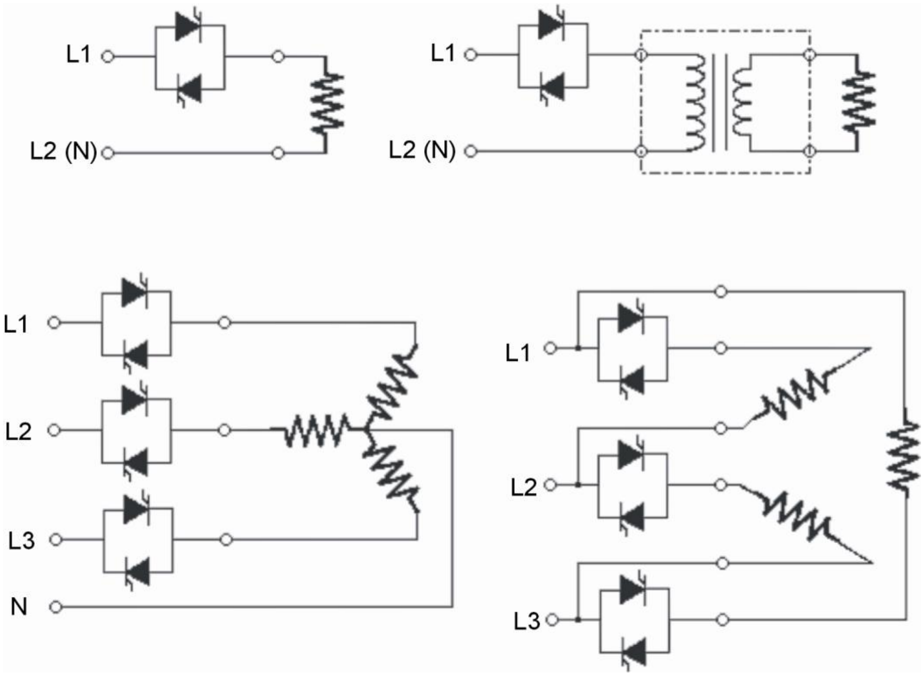


**Fig. 3.12: single-phase and three-phase connections of a resistance furnace.**

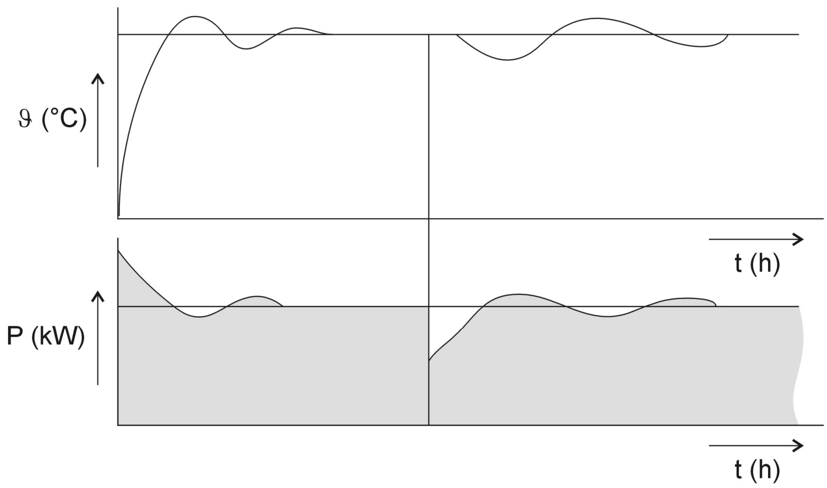


**Fig. 3.13: Continuous control of a resistance furnace**

Maximum uniformity in most of resistance furnaces ±5 °C is reached in the internal space. Some processes requiring higher accuracy and uniformity of heating are equipped with heating strips which are controlled separately.

Basically, the following measures are taken to obtain the higher uniformity of temperature with furnaces:

- improvement in heat insulation of the entire furnace against heat losses, especially sealing of doors and other measures,
- more efficient deployment of heating resistance elements in furnaces,
- better and more sensitive controlling of heating element groups,
- circulation of the internal atmosphere in shaft and tunnel furnaces,
- placement of circulation liners or adjustable flaps for controlling of the atmosphere flow in the resistance furnace,
- in resistance furnaces with medium temperatures, the calorifier heating (circulation of the atmosphere through heating elements) is performed.

## 3.3. The calculations of resistance electro-thermal equipment for indirect resistance heating

### 3.3.1. The basics of the design and calculation of resistance furnaces

The following aspects are decisive for the correct determination of the furnace type for the required way of heat processing of charge,

- technological requirements for heat processing of the charge,
- the charge type and the charge size,
- the weight of the charge for processing per a time unit,
- the course of the thermal mode, maximum temperature (heating speed, final temperature, dwell time at the specific temperature, cooling speed, etc.),
- uniformity and accuracy of temperature maintenance,
- natural or controlled atmosphere in the furnace,
- intermittent or continuous operation,
- available space,
- furnace price.

For unit or small batch production, a furnace or a group of furnaces with stable charge should be selected; for mass production, continuous furnaces or fully automatic continuous furnace lines are more suitable.

The furnace design requires lots of technical compromises, experience and economic considerations. Usually, preliminary basic parameters of the furnace (its size, mechanization, power input and others) shall be determined for the specific purpose according to experience. Then the design of the furnace is drafted, including the selection of heat insulations, their thickness, etc. This first approximate draft is checked against more detailed calculations. The appropriate corrections to it shall be performed. Then, another draft, which is more accurate, is drawn. This second draft is checked more precisely as to the calculations. The dimensions of the furnace as to the respective process technology, for which the furnace is intended, heat insulation and heat losses, power input, efficiency, temperature courses, capacity and other parameters are subject to check.

### 3.3.2. The calculation of the total power input of the furnace

Power losses $P_z$ of the furnace in a steady state shall be calculated for the basic design draft of the furnace at the known temperature or distribution of temperature in the furnace.

Power losses are determined by losses:

1) through each wall of the furnace,
2) through leaks (e.g. through doors and cars at furnaces with stationary charge),
3) at the input and output of continuous furnaces,
4) during opening and closing of doors,
5) taking heat out of continuous furnaces by transportation mechanisms (belts, chains, etc.),
6) for the heating of muffles, pallets, work rests, etc.

Power losses can be divided into idle power losses $P_{z0}$ (it does not depend on the operation of the charged furnace) and losses $P_{zv}$ associated with the operation of the charged furnace.

The following applies

$$P_{z} = P_{z0} + P_{zv} \qquad (3.12)$$

$P_{z0}$ is related to the above-mentioned items 1, 2 and 3. $P_{zv}$ with the items 4, 5 and 6. The calculation of $P_{z0}$ and $P_{zv}$ is carried out according to general principles of heat propagation in the steady state and according to the specific heat capacity, temperature and weight for losses through pellets, muffles, work rests, etc.

**Useful power**

Energy is needed to heat charge with a weight of *m*, specific capacity of *c* from the temperature of $\vartheta_0$ to the temperature of $\vartheta_k$:

$$W_u = \int_{\vartheta_0}^{\vartheta_k} c \cdot m \cdot d\vartheta \qquad (3.13)$$

Upon introduction of mean specific heat capacity $c_{av}$, the relation (3.13) is simplified as follows:

$$W_u = c_{av} \cdot m \cdot (\vartheta_k - \vartheta_0) \qquad (3.14)$$

If the heating time $t_{ohř}$ is mentioned in the problem, the result will be the useful input

$$P_u = \frac{W_u}{t_{ohř}} \qquad (3.15)$$

and the theoretical required power input of the furnace is specified as follows:

$$P_p = P_z + P_u \qquad (3.16)$$

Since it is necessary to take account of some inaccuracy of the calculation, reserve for voltage drop in the network, aging of heating elements, increase in furnace losses, etc., the specific safety factor $k_b$ = 1.2 to 1.7 must be applied. Then the furnace input is

$$P_p = k_b \cdot (P_z + P_u) \qquad (3.17)$$

In common practice, we do not know the charge heating time $t_{ohř}$; therefore, it must be calculated.

### 3.3.3. The calculation of the charge heating time

Heat is transferred to the charge mainly by radiation and convection. Transfer by means of heat conduction is usually neglected, which represents a specific reserve. The following relation based on Newton's law applies to convective heat transfer

$$P_u = \alpha \cdot S \cdot (T_p - T_v) \qquad (3.18)$$

$$\alpha = \alpha_s + \alpha_k \qquad (3.19)$$

where:

$P_u$ ......... is useful power transferred to the charge through the surface $S$, *α* ............ is the resulting coefficient of heat transfer from the furnace to the charge,

$\alpha_s$ .......... radiation heat transfer coefficient,

$\alpha_k$........... convective heat transfer coefficient (by convection),

$T_p$........... furnace temperature,

$T_v$........... charge temperature.

The following relation was derived for the equivalent radiation heat transfer coefficient within a certain smaller range of temperatures $T_p$ a $T_v$

$$\alpha_s = \frac{\sigma_č \cdot \varepsilon \cdot \left(T_p^4 - T_v^4\right)}{T_p - T_v} \qquad (3.20)$$

where *ε* is the emissivity of the charge surface and $\delta_č$ is the Stefan-Boltzmann constant (($\delta_č$ = 5.674·10$^{-8}$ W·m$^{-2}$·K$^{-4}$).

The inner surface of the furnace is many times larger than the surface of the charge, the blackness degree of the furnace inner surface is *ε* = 0.8 to 0.9.

The mean value of $\alpha_k$ coefficient for furnaces without forced circulation of the atmosphere is about 15 W·m$^{-2}$·K$^{-1}$. Then it applies that

$$\alpha = \alpha_s + 15 \qquad (3.21)$$

The charge accepts heat through the surface *S*. This surface forms only a part of the total charge surface $S_v$ according to its placement in the furnace and according to the mutual “shielding” of the charge parts.

$$S = k_v + S_v \qquad (3.22)$$

where $k_v$ < 1 is a reduction factor. It is usually determined according to practical experience.

For the calculation, we also assume that the charge is thermally thin. For such charge, the temperature of the charge interior follows the temperature of its surface without almost any time delay. At any moment, the charge can be considered as uniformly heated. For thermally-thin charge, it applies that Bi < 0.25.

$$\mathrm{Bi} = \frac{\alpha \cdot s}{2 \cdot \lambda} \qquad (3.23)$$

where Bi is the Biot similarity criterion, s is the representative thickness of a charge wall which determines its “solidity”. If Bi > 0.5, the charge is thermally-solid. The inside of the charge is cooler than its surface at the specific time of heating.

For continuous furnaces, while heating smaller components for mass production (e.g. forgings and castings for passenger cars), the Bi < 0.25 condition is usually met. In furnaces with continuous charge, the thermally-solid charge (Bi > 0.5) is usually heated. The heating time is calculated like heating of a thick board, cylinder etc.

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# 4. Electro-Thermal Induction Equipment

## 4.1. The principles of heat generation in induction equipment

Each induction equipment consists of a coil, through which alternating current passes, a source and charge, which receives electromagnetic waves emitted by the coil. Basically, it is an air transformer, where the coil is the primary side, whereas the charge is the secondary side, short connected. By passing of current through a flat emitter, flat electromagnetic waves occur in its surroundings. By passing of current through a cylindrical emitter, cylindrical electromagnetic waves occur in the surroundings of the emitter. The cylindrical coil, through which alternating current passes, emits cylindrical waves into its cavity [4], [8].

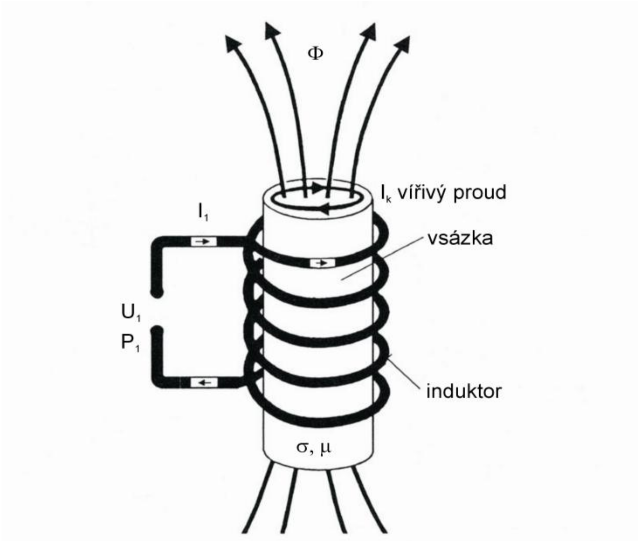


**Fig. 4.1: The principles of induction equipment**

If we put a cylindrical electrical conductive charge in a coil in alignment, then the incident electromagnetic waves enter the charge through the surface and evoke induce currents; their effect heats up the charge. The penetrating electromagnetic waves are suppressed and their energy changes into thermal energy. The radiation penetration depth is dependent on the frequency.

$$\delta = \sqrt{\frac{2}{\omega \cdot \mu \cdot \sigma}} = \sqrt{\frac{2}{2\pi \cdot f \cdot \mu \cdot \sigma}}_r \qquad (4.1)$$

where $\delta$ is the penetration depth (m), $f$ is the frequency (Hz), $\mu$ is permeability ($H \cdot m^{-1}$), $\sigma$ is conductivity ($S \cdot m^{-1}$).

The dependence of the penetration depth of electromagnetic waves on the frequency is shown in Fig. 4.2.

| | Penetration depth (mm) | | | | | |
|---|---|---|---|---|---|---|
| | Cu | | Al | | Steel | |
| Frequency (Hz) | 20 °C | 1100 °C | 20 °C | 660 °C | 20 °C | 800 °C |
| 50 | 9.5 | 31.8 | 12.2 | 31.5 | 8 | 71,2 |
| 1000 | 2.1 | 7.1 | 2.7 | 7 | 1.8 | 15.9 |
| 10000 | 0.67 | 2.25 | 0.86 | 2.2 | 0.56 | 5 |
| 1000000 | 0.067 | 0.22 | 0.086 | 0.22 | 0.056 | 0.5 |

**Tab. 4.1: The dependence of the penetration depth on the frequency**

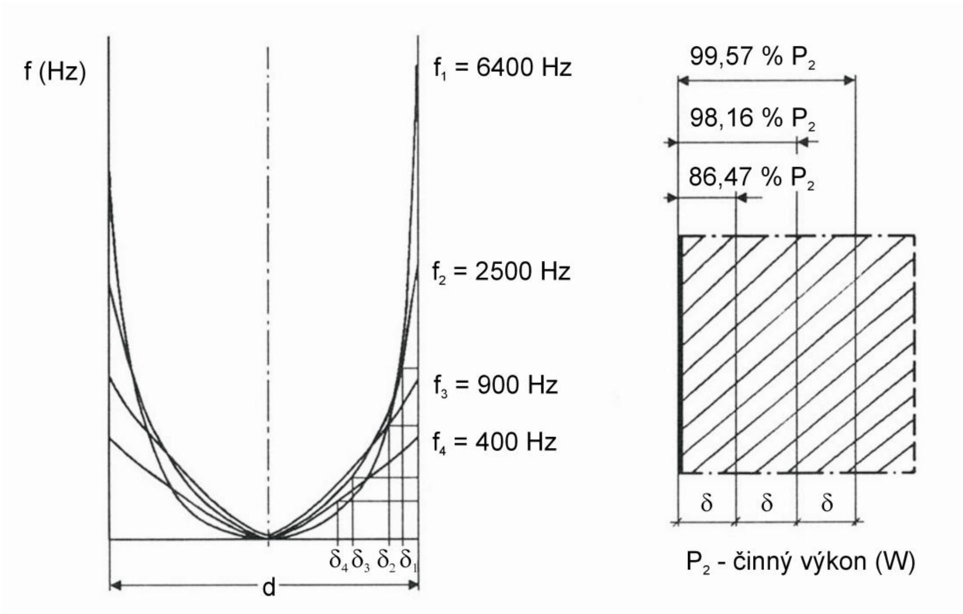


**Fig. 4.2: The dependence of the penetration depth of electromagnetic waves on the frequency**

Electric efficiency of induction heating depends on the *d/δ* ratio. Therefore, sizes of the diameter, charge thickness to the penetration depth according to Fig. 4.3. Also the material of the heated object and its temperature influence the resulting induction heating efficiency.

| Material | Temperature (°C) | $\eta_{el. max}$ (%) |
|---|---|---|
| | 20 | 50 |
| Cu | 1100 | 77 |
| AL | 20 | 56 |

| | | |
|---|---|---|
| | 660 | 77 |
| Steel | 20 | 93 |
| | 800 | 88 |

**Tab. 4.2: The dependence of the induction heating efficiency on the material**

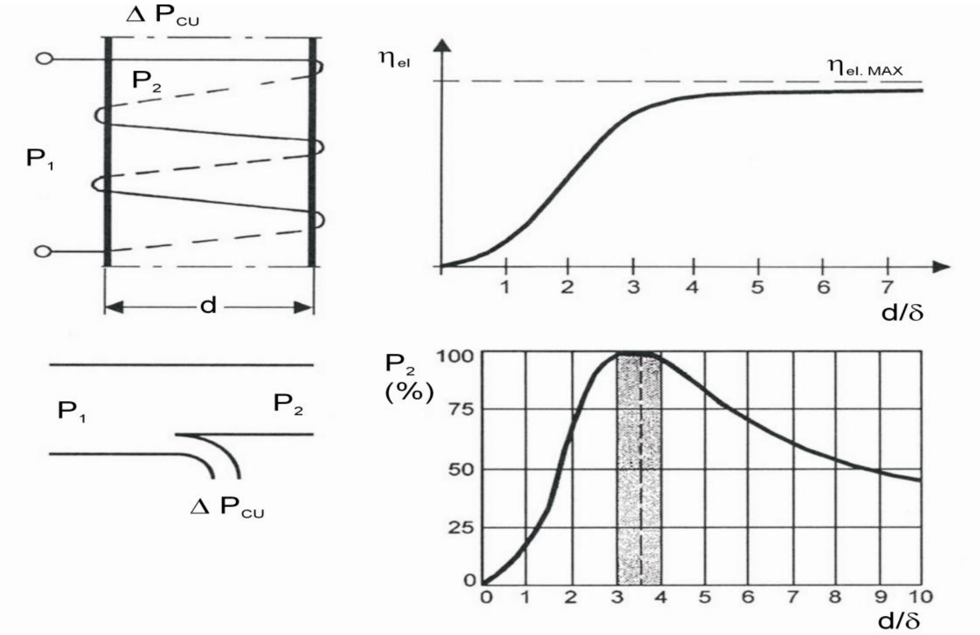


**Fig. 4.3: Electrical induction heating efficiency**

# 4.2. Electrical diagram of induction equipment

## 4.2.1. Two aligned electric circuits

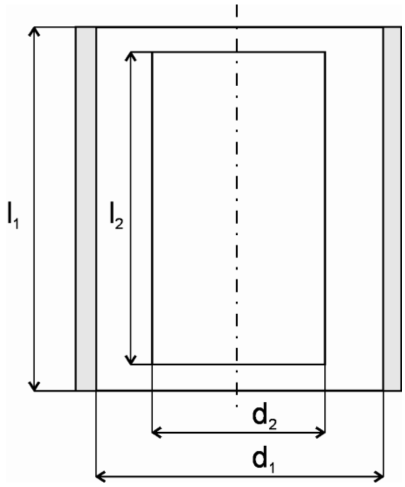


**Fig. 4.4: Two aligned electric circuits**

Two electrical circuits according to Fig. 4.4, a coil and charge are under consideration. The inner diameter of the coil is labeled $d_1$, the charge diameter is $d_2$ and the penetration depths are $\delta_1$, $\delta_2$ . Lengths are $l_1$, $l_2$ . In these considerations, equivalent of zero thickness is taken instead of the actual electric circuits with a current layout. If the penetration depth is significantly smaller than the respective radius, the equivalent diameters are determined by the relations for the coil $d_c = d_1 + \delta_1$, provided that $(\delta_1 << r_1)$, for the charge $d_v = d_2 - \delta_2$ provided that $(\delta_2 << r_2)$.

Lengths $l_1$, $l_2$ remain the same. Instead of fig. 4.4, the equivalent layout according to Fig. 4.5 will be taken into account. Its wiring diagram is shown in Fig. 4.6.

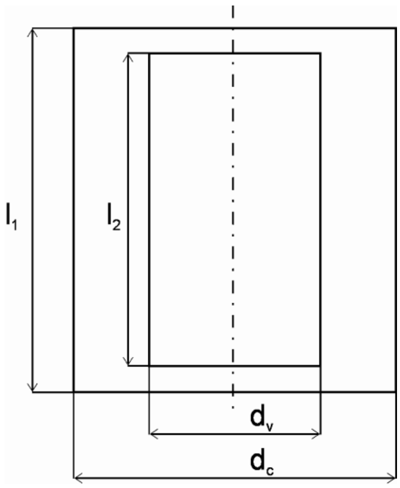


**Fig. 4.5: The equivalent of two aligned electric circuits.**

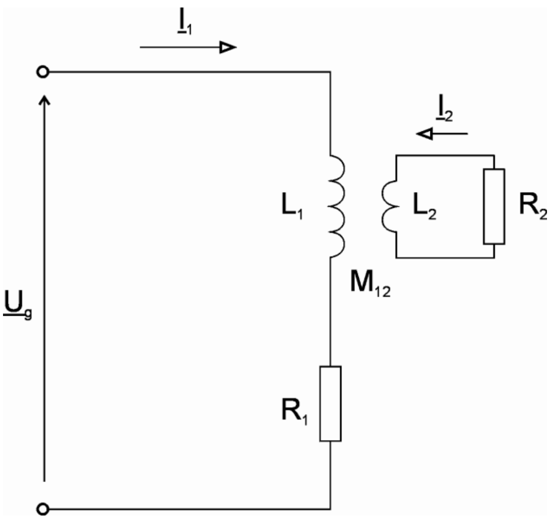


**Fig. 4.6: Electrical diagram**

In Fig. 4.6 $L_1$, $L_2$ (H) and $R_1$, $R_2$ (Ohm) stands for the actual inductance and the resistances of the coil and the charge, $M_{12}$ (H) is the mutual inductance of both the circuits $M_{12} = k_{12}\,(L_1 \cdot L_2)^{1/2}$; the currents in these circuits are labeled $\underline{I}_1$ and $\underline{I}_2$. Alternating current at frequency $f$ (Hz) and voltage $\underline{U}_g$ (V) is supplied to the terminals of the furnace coil. Let us derive the effect of the second circuit on the first circuit. The basis is two basic equations of these circuits; each of them means that the supplied voltage equals to the sum of drops (the second Kirchoff law).

$$\underline{U}_g = (R_1 + j\omega L_1)\cdot \underline{I}_1 + j\omega M_{12}\cdot \underline{I}_2 \qquad (4.2)$$

$$0 = (R_2 + j\omega L_2) \cdot \underline{I}_2 + j\omega M_{12} \cdot \underline{I}_1 \qquad (4.3)$$

$I_2$ is derived from the second equation and it is put into the first equation

$$\underline{U}_g = [(R_1 + p_{12}^2 R_2) + j\omega(L_1 - p_{12}^2 L_2)] \cdot \underline{I}_1 = (R_I + j\omega L_I) \cdot \underline{I}_1 \qquad (4.4)$$

The indicated mathematical adjustment shows that we have converted charge values (impedance $Z_2 = R_2 + j\omega L_2$) to the first circuit. The resulting resistance $R_I$ in the first circuit is determined by the sum of the actual resistance of the coil and the transferred resistance of the charge. The resulting inductance $L_I$ is determined by the actual inductance of the coil and the transferred inductance of the charge.

$$R_I = R_1 + p_{12}^2 R_2 \ ; \quad L_I = L_1 - p_{12}^2 L_2 \ ; \quad p_{12}^2 = \frac{\omega^2 \cdot M_{12}^2}{R_2^2 + \omega^2 \cdot L_2^2} \qquad (4.5)$$

After transferring of the secondary side, the electrical diagram according to Fig. 4.6 has the shape according to Fig. 4.7.

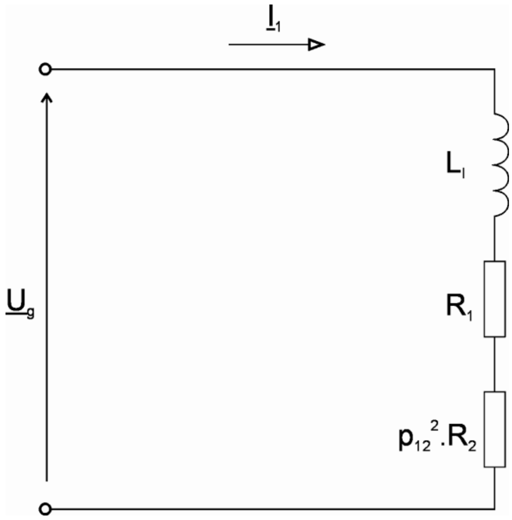


**Fig. 4.7: The electrical diagram transferred to the primary side**

### 4.2.2. Three aligned electric circuits

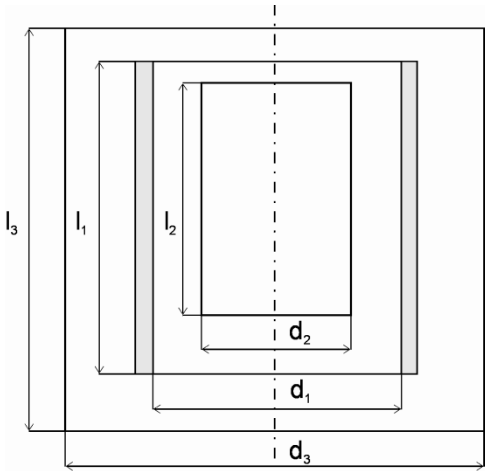


**Fig. 4.8: Three aligned electric circuits**

This is an induction device with a shielding jacket and the three aligned electric circuits must be taken into account according to Fig. 4.8. The inner diameter of the coil is labeled $d_1$, the charge diameter is $d_2$ and the shielding diameter is $d_3$. Lengths are $l_1$, $l_2$ a $l_3$. Instead of the real circuits with a current layout, again the equivalent cylinders of zero thickness with the following coil diameters are taken into account: $d_c = d_1 + \delta_1$ provided that $(\delta_1 << r_1)$, for the charge $d_v = d_2 - \delta_2$ provided that $(\delta_2 << r_2)$, for shielding: $d_s = d_3 + \delta_3$ provided that $(\delta_3 << r_3)$.

The lengths $l_1$, $l_2$ remain the same. Instead of Fig. 4.8, we shall take the equivalent arrangement according to Fig. 4.9 into account. Its electric diagram is shown in Fig. 4.10.

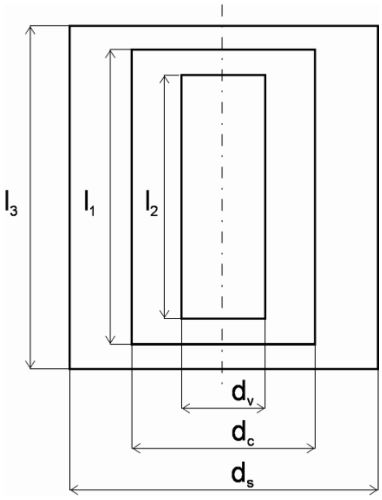


**Fig. 4.9: The equivalent arrangement of the three aligned electric circuits**

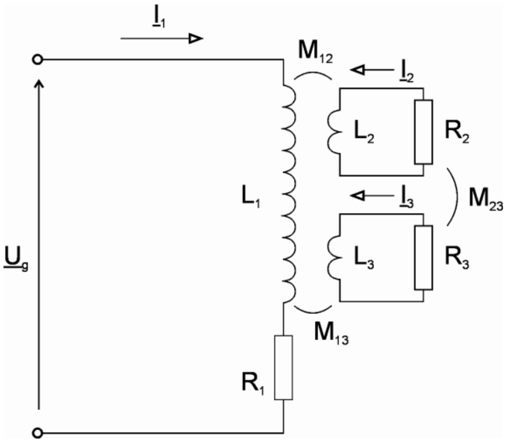


**Fig. 4.10: Electrical diagram**

In Fig. 4.10, $L_1$, $L_2$ , $L_3$ (H) and $R_1$, $R_2$ , $R_3$ (Ω) stand for the actual inductance and resistances of the coil, charge and shielding, $M_{12}$, $M_{13}$, $M_{23}$, (H) stand for the mutual inductance. The currents in these circuits are labeled $\underline{I}_1$, $\underline{I}_2$ and $\underline{I}_3$. Alternating current at frequency *f* (Hz) and voltage $\underline{U}_g$ (V) is supplied to the terminals of the furnace coil. Let us derive the effects of the second and the third circuits on the first circuit. The basis is three fundamental equations of these circuits. Each equation means that the supplied voltage equals to the sum of voltage drops. In order to write in a shorter way, let us introduce impedances.

$$\underline{Z}_1 = R_1 + j\omega L_1 \ ; \quad \underline{Z}_2 = R_2 + j\omega L_2 \ ; \quad \underline{Z}_3 = R_3 + j\omega L_3 \qquad (4.6)$$

$$\underline{U}_g = \underline{Z}_1 \cdot \underline{I}_1 + j\omega M_{12} \cdot \underline{I}_2 + j\omega M_{13} \cdot \underline{I}_3 \qquad (4.7)$$

$$0 = \underline{Z}_2 \cdot \underline{I}_2 + j\omega M_{12} \cdot \underline{I}_1 + j\omega M_{23} \cdot \underline{I}_3 \qquad (4.8)$$

$$0 = \underline{Z}_3 \cdot \underline{I}_3 + j\omega M_{13} \cdot \underline{I}_1 + j\omega M_{23} \cdot \underline{I}_2 \qquad (4.9)$$

Upon solving of these equations, we will obtain the unknowns $\underline{I}_1$, $\underline{I}_2$ and $\underline{I}_3$. Let us determine $\underline{I}_3$ from the third equation and put it into the first and the second equations. To simplify writing, we have introduced auxiliary constants.

$$\underline{V}_1 = \left( \underline{Z}_1 + \frac{\omega^2 \cdot M_{13}^2}{\underline{Z}_3} \right)$$

$$\underline{V}_2 = \left( \underline{Z}_2 + \frac{\omega^2 \cdot M_{23}^2}{\underline{Z}_3} \right)$$

$$\underline{V}_3 = \left( \frac{\omega^2 \cdot M_{13} \cdot M_{23}}{\underline{Z}_3} + j\omega M_{12} \right)$$

The equations (4.7) and (4.8) are now as follows:

$$\underline{U}_g = \underline{V}_1 \cdot \underline{I}_1 + \underline{V}_3 \cdot \underline{I}_2 \qquad (4.10)$$

$$0 = \underline{V}_2 \cdot \underline{I}_2 + \underline{V}_3 \cdot \underline{I}_1 \quad \text{z toho} \quad \underline{I}_2 = -\frac{\underline{V}_3}{\underline{V}_2} \cdot \underline{I}_1 \qquad (4.11)$$

$$\underline{U}_g = \left[ \underline{V}_1 - \frac{\underline{V}_3^2}{\underline{V}_2} \right] \cdot \underline{I}_1 \qquad (4.12)$$

Without appropriate adjustments, we would obtain a complex and confusing result. Therefore, we should simplify the preceding expressions, neglecting some quantities, whose values are negligible compared to the others. For melting furnaces, it usually applies that

$R_1 < \omega L_1$, $R_3 < \omega L_3$

In this consideration, $R_1$ can be neglected against $\omega L_1$, as well as $R_3$ can be neglected against $\omega L_3$. In the calculation of the furnace, $R_1$ must be, however, included in an appropriate way. Impedance expressions will then be

$$\underline{Z}_1 \cong j\omega L_1 \, ; \quad \underline{Z}_3 \cong j\omega L_3 \qquad (4.13)$$

For the charge, $R_2$ against $\omega L_2$ cannot be neglected, therefore the following must be taken into account.

$$\underline{Z}_2 = R_2 + j\omega L_2$$

The expressions $\underline{V}_1$ to $\underline{V}_3$ will be

$$\underline{V}_1 = j\omega L_1 \cdot (1-\kappa_{13}^2)$$
$$\underline{V}_2 = R_2 + j\omega L_2 \cdot (1-\kappa_{23}^2)$$
$$\underline{V}_3 = j\omega\sqrt{L_1 \cdot L_2} \cdot (\kappa_{12} - \kappa_{13} \cdot \kappa_{23})$$

Further we can neglect

$$\left(\frac{R^2}{\omega L^2}\right)^2 \quad \text{proti} \quad (1-\kappa_{23}^2)^2$$

After further adjustments, the expression for voltage will have its final form:

$$\underline{U}_g = \begin{bmatrix} j\omega L_1 \cdot (1-\kappa_{13}^2) - j\omega L_1 \cdot \dfrac{(\kappa_{12} - \kappa_{13} \cdot \kappa_{23})^2}{1-\kappa_{23}^2} + \\ + \dfrac{L_1}{L_2} \cdot \dfrac{(\kappa_{12} - \kappa_{13}\kappa_{23})^2}{(1-\kappa_{23}^2)^2} \cdot R_2 \end{bmatrix} \cdot \underline{I}_1 = \tag{4.14}$$
$$= \left[\frac{L_1}{L_2} \cdot \Psi_{12}^2 \cdot R_2 + j\omega L_1 \cdot (1-\varepsilon)\right] \cdot \underline{I}_1$$

We have used these symbols for the purpose of shortened writing

$$\Psi_{12}^2 = \frac{\kappa_{12} - \kappa_{13} \cdot \kappa_{23}}{1-\kappa_{23}^2} \tag{4.15}$$

$$\varepsilon = \frac{\kappa_{12}^2 + \kappa_{13}^2 - 2 \cdot \kappa_{12} \cdot \kappa_{13} \cdot \kappa_{23}}{1-\kappa_{23}^2} = \frac{\kappa_{12} - \kappa_{13} \cdot \kappa_{23}}{1-\kappa_{23}^2} \cdot \kappa_{12} + $$
$$+ \frac{\kappa_{13} - \kappa_{12} \cdot \kappa_{23}}{1-\kappa_{23}^2} \cdot \kappa_{13} = \kappa_{12} \cdot \Psi_{12} + \kappa_{13} \cdot \Psi_{13} \tag{4.16}$$

The performed mathematical adjustment means that we have transferred the impedance values from the second and the third circuits to the coil circuit. The resulting impedance has a real component $R_l'$ and an imaginary component $\omega L_l'$. The electric diagram in Fig. 4.10 will be replaced by a new diagram in Fig. 4.11.

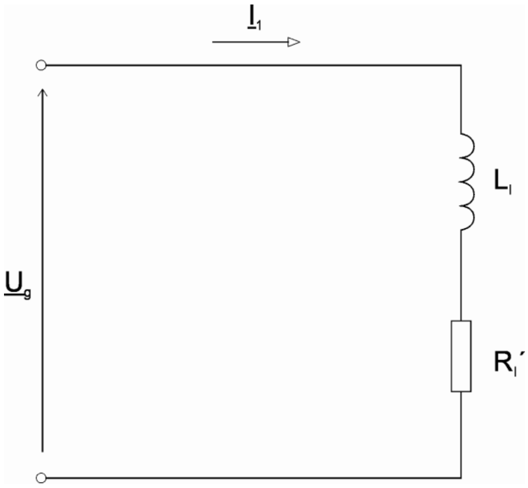


**Fig. 4.11: Equivalent electric diagram**

The real element of the expression for $\underline{U}_g$ indicates the resulting active resistance $R_I'$ in the first circuit. Since, according to the assumption that we have made, we presumed that $R_1$=$R_3$=0, $R_I'$ stood only for the active resistance of the actual charge transferred to the first circuit. The other element of the expression indicates the resulting reactance ω$L_I$ in the first circuit, i.e. the actual reactance ω$L_1$ of the furnace coil minus the transferred reactances ω$L_2$ and ω$L_3$ of the charge and shielding.

$$\underline{U}_g = (R_I' + j\omega L_I) \cdot \underline{I} \tag{4.17}$$

$$R_I' = \frac{L_1}{L_2} \cdot \Psi_{12}^2 \cdot R_2 \tag{4.18}$$

$$L_I = L_1 \cdot (1 - \varepsilon) \tag{4.19}$$

**Current in all the three circuits**

The current $\underline{I}_1$ in the first circuit from the relation for the voltage $\underline{U}_g$

$$\underline{I}_1 = \frac{\underline{U}_g}{\left[\frac{L_1}{L_2} \cdot \Psi_{12}^2 \cdot R_2 + j\omega L_1 \cdot (1-\varepsilon)\right]} = \frac{\underline{U}_g \cdot \left[\frac{\Psi_{12}^2}{Q_2} - j(1-\varepsilon)\right]}{\omega L_1 \cdot \left[\left(\frac{\Psi_{12}^2}{Q_2}\right)^2 + (1-\varepsilon)^2\right]} \tag{4.20}$$

We have introduced the symbols $Q_1$ and $Q_2$ for the “circuit quality”

$$Q_1 = \frac{\omega L_1}{R_1} \quad a \quad Q_2 = \frac{\omega L_2}{R_2}$$

The ratios for melting furnaces are such that we make a mistake not exceeding 1 % if we neglect the first element against the second one in the denominator, and then we divide the numerator and the denominator by the expression (1 – $\varepsilon$). Then

$$\underline{I}_1 = \frac{\underline{U}_g}{\omega L_1 \cdot (1-\varepsilon)} \cdot \left[ \frac{\Psi_{12}^2}{(1-\varepsilon) \cdot Q_2} - j1 \right] \tag{4.21}$$

The current $\underline{I}_2$ from the expression (4.11)

$$\underline{I}_2 = -j \cdot \sqrt{\frac{L_1}{L_2}} \cdot \frac{(\kappa_{12} - \kappa_{13} \cdot \kappa_{23}) \cdot \left[ \frac{R_2}{\omega L_2} - j \cdot (1-\kappa_{23}^2) \right]}{\left( \frac{R_2}{\omega L_2} \right)^2 + (1-\kappa_{23}^2)^2} \cdot \underline{I}_1 \tag{4.22}$$

Again, we can neglect the first element against the second one in the denominator. Finally

$$\underline{I}_2 = \frac{\underline{U}_g \cdot \Psi_{12}}{\omega \cdot \sqrt{L_1 \cdot L_2} \cdot (1-\varepsilon)} \cdot \left[ -\frac{1}{(1-\varepsilon) \cdot Q_2} \cdot \frac{1-\kappa_{13}^2}{1-\kappa_{23}^2} - j1 \right] \tag{4.23}$$

The current $\underline{I}_3$ comes out from the expression (4.9) for voltage drops in the third loop of the equivalent diagram.

$$\underline{I}_3 = -\frac{j\omega}{\underline{Z}_3} \cdot \left( M_{13} \cdot \underline{I}_1 + M_{23} \cdot \underline{I}_2 \right) \tag{4.24}$$

Let us substitute $\underline{I}_1$ and $\underline{I}_2$ from the derived relations.

$$\underline{I}_3 = -\frac{\underline{U}_g \cdot \Psi_{12}}{\omega \cdot \sqrt{L_1 \cdot L_3} \cdot (1-\varepsilon)} \cdot \left[ \frac{1}{(1-\varepsilon) \cdot Q_2} \cdot \frac{\kappa_{23} - \kappa_{12} \cdot \kappa_{13}}{1-\kappa_{23}^2} + j \frac{\kappa_{13} - \kappa_{12} \cdot \kappa_{23}}{\kappa_{12} - \kappa_{13} \cdot \kappa_{23}} \right] \tag{4.25}$$

## 4.2.3. Resonant circuit

Let us consider three electrical circuits, a coil, charge and a shielding jacket according to Fig. 4.8. Having transferred the charge impedances and shielding into the coil circuit, we obtained the expression for the voltage of the generator. The resulting electrical diagram is shown in Fig. 4.10. The resulting inductance and the resulting resistance are determined by the following expressions

$$L_I = L_1 \cdot (1-\varepsilon)\,; \quad R_I' = \frac{L_1}{L_2} \cdot \Psi_{12}^2 \cdot R_2 \qquad (4.26)$$

The real part $R_I'$ of the resulting impedance means only the active resistance $R_2$ of the actual charge transferred to the first circuit, because we neglected the real components $R_1$ and $R_3$ in impedances $Z_1$ and $Z_3$ while deriving. The actual resulting active resistance of the coil $R_I$ is, however, determined by the sum of the transferred resistance $R_I$ and the actual resistance $R_1$ of the coil $R_I = R_I' + R_1$. The $R_1$ value for the furnaces of this type is usually approx. 4 times smaller than $R_I'$. The resulting reactance $\omega L$ for steel-processing furnaces is usually approx. 10 times higher than $R_I'$; this ratio is even higher for more conductive materials. It shall be a minor mistake if we take the $R_I$ resistance instead of $R_I'$ in the resulting impedance $Z_I = R_I + j\omega L_I$. For better clearness, let us design a vector diagram for the impedance $Z_I'$ and $Z_I$ Fig. 4.12. The difference in both the impedances is really negligible.

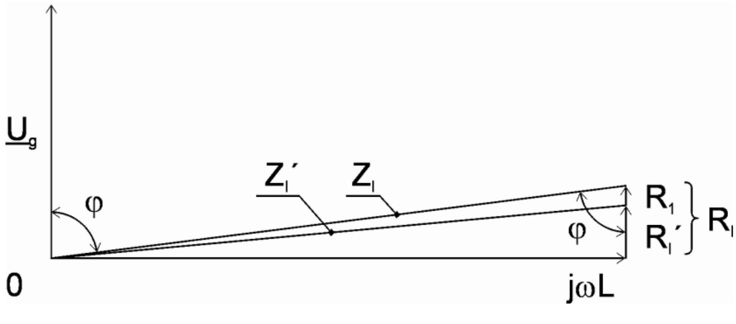


**Fig. 4.12: The vector diagram for the $Z_I'$ impedance**

For the melting furnaces under consideration, the active losses $P_3$ in shielding are only 0.73 % out of the power input $P_c$ to the coil. The resistance $R_3$ transferred to the coil is also lower than 1 % of the total resistance $R_I$; therefore, we can neglect it and we no longer take it into account. In the resulting diagram Fig. 4.10, we will draw the actual resistance of the $R_1$ coil. As a result, we will obtain a new diagram.

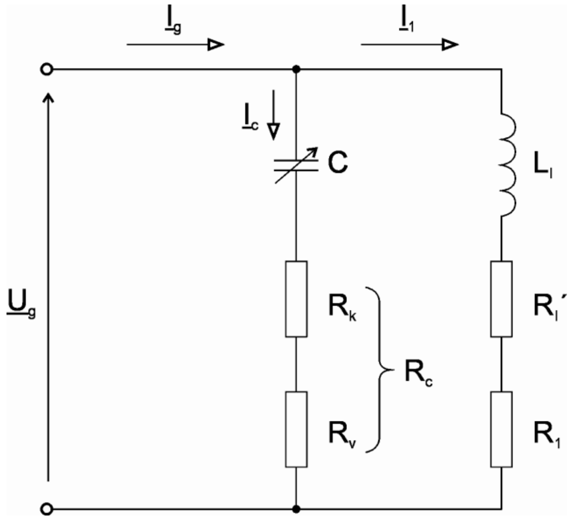


**Fig. 4.13: The resulting electric diagram**

It is obvious from the impedance diagram that the $Z_1$ impedance has a relatively small real component, which means that the power factor cosφ of the furnace coil is small.

$$\cos\varphi = \frac{R_I}{Z_I} = \frac{R_I}{\sqrt{{R_I}^2 + (\omega L_I)^2}} = \\ = \frac{R_I}{\omega L_I} \cdot \frac{1}{\sqrt{\frac{1}{Q_I^2} + 1}} = \frac{1}{\sqrt{1 + Q_I^2}} \cong \frac{1}{Q_I} \tag{4.27}$$

The symbol $Q_1$ stands for the resulting "quality" of the circuit.

$$Q_I = \frac{\omega L_I}{R_I}$$

The current $\underline{I}_1$ in the furnace coil has its idle component, which is much bigger than the active component. In order to relieve the source of the supply of idle magnetizing current, we connect appropriately dimensioned capacity *C,* which supplies the magnetizing component of the $\underline{I}_1$ current to the coil in a parallel way. The actual frequency of the circuit ($L_l$, $R_l$, $C$) then equals the supplied frequency *f* (Hz). In calculations, dielectric losses in the *C* capacitor are usually replaced by losses in the appropriate resistance $R_k$, which is connected to the capacitor either in the parallel way, or in the series. In Fig. 4.13, we indicated the series resistance $R_c$, which includes dielectric losses $P_k$ in the *C* capacitor and further, relatively small losses $P_v$ in the connection strip line between the furnace coil and the capacitor battery, $R_c = R_k + R_v$. The size of the resistance $R_c = R_k + R_v$ in the diagram Fig. 4.13 is determined by the fact that upon passage of the $\underline{I}_c$ current, 5 % of the power input $P_g$ is consumed in it. For steel-processing heating equipments, the dielectric losses and losses in the connecting power line are lower; approx. 3 % of the generator $P_g$. Then the $\underline{I}_c$ current is approximately the same as if there was no $R_c$ resistance.

$$\underline{I}_c = j\omega C \cdot \underline{U}_g \qquad (4.28)$$

For the purpose of a simple calculation of the induction equipment, we make a tolerable mistake lower than 1 %, because we move the $R_c$ resistance in the diagram Fig. 4.13 from the capacitor branch to the coil branch. All active resistances Fig. 4.14 are included in the coil branch.

$$R_I = R_I' + R_1 + R_c \qquad (4.29)$$

**Deriving of the required capacity**

In Fig. 4.14, there is a circuit ($L_I$, $R_I$, $C$) with two parallel branches, supplied from the generator source with increased frequency *f* (Hz), $\omega = 2\pi f$. The capacity *C* can be set to such a value, so that the circuit is fine-tuned for the supplied frequency. This means that the actual circuit frequency equals to the supplied frequency *f*. Should it be the case, the source supplies the circuit ($L_I$, $R_I$, $C$) only with the active power input. The current $\underline{I}_g$ does not have the idle component. Generally, the current $\underline{I}_g$ from the source is divided into both branches, through which the currents $\underline{I}_1$ and $\underline{I}_c$ pass.

$$\underline{I}_g = \underline{I}_1 + \underline{I}_c = \frac{\underline{U}_g}{R_I + j\omega L_I} + j\omega C \cdot \underline{U}_g$$

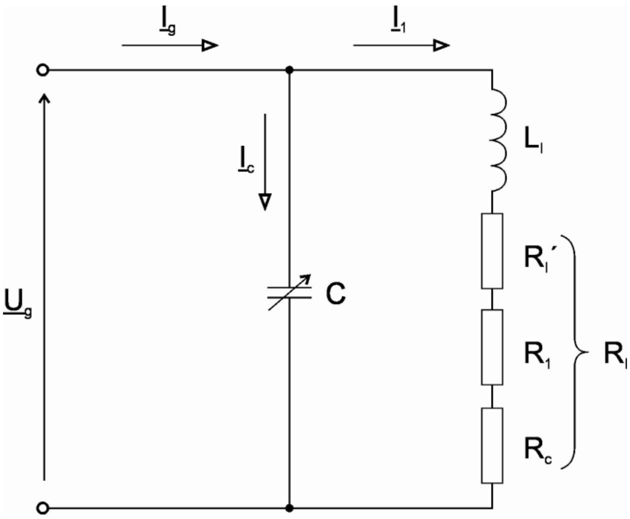


**Fig. 4.14: Electrical diagram for derivation of the required capacity**

After the adjustment to the real and the imaginary components

$$\underline{I}_g = \frac{\underline{U}_g}{R_I{}^2 + (\omega L_I)^2} \cdot R_I + j \frac{\underline{U}_g}{R_I{}^2 + (\omega L_I)^2} \cdot \\ \cdot (\omega \cdot C \cdot R_I{}^2 + \omega \cdot C \cdot \omega^2 \cdot L_I{}^2 - \omega \cdot L_I) = \underline{I}_{gr} + j\underline{I}_{gi} \tag{4.30}$$

If the circuit is not fine-tuned, the current $\underline{I}_g$ has the active and the idle components. The required capacity $C$ for fine-tuning will come out from the condition that the idle component $\underline{I}_{gi} = 0$. If the expression in the brackets is equal to zero, then we will obtain

$$C = \frac{L_I}{R_I{}^2 + (\omega L_I)^2} = \frac{L_I}{(\omega L_I)^2} \cdot \frac{1}{1 + \frac{1}{Q_I{}^2}} = \frac{1}{\omega^2 L_I} \cdot \frac{Q_I{}^2}{Q_I{}^2 + 1} \tag{4.31}$$

It is obvious from this expression, how much the required capacity $C$ depends on the circuit quality $Q_I$ . If $R_I^2 << (\omega L_I)^2$ or $Q_I^2 >> 1$ (if we neglect losses in the circuit), we will obtain the well-known Thompson relation:

$$C = \frac{1}{\omega^2 L_I}; \quad \omega^2 = \frac{1}{C \cdot L_I} \tag{4.32}$$

If the supplied frequency is constant, $\omega = 2\pi f$ = const., the capacity $C$ required for fine-tuning decreases provided that the active resistance $R_I$ grows. On the contrary, it reaches the maximum value $C$, if $R_I = 0$. Let us get back to the expression for the $C$ capacity. If the capacity $C$ is of the right size, the circuit ($L_I$, $R_I$, $C$) is in resonance for the supplied frequency. The circuit takes only the active power from the source, the $\underline{I}_g$ current has only the active component $\underline{I}_{gr}$. Its size can be determined from the expression for the $\underline{I}_g$ current.

$$\underline{I}_{gr} = \frac{\underline{U}_g}{R_I{}^2 + (\omega L_I)^2} \cdot R_I \tag{4.33}$$

The fine-tuned furnace circuit behaves as the active load resistor $R_z$, whose value can be obtained from the previous relation.

$$R_z = \frac{\underline{U}_g}{\underline{I}_{gr}} = \frac{R_I{}^2 + (\omega L_I)^2}{R_I} = R_I \cdot (1 + Q_I{}^2) \tag{4.34}$$

If $_I^2 << (\omega L_I)^2$; $Q_I^2 >> 1$ for melting furnaces, we can simplify the expression for the calculation of $R_Z$

$$R_z \cong \frac{(\omega L_I)^2}{R_I} = R_I \cdot Q_I^{\ 2} \cong \omega L_I \cdot Q_I \qquad (4.35)$$

If we connect the expression (4.35) for $R_z$ and the expression (4.32) for the $C$ capacity, we obtain another expression for $R_z$

$$R_z = \frac{L_I}{C \cdot R_I} \qquad (4.36)$$

Further, we can derive from the expression for $\underline{I}_{gr}$, how many times larger the oscillating current $\underline{I}_1$ is in the fine-tuned circuit compared to the field current $\underline{I}_g$ from the generator.

$$\underline{I}_g = \underline{I}_{gr} = \frac{\underline{U}_g \cdot R_I}{R_I^{\ 2} + (\omega L_I)^2} = \frac{\underline{U}_g}{R_I + j\omega L_I} \cdot \frac{R_I}{R_I - j\omega L_I} = \underline{I}_1 \cdot \frac{1 + jQ_I}{1 + jQ_I^{\ 2}} \qquad (4.37)$$

Absolute values

$$\left|I_g\right| = \left|I_{gr}\right| = I_1 \cdot \frac{\sqrt{1+Q_I^{\ 2}}}{1+Q_I^{\ 2}} = \frac{I_1}{\sqrt{1+Q_I^{\ 2}}} = \frac{I_1}{Q_I}$$

For melting furnaces, $Q_I$ = 12, so $Q_I^2$ >> 1. The current $I_1$ in the furnace coil is very approximately $Q_I$ times higher than the current $I_g$ from the source. Yet, let us compare the current $I_1$ in the furnace coil and the current $I_c$ passing through the capacity $C$.

$$\underline{I}_c = \underline{I}_g - \underline{I}_1 = \frac{\underline{U}_g \cdot R_I}{(R_I + j\omega L_I) \cdot (R_I - j\omega L_I)} - \frac{\underline{U}_g}{R_I + j\omega L_I} = \underline{I}_1 \cdot \left( \frac{R_I}{R_I - j\omega L_I} - 1 \right) = \underline{I}_1 \cdot Q_I \cdot \frac{(-Q_I + j1)}{1 + Q_I^{\ 2}} \qquad (4.38)$$

$$\left|I_c\right| = I_1 \cdot \frac{Q_I}{\sqrt{1+Q_I^{\ 2}}} \cong I_1$$

If we can neglect 1 against $Q_I^2$ (for melting furnaces), the current $I_c$ passing through the capacity $C$ is almost as high as the current $I_1$ in the furnace coil. For example, already for $Q_I$ = 10 holds that $I_c$ = 0.995 $I_1$. The older method of induction furnace tuning using two identical ammeters placed on the switchboard, usually one over the other, is based on this fact. Furnacemen can monitor the condition of tuning from greater distances and they can intervene if necessary. The currents $I_1$ = $I_c$ in the oscillating circuit feature relatively high values. If the generator supplies power of $P_g$ = 500 kW, $U_g$ = 1000 V to the furnace, the power from the source in the fine-tuned condition $I_g$ = 500 A. If the circuit quality is $Q_l$ = 12, the current $I_1$ = $I_c$ = 6000 A. For such high currents, it is suitable to connect the coil and the capacity through strip line [1], [2].

## 4.3. Induction crucible furnaces

**Induction crucible furnaces with a non-conductive crucible**

There is a crucible in the coil; it is rammed from ceramic material. Such a furnace features either a copper shielding jacket, or a steel jacket, which carries bunches of transformer sheets on its inner side.

**Induction crucible furnace with a conductive shielding jacket**

The diagram of this furnace is shown in Fig. 4.16.

The ceramic crucible 4 of a cylindrical shape, which is electrically non-conductive, containing the charge 2, is wound by the coil 1. The coil is usually wound from a copper tube with a rectangular cross-section. The current of increased frequency (500 to 1000 Hz) or current of the network (mains) frequency (50 Hz) passes through the coil. The inner surface of the coil emits electromagnetic waves to its cavity; the waves impact perpendicularly on the charge surface; the charge absorbs them and the electromagnetic energy changes into heat. A magnetic flux passes through the coil cavity; the flux is closed inside the coil. Necessary precautions must be taken, so that the intensity of the magnetic field inside the furnace is as low as possible to prevent heating of furnace load-bearing structures. It can be achieved either by providing the furnace with the shielding jacket from well-conductive material with the appropriate diameter, or by placing the sheet-iron core, through which the magnetic flux closes, inside the coil. Induction crucible furnaces always feature shielding, either in the form of a conductive shielding jacket, or as a core from transformer sheets inside the coil. If we calculate smaller furnaces, we can neglect the effects of shielding and we can consider the furnace to be without any shielding. In calculations, only the coil and the charge are taken into consideration, i.e. only the two mutually acting electric circuits. The calculation is thus simplified and shortened.

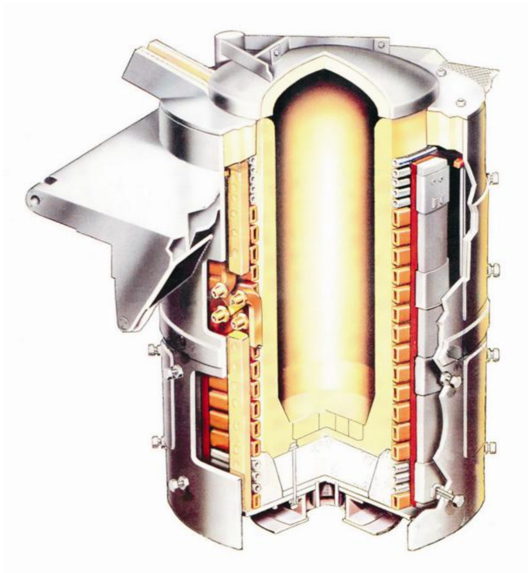

**Fig. 4.15: Induction crucible furnace**

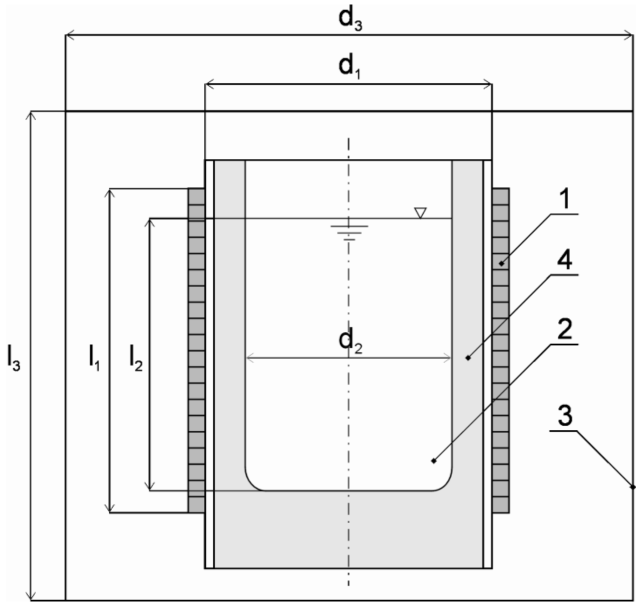


**Fig. 4.16: Induction crucible furnace with a conductive shielding jacket**

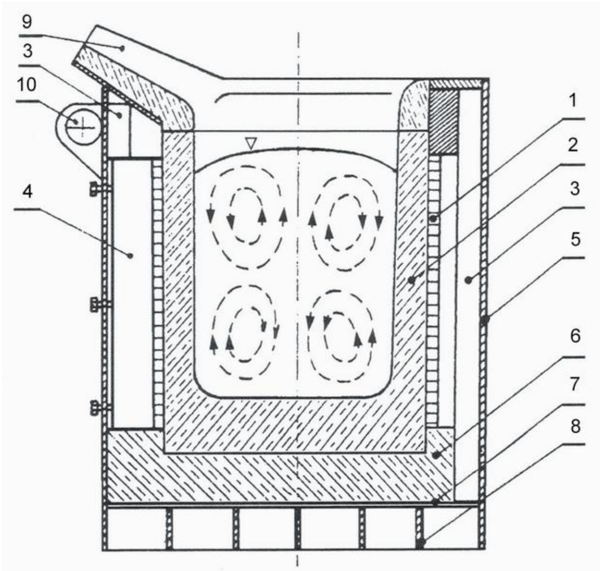


**Fig. 4.17: The diagram of an induction crucible furnace with a conductive shielding jacket**

1 is a circular heating coil The coil is wound as a single-layer coil, most often with a hollow copper conductor of a rectangular cross-section. Cooling water flows through the cavity. 2 is a ceramic rammed crucible of the furnace. Most often, it is made from silica sand. 3 - bunches of transformer sheets. They are arranged vertically along the entire inner perimeter of the furnace jacket 5. These bunches conduct the magnetic flux outside the coil, so that it does not interfere with the jacket 5 or other structural parts of the furnace. 4 - medium refractory concrete beams for the coil 1, 5 - steel jacket of the furnace, 6 - brick ceramic lining on the bottom of the furnace, 7 - shielding copper sheet preventing the penetration of the magnetic flux of the coil to the grating bottom of the furnace; 8 - grating furnace bottom, 9 - furnace lip and 10 - axis, around which the furnace revolves while tipping or during tapping. The furnace is usually tipped by hydraulic cylinders.

**An induction crucible furnace with an iron core inside the coil**

If the induction crucible furnace is fitted with a conductive shielding jacket, the intensity of the magnetic field inside the jacket drops significantly. A similar effect can also be reached if we put an iron core from multiple bunches of transformer sheets instead of the shielding conductive jacket outside the coil according to Fig. 4.18. The bulk of the magnetic flux induced by the coil will be closed inside the coil through a magnetically well-conductive path, i.e. by bunches of transformer sheets fixed on the inner part of the furnace jacket from the boiler plate. The inclusion of the magnetically conductive bunches decreases the magnetic resistance for the magnetic flux induced by the coil. As a result, the flux somehow increases. The inductance of the furnace coil, i.e. the magnetic flux for the unit current, increases. The actual inductance and the mutual inductance $M_{12}$ increase as well. The furnace with the shielding jacket requires a coil with a higher number of turns and larger capacitor battery than the furnace with the core. The useful power is lower and the losses in the coil are higher for the furnace with the jacket than the furnace with the core. The values for the furnace without shielding are usually between the values of I and III and they are closer to the values of III. The electrical efficiency of the furnace with the jacket is significantly lower than the efficiency of the furnace with the core. The most important result of the performed comparison is the finding that the efficiency of the furnace with the iron core is higher by almost 5% than the efficiency of the furnace with the shielding jacket. It brings significant savings of electric energy in the operation, especially with large furnaces with continuous operation. The induction crucible furnace with the iron core is, therefore, technically more advanced solution. The manufacturing of this furnace is, however, more expensive, and if crucible break-through occurs during operation, the damages to the furnace are usually worse and sometimes the furnace is destroyed completely.

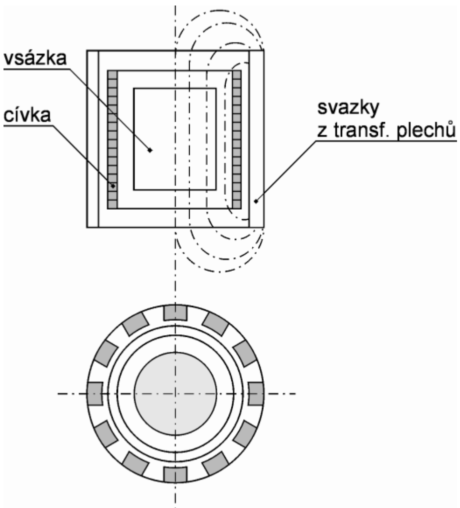


**Fig. 4.18: An induction crucible furnace with an iron core inside the coil**

**An induction crucible furnace with a conductive crucible**

The induction furnace with a non-conductive crucible features low electric efficiency in melting of well-conductive materials, such as copper and its alloys, aluminium and its alloys, etc. The electrical efficiency grows significantly if the furnace is equipped with a conductive crucible, as shown in the diagram Fig. 4.19 (for melting of Al and Cu). If there are lower temperatures, which can be taken into account for melting of aluminium and its alloys, the crucible is made of cast steel. Graphite crucibles are used for higher temperatures. They are made of the mixture of fireclay (chamotte) and graphite. The more graphite it contains, the more conductive the crucible is. However, high conductivity is not desirable. There is a ceramic insulation layer between the crucible and the coil; it reduces the loss heat flux from the hot crucible to the water-cooled coil. If we introduce current into the coil, the electromagnetic waves emitted by the internal surface of the coil impact to the outer surface of the conductive crucible, they enter its wall, induce current in the wall and through its passage, the crucible is heated. The crucible then passes the heat to the charge inserted into its cavity. Usually the bulk of the impacting electromagnetic energy transforms into heat in the crucible wall, the rest of the energy then passes through the wall and heats the charge up directly. The decisive is the mutual ratio between the thickness of the wall $\check{s}_3$ and penetration depth $d_3$. If the penetration depth is small compared to the wall thickness, all electromagnetic energy is absorbed into the crucible wall. The calculation is the same as for two aligned cylinders of the furnace with a non-conductive crucible. In the calculation, the crucible is considered to be the charge. If it does not apply that $d_3 << \check{s}_3$, a significant part of the electromagnetic energy passes through the crucible wall to the charge.

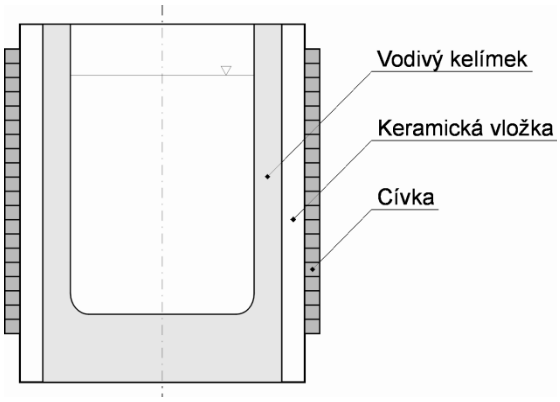


**Fig. 4.19: An induction crucible furnace with a conductive crucible**

**Induction heat-soaking equipment**

In plants, where components are made by hot forging or pressing, the important issue is uniform heating of the material. Metal roller products of a cylindrical or rectangular shape, most often the steel ones, must be heated to the forging temperature (steel 1150 °C to 1250 °C). In older forging shops, fuel furnaces heated by gas, pulverized coal or diesel are used for heat-soaking of roller products. Uniform heat-soaking in the entire cross-section, however, lasts too long and during the time, surface oxidation of the material occurs. Several per cents of material are damaged. Besides, scales damage swages in further operation and they shorten their lifespan. On the contrary, heat-soaking in the induction furnace lasts a very short time, so practically no oxidation occurs on the surface. Economic benefits are the reason for quick introduction of induction heat-soaking in newly-built forging shops. The induction furnace (induction heater) for rolled products usually features a cylindrical shape; see the diagram in Fig. 4.20.

The main part of the induction heater is an induction coil 1 with a normal length of approx. 1 m. Its diameter is selected according to the diameter of the rolled products which shall be induction-heated.. In the inductor, there is usually a larger number (4 to 7) of rolled products of the same length.

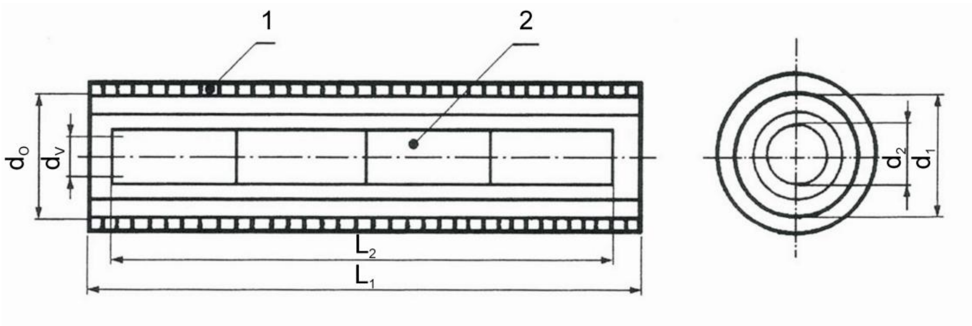


**Fig. 4.20: The diagram of induction heat-soaking**

**The selection of the frequency and the heat-soaking time**

Uniform heat-soaking occurs when the heat from the top layer, where it develops, is spread through conduction to the heart of the rolled product. From this perspective, it is more beneficial that the layer, in which the heat develops, is not too thin; in other words: the frequency must not be too high. Therefore, it is necessary to select suitable frequency, and thus a layer, where the heat is developed, so that energy absorbing with good efficiency occurs, but overheating of the surface is prevented. The optimal frequency is the one, at which $r_2 = (2.5 \text{ to } 3.0) \cdot \delta_2$. The penetration depth of the heat-soaked material, i.e. magnetic material in the case of steel, is taken into consideration. In practice, the frequency for induction heat-soaking of steel is chosen depending on the diameter of the rolled products according to Tab. 4.3. It is evident from the table, that each frequency can be used for various diameters in the significant range.

| f (Hz) | 50 | 500 | 1000 | 2000 | 4000 | 8000 |
|---|---|---|---|---|---|---|
| d (mm) | 160÷500 | 80÷280 | 50÷180 | 35÷120 | 22÷70 | 15÷50 |

**Tab. 4.3: The selection of the frequency for induction heat-soaking of steel.**

It is evident from the diagram in Fig. 4.21 that the specific consumption in kWh for heat-soaking of 1 kg of steel to 1200 °C grows if we are closer to the lower limit of the diameter range. The material becomes electromagnetically transparent. For the steel diameter $d_2$ = 10.0, the specific energy consumption from the mains is approx. 0.40 kWh·kg$^{-1}$ at the frequency of 1000 Hz. The time *t* required for uniform heat-soaking of steel is not usually calculated, but it is taken from the respective diagram according to Fig. 4.22.

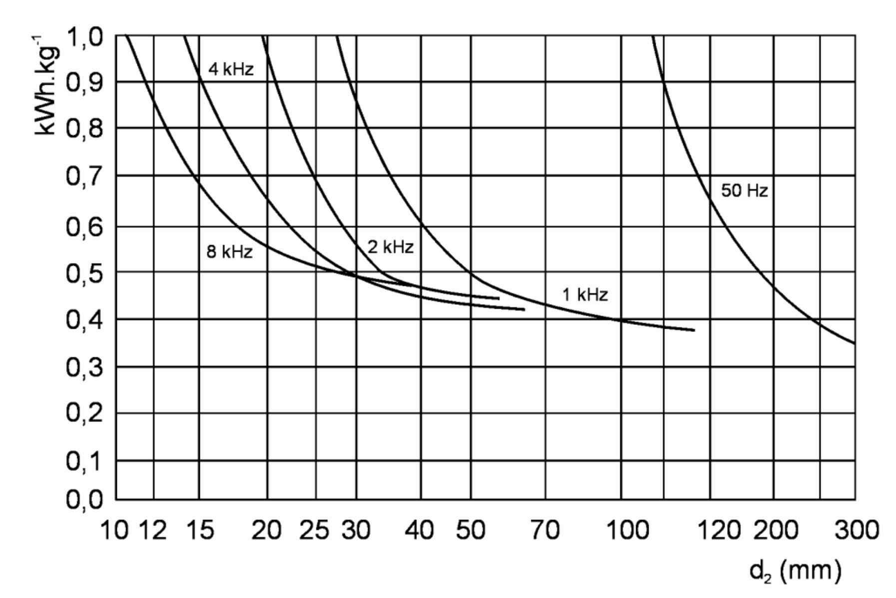


**Fig. 4.21: The diagram of the specific consumption**

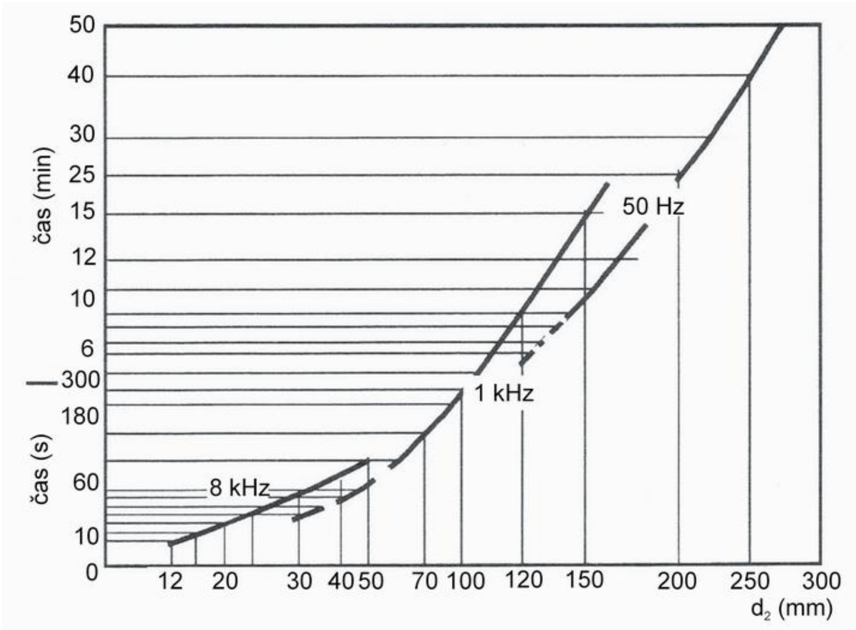


**Fig. 4.22: The diagram of the time for heat-soaking of a steel rolled product**

Heat-soaking uniformity of the rolled product is sufficient provided that the temperature difference between the surface temperature and the temperature in the axis at the end of the heat-soaking process does not exceed 100 °C. Inequality will usually be even lower within several tens of seconds between the end of heat-soaking and the forging operation in the press. We can read from Fig. 4.22 that the appropriate time for heat-soaking of the rolled product with a diameter of $d_2$ = 100 mm is approx. 7 min. at the considered frequency of 1000 Hz. The heat-soaking time can be reduced by one third in the case of the so-called fast heat-soaking. During the fast heat-soaking, the part of the coil at the beginning of the inductor, through which cold rolled products come out, is fitted with packed turns. In this part, there are more turns $N_{11}$ per 1 cm of the coil length. Thus the intensity of the magnetic field $H_1 = N_{11} I_1$ (A·cm$^{-1}$) grows in the input part of the inductor. As the developed heat depends on the second square $H_1^2$, the pieces inserted into this part of the inductor warm faster and the total heat-soaking time is thus reduced. The big difference in temperatures on the surface and in the axis of the rolled product is then balanced during the further course of slower heat-soaking [6].

## 4.4. Induction equipment for surface heating

High frequencies of the orders of $10^4$ to $10^6$ Hz are used for surface heat-soaking of objects to the depth of the order of 10 mm. In the industry, induction heat-soaking devices, inductors, are used for the following purposes [4]:

- Hardening
- Soldering/Brazing
- Welding
- Refining remelting

The depth of the heat-soaked layer depends on the frequency on the basis of the following Tab. 4.4.

| Frequency *f* (kHz) | Penetration depth $\delta$ (mm) (steel 1000 °C) | The depth of the heated layer $g = (2 \text{ to } 3)\ \delta$ (mm) |
|---|---|---|
| 10 | 5.00 | 10 - 15 |
| 100 | 1.60 | 3.2 - 5 |
| 1 000 | 0.50 | 1 - 1.5 |
| 10 000 | 0.16 | 0.3 - 0.5 |
| 30 000 | 0.09 | 0.2 - 0.3 |
| 100 000 | 0.05 | 0.1 - 0.15 |

**Tab. 4.4: The depth of the heat-soaked layer depending on the frequency**

**Hardening**

For hardening, the specific power in the range of 1 to 20 kW kW·cm$^{-2}$ is used. The optimum frequency is calculated from the following relation

$$\frac{0{,}015}{d^2} < f < \frac{0{,}25}{d^2} \qquad (4.39)$$

where *d* is the depth of the hardening zone (mm), *f* is frequency (kHz)

In Fig. 4.23 and in Tab. 4.4, you can see the depth of the hardening zone *d* depending on the frequency and hardness of the steel used. The curve 1 is for *f* = 400 kHz, 2 is for *f* = 10 kHz and 3 is for *f* = 4 kHz. The energy ratios at hardening are expressed by a nomograph in Fig. 4.24. Methods of cooling of a heated surface layer at hardening are schematically shown in Fig. 4.25.

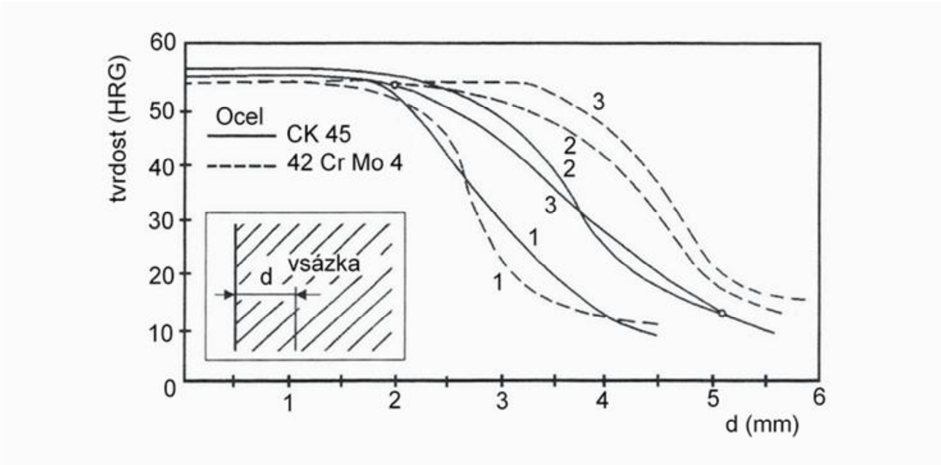


**Fig. 4.23: The depth of the hardening zone**

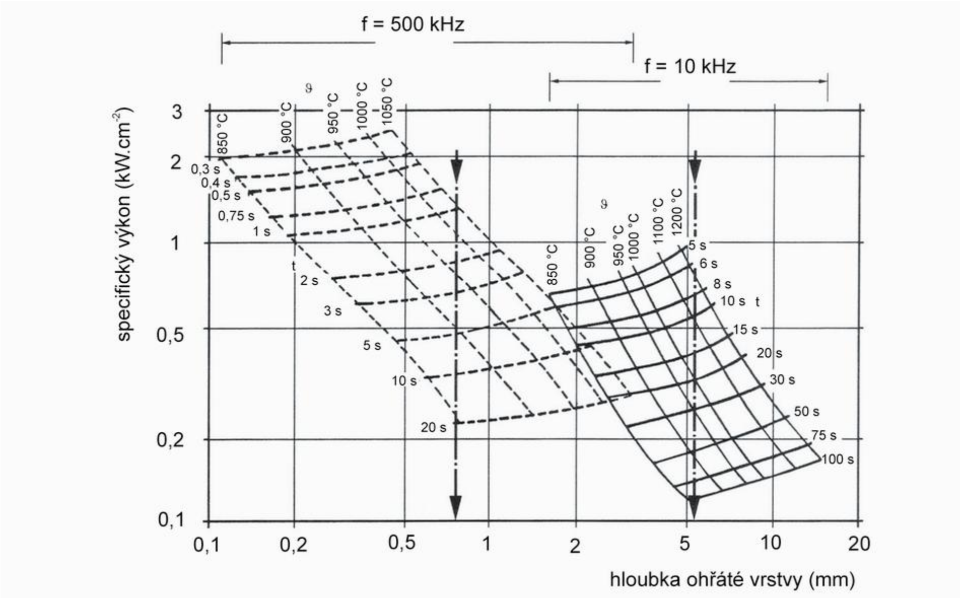


**Fig. 4.24: Frequency dependence of the heated layer during hardening**

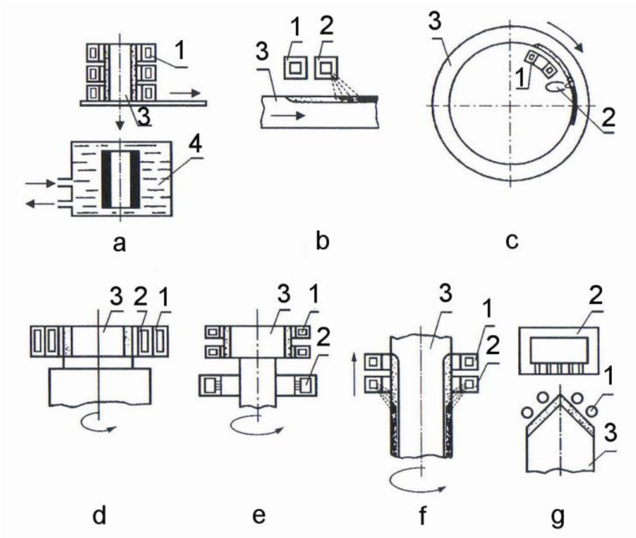

**Fig. 4.25: The methods of cooling of the heated surface layer during hardening**

In fig. 4.25, 1 is the inductor, 2 is the shower, 3 is the charge and 4 is the water tank.

**Soldering/Brazing**

For soldering, we use the frequency range from 2 kHz to 2.5 MHz. The soldering principle using a special inductor is shown in Fig. 4.26 and Fig. 4.27 a special inductor for soldering of three different shapes at the same time. Soldering uses temperatures from 150 to 450 °C and power from 0.5 to 5 kW. Brazing uses temperatures from 450 to 1050 °C and power from 3 to 30 kW. In fig. 4.26, 1 is the inductor, 2 is the solder.

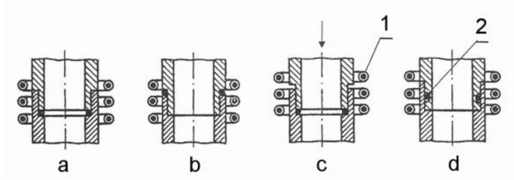


**Fig. 4.26: The principle of induction brazing**

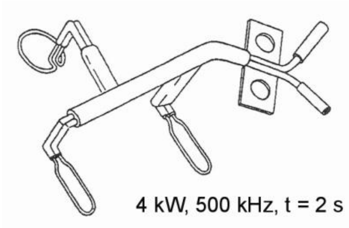


**Fig. 4.27: Special inductor for brazing of three shapes at the same time**

**Welding of tubes**

The principle of tube welding using a moving inductor is shown in Fig. 4.28. Sources with a frequency range from 8 to 500 kHz and power from 50 to 700 kW are used according to different heat-soaking depth.

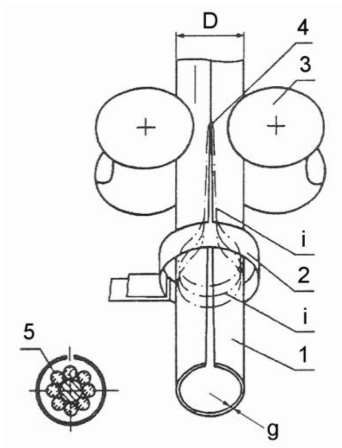


**Fig. 4.28: The principle of induction welding of tubes**

1 - tube, 2 - inductor, 3 - rollers, 4 - weld, 5 - magnetic core, i = induced current, *v* =

15 - 100 m·min$^{-1}$, $f$ = 8 - 500 kHz, $P$ = 50 - 700 kW, g = 0.4 - 12 mm, D = 8 -500 mm.

**Refining remelting**

The principle of refining remelting is shown in Fig. 4.29; 1 is the inductor and 2 is the charge (Si). Sources with a frequency range from 400 kHz to 5 MHz at power values from 10 to 50 kW are used.

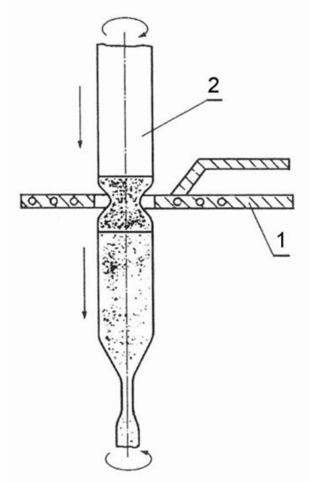


**Fig. 4.29: The principle of refining remelting**

## 4.5. Channel-type induction furnaces

**The design of a channel-type induction furnace**

The channel-type induction furnace is actually a transformer with a closed iron core and with a primary coil connected to the mains. The channel filled with melted metal is a secondary side of the transformer. Basically, it is a short coil Fig. 4.30.

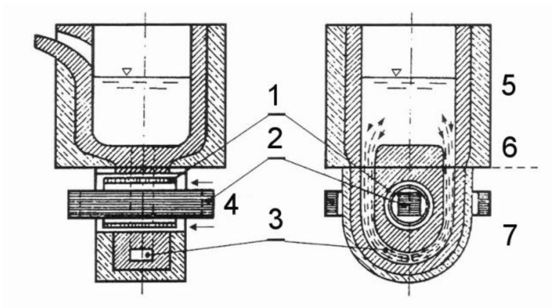


**Fig. 4.30: The channel-type induction furnace**

1 - air-cooled heating coil (indicated by the arrows), 2 - core consisting of transformer sheets of a jacket type; its middle column includes the heating coil 1,

3 - channel surrounding the heating coil as a short coil, 4 - cooling air 5 - furnace tank, 6 - dividing gap, 7 - inductor.

Electrical channel-type induction furnaces are used for melting of non-ferrous metals, especially copper and its alloys, aluminium and its alloys, or for reheating of pre-melted alloy, e.g. in the cupola furnace. While power to the furnace with an opened channel was increased, it was found out, that upon exceeding a certain critical value of current in the charge, an undesirable effect, the so-called "pinch effect" occurred. Due to electrodynamic forces acting in radial planes in all directions perpendicularly to the surface of a liquid conductor, the continuous ring of molten metal is interrupted.. At this moment, electromagnetic action ends, the ring joins again and the phenomenon repeats. Surges preventing the proper operation of furnaces occur. This phenomenon can be partly coped with using the suitable structure of the furnace coil. The solution with a covered channel recessed in the furnace bottom proved to be even better. The hydrostatic pressure of molten metal prevents the occurrence of the pinch effect significantly. About a third of the charge filling the channel and the furnace bottom is left in the furnace during casting, so that heat can develop. Then the furnace is filled with charge, which is melted by dipping into the pre-heated bath at the furnace bottom. For these furnaces, heat develops only in the charge in the channel. By action of electrodynamic pressure, the metal is continuously pushed out of the channel to the hearth and the cooler metal from the hearth flows into the channel. Thus the heat from the channel is transferred to the whole charge in the hearth. If the channel surface is vertical, the temperature difference also supports intensive penetration of the hot metal from the channel because the warmer metal is lighter.

## 4.5.1. Connection of the channel-type furnaces to the mains

**Single-phase furnaces**

Single-phase furnaces usually feature one channel. They should be connected to phase or delta voltage of the 3 x 400 V, 50 Hz mains. The single-phase furnaces are designed up to the apparent power input of 150 kVA. There are usually multiple similar furnaces in the metallurgical works. Load in all three phases is almost balanced upon their alternating connections to various phases. The connection of a single-phase induction furnace with one channel is schematically shown in Fig. 4.31.

The furnace coil features inductance $L_1$, resistance $R_1$, the charge (short coil filled with molten metal) values are $L_2$, $R_2$. After converting the charge values to the coil, we will obtain the resulting inductance $L_l$ and the resulting resistance $R_l$, diagram in Fig. 4.32.

The furnace is connected to delta voltage $U_1$. It loads the mains non-symmetrically and in addition to the active power it also takes the reactive power; the more, the lower the power factor $\cos\varphi$ is. For furnaces for melting of resistance alloys, $\cos\varphi$ = 0.8; for aluminum melting, it is about 0.3. In order to relieve the mains of the supply of the reactive power, a suitably large capacity $C$ should be parallel-connected to the coil. Taking into account Fig. 4.32, we can form the expression for the current $I_S$

from the mains ($I_C$ is the capacitor current, $I_1$ is the current in the furnace coil - losses in the capacitor should be neglected).

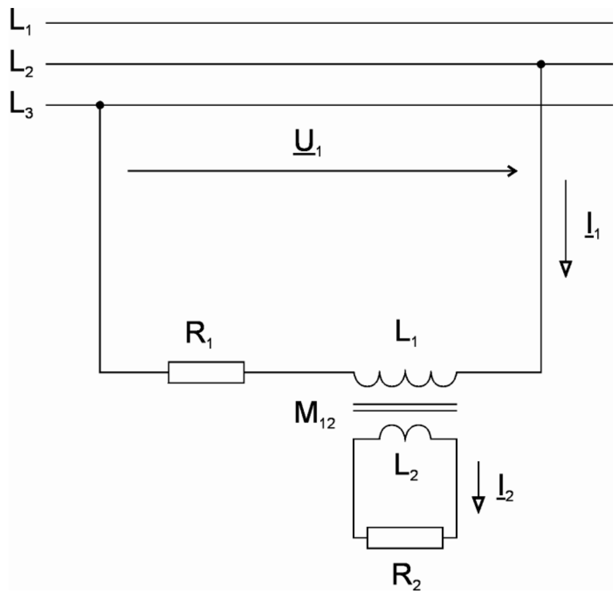


**Fig. 4.31: Connection of a single-phase induction furnace**

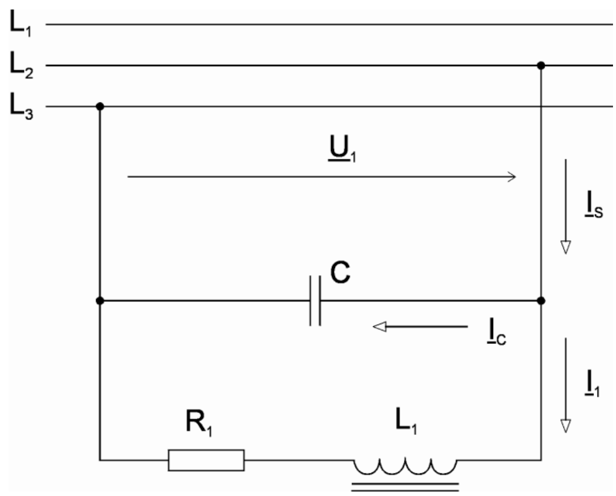


**Fig. 4.32: A diagram of a single-phase induction furnace after conversion of charge values**

$$\underline{I}_S = \underline{I}_1 + \underline{I}_C = \frac{\underline{U}_1}{R_I + j\omega L_I} + j\omega C \cdot \underline{U}_1 \qquad (4.40)$$

$$\underline{I}_S = \frac{\underline{U}_1}{R_I^{\,2} + (\omega L_I)^2} \cdot \left[ R_I + j(\omega C R_I^{\,2} + \omega C \cdot \omega^2 L_I^{\,2} - \omega L_I) \right] \qquad (4.41)$$

The required capacity for full compensation of the reactive power is based on the condition that the current $I_S$ does not have the reactive component. The oscillatory circuit ($L_I$, $C$, $R_I$) will be "fine-tuned" for the supplied frequency $f$ (Hz). The imaginary element of the expression (4.41) will be equal to zero. Out of which

$$C = \frac{L_I}{R_I^{\,2} + (\omega L_I)^2} \qquad (4.42)$$

For the channel-type furnaces, it is usually compensated to the resulting cosφ = 0.95. A part of capacitor battery is saved; the current $I_S$ from the mains is bigger than it would be at the total compensation. In some cases it is desirable to take such measures, so that the single-phase induction furnace loads all the three phases symmetrically. In particular, we connect the capacitor $C$ to the furnace in parallel.

It is evident from the expression for $I_S$ that the induction furnace draws only the active current.

$$I_S = \frac{U_1}{R_I^{\,2} + (\omega L_I)^2} \cdot R_I \qquad (4.43)$$

The fine-tuned circuit of the furnace ($L_I$, $C$, $R_I$) behaves to the mains as the active load resistance $R_Z$.

$$R_Z = \frac{U_1}{I_S} = \frac{R_I^{\,2} + (\omega L_I)^2}{R_I} \qquad (4.44)$$

Instead of the diagram in Fig. 4.32, we have obtained the diagram in Fig. 4.33.

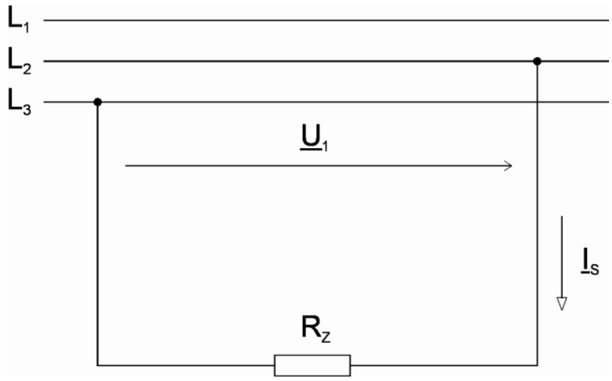


**Fig. 4.33: The equivalent scheme of the single-phase furnace**

Further, we will consider the load resistance $R_Z$ instead of the fine-tuned induction furnace. The symmetric loading of the three-phase network fine-tuned by the single phase furnace can be achieved using the so-called symmetrization device (bazooka). It is an artificial load comprising three branches. In one branch, there is resistance $R_Z$, replacing the fine-tuned induction furnace. In the second branch, the inductance $L$ has the appropriate size, and the third branch features appropriately large $C$ capacity. These three branches of the symmetrization device can be delta- or star connected. In both the cases, the accurately symmetric loading of all three phases at cosφ = 1.0 can be achieved with the correct phase sequence. In Fig. 4.34, the delta-connected symmetrization device (bazooka) is shown.

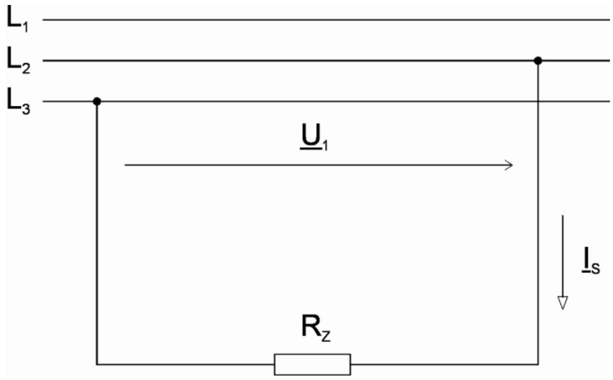


**Fig. 4.34: The delta-connected bazooka**

The following applies to the size of the required inductance $L$ and capacity $C$

$$L = \frac{\sqrt{3} \cdot R_Z}{\omega}; \; C = \frac{1}{\omega \cdot \sqrt{3} \cdot R_Z}; \; \omega L = \frac{1}{\omega C} = \sqrt{3} \cdot R_Z \qquad (4.45)$$

The symmetrization can be achieved only if the phases are in the correct sequence according to Fig. 4.34. In the diagram in Fig. 4.35 on the following page, there are currents and voltages indicated in each part of the symmetrization equipment (bazooka). The active power $P_2$ withdrawn by the furnace, and the reactive power $Q$ in the inductance $L$ or in the capacitor $C$ will be:

$$P_2 = U_2 \cdot I_2 = \frac{U_2^{\,2}}{R_Z} \qquad (4.46)$$

$$Q_2 = \frac{U_2^{\,2}}{\sqrt{3} \cdot R_Z} = \frac{P_2}{\sqrt{3}} \qquad (4.47)$$

Ammeters shall be inserted to all three supplies while the furnace with the symmetrization equipment (bazooka) is installed. At the correct sequence of phases, all the three ammeters show the same deflection. If the phases are not in the correct order, the ammeters show higher current at the phases $L_1$ and $L_3$ than at the phase $L_2$. If we exchange any two of the supplies, we will obtain the correct phase sequence. The star connection of the symmetrization equipment (bazooka) is shown in Fig. 4.36. Currents and voltages are stated in the diagram in Fig. 4.37.

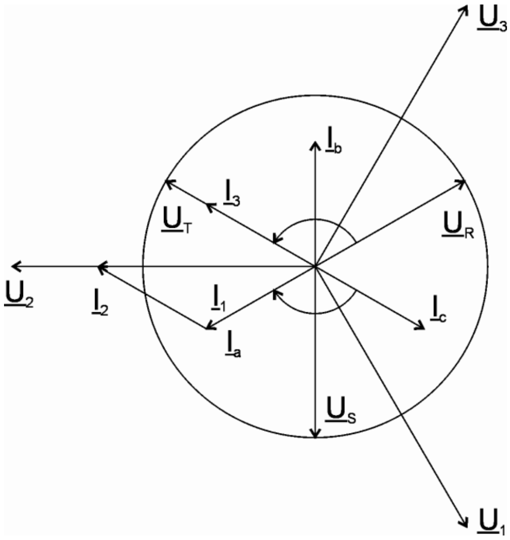


**Fig. 4.35: The phasor diagram of the symmetrization equipment (bazooka) in the delta-connection.**

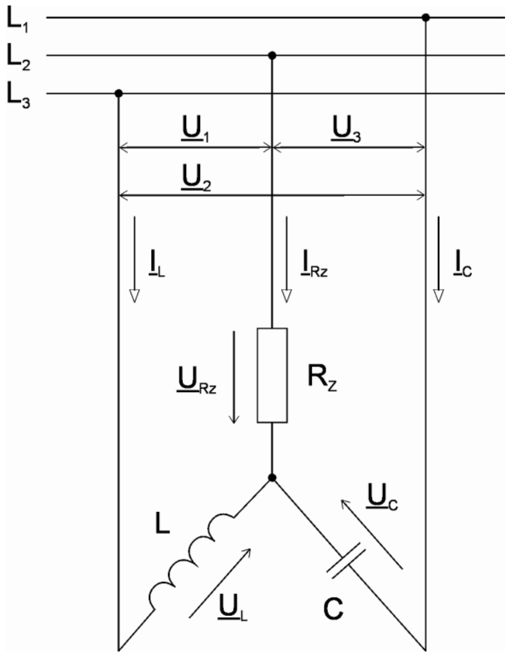


**Fig. 4.36: The star-connected bazooka.**

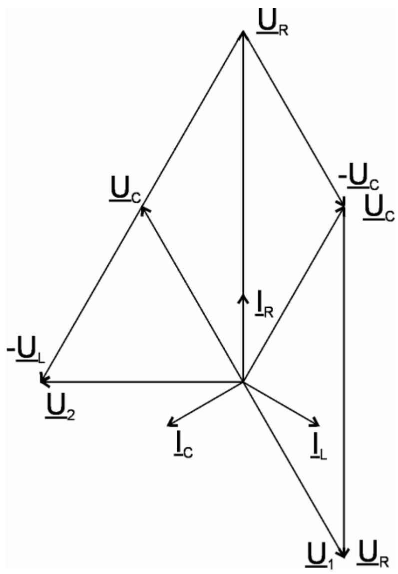


**Fig. 4.37: The phasor diagram of the star-connected bazooka**

Increased voltage $U_R$ occurs in the furnace; the voltage equals to the triple of the phase voltage, which is convenient for larger furnaces.

$$L = \frac{R_Z}{\sqrt{3} \cdot \omega}\ ;\ C = \frac{\sqrt{3}}{\omega \cdot R_Z}\ ;\ \omega L = \frac{1}{\omega C} = \sqrt{3} \cdot R_Z \qquad (4.48)$$

If we compare with the equation (4.45), we can see, that with the symmetrization star-connection, the required inductance is three times lower and that the required capacity is three times higher compared to the delta-connection - in both the cases, it is related to the load resistance $R_Z$ . Since the load resistance $R_Z$ with the star-connection is three times higher for the furnace of the same power input (the voltage in the furnace is 3.5times higher), the actual values of *L* and *C* are exactly the same in both the cases.

**Two-phase furnaces**

These are induction furnaces with two or up to four channels, out of which two are parallel-connected and they have a common furnace transformer for both the channels. The symmetric distribution to all three phases can be achieved either by the use of two furnace transformers in the Scott connection, or a special symmetrization connection can be used. In Fig. 4.38, there is a furnace with two identical channels in the Scott connection.

Each of the two channels has its own furnace transformer connected to the three-phase mains. Between the points A and B in Fig. 4.38. the so-called “main” transformer is connected ; the transformer features the primary winding divided into two identical parts. The number of turns in each part is marked $N_1/2$. An “auxiliary” transformer with the turn number of $3^{1/2}/2$ is connected between the third phase (pos. C) and the winding center of the main transformer. $N_1$. Both the channels are exactly the same. Currents withdrawn from each phase are same with this arrangement. Even the currents induced in both the channels are the same, thus also the amount of heat developed in them.

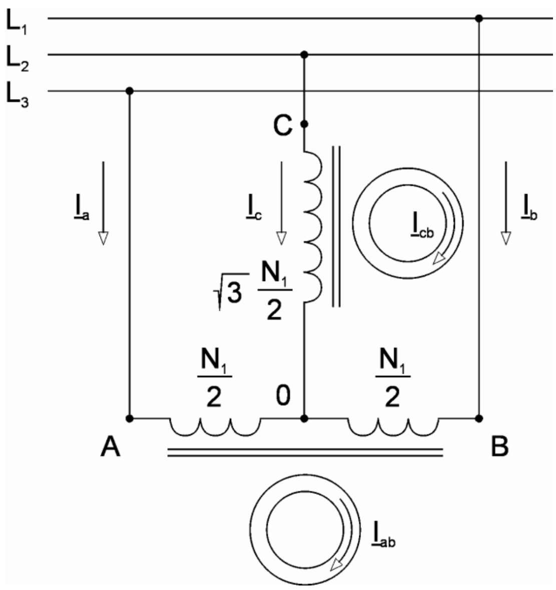


**Fig. 4.38: Connection of a two-stage induction furnace**

$$|I_a| = |I_b| = |I_c| \tag{4.49}$$

$$|I_{cb}| = |I_{ab}| = \frac{\sqrt{3}}{2} \cdot N_1 \tag{4.50}$$

We can also use the symmetrization connection according to Fig. 4.39 for the symmetrical connection of the induction two-channel furnace to the three-phase mains.

Both the furnace transformers are the same with the symmetrical connection according to Fig. 4.39. This is an advantage against the Scott connection. The symbols $L_I$, $R_I$ indicate the resulting inductance and the resulting resistance of the furnace coil in the first branch after conversion of the values of $L_2$, $R_2$ of the charge (channel) to the first circuit. Similarly $L_{III}$, $R_{III}$ in the third branch. As both the furnace transformers and both the channels are identical, the equation applies:

$$L_I = L_{III} \;\; ; \;\; R_I = R_{III} \tag{4.51}$$

First, it is necessary to “fine-tune” the circuit of each furnace by assigning of the parallel resonance capacity.

$$C_{1rez} = C_{3rez} = \frac{L_I}{R_I^{\,2} + (\omega L_I)^2} \tag{4.52}$$

The vector diagram of voltage and current is shown in Fig. 4.40.

The correct phase sequence must be followed. It can be checked during installation using three ammeters at the inlets. At the incorrect phase sequence, the $I_b$ current is two times higher than $I_a = I_c$ . To obtain the correct sequence, it is enough to interchange any two inlets.

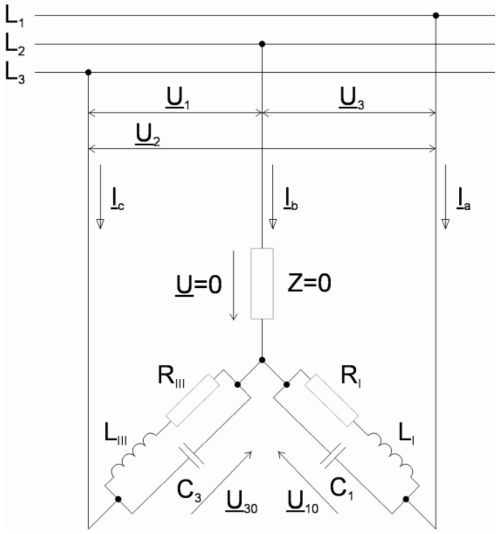


**Fig. 4.39: The symmetrization connection of a two-channel furnace to a three-phase mains.**

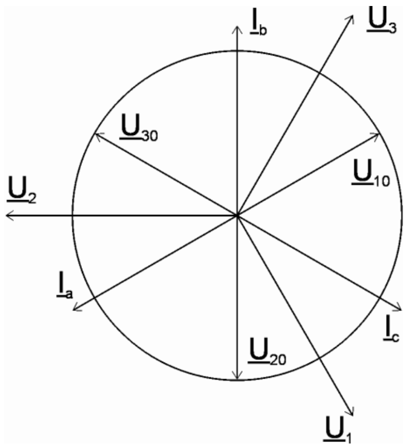


**Fig. 4.40: The phasor diagram of a two-phase induction furnace**

**Three-phase furnaces**

Three-phase furnaces with three or six channels, out of which two channels are always parallel-connected to the common core. These furnaces feature a three-phase furnace transformer, either in the core or shell designs. Each of the three cores bears a furnace coil, around which there is either one, or two identical parallel channels. All the three coils are connected to the mains; either delta-connected (the coils are for the line voltage), or star-connected (the coils are for the phase (line-to-ground) voltage). Loading of all the three phases is symmetrical; to improve cosφ, a three-phase capacitor battery must be connected. In Fig. 4.41, there is a three-phase furnace with three channels and a three-phase transformer intended for melting of non-ferrous metals. This furnace loads the three-phase network symmetrically [2].

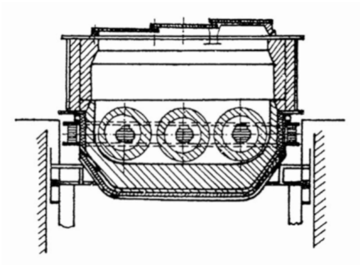

**Fig. 4.41: The three-phase furnace with three channels**

# 4.6. Electrical sources for feeding of induction furnaces

**Mains supply 3 x 400/230 V, 50 Hz**

As it has already been mentioned above, there are some situations, when induction crucible furnaces are fed with current at a frequency of 50 Hz. Similarly, induction heaters are connected to the mains 50 Hz, if they are intended for heat-soaking of cylindrical bodies with larger diameters; for steel, the diameters range from 160 up to 500 mm.

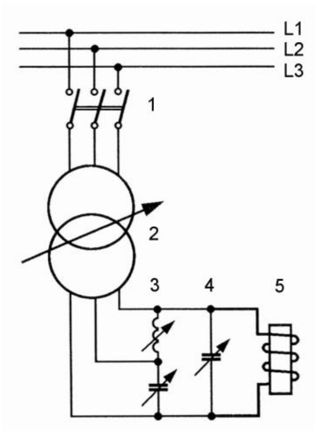


**Fig. 4.42: Connection for mains supply 3 x 400/230 V, 50 Hz.**

The principle of connection of electric circuits is shown in Fig. 4.42, where 1 is a power switch, 2 is a regulating transformer, 3 is a symmetrization circuit converting single-phase loading to three-phase symmetrical loading, 4 is a regulating capacitor battery compensating reactive power of the heating coil, 5 is induction heating.

**Medium-frequency power sources**

The connection principle is shown in Fig. 4.43, there 1 is a power switch, 2 is a frequency converter, 3 is a capacitor battery and 4 is induction heating.

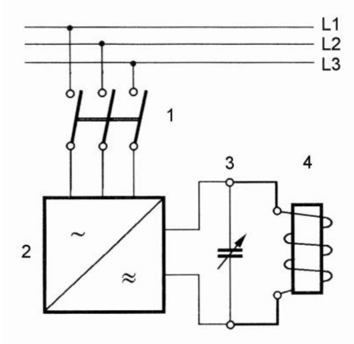


**Fig. 4.43: The connection of a medium-frequency power source.**

**Thyristor frequency converters for medium-frequency induction heating**

Rotation converters have been used as energy sources for induction heating so far; they are geared template generators driven by an asynchronous engine. Recently, static converters with thyristors have been used. The connection principle of a thyristor frequency converter is shown in Fig. 4.44.

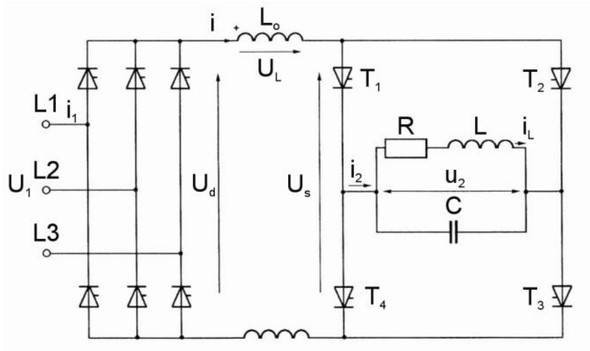


**Fig. 4.44: The connection of a thyristor frequency converter**

The thyristor converter consists of a rectifier, a choke and a current inverter. The rectifier is a fully-controlled three-phase bridge with a possibility of inverter operation. The inverter works as a current inverter. It is controlled by load, i.e. directly by the heating circuit. This oscillating circuit comprises elements of inductor coil - *R, L* and the compensation capacitor *C*. This capacitor is also intended as a commutating capacitor for thyristors of inverters $T_1$ through $T_4$. The inter-circuit choke $L_o$ features high inductance (several mH), and has the following three functions:

1) Separating - before the choke, there is oscillating circuit voltage switched through thyristors. The choke catches the immediate voltage differences between the rectifier and the inverter.

2) Smoothing - one of the tasks of the choke is to smooth direct current from the rectifier, even if it is phase-controlled.

3) Restrictive - the choke limits the increase in short-circuit current to the permissible value of cumulative current for thyristors, where there is short-circuit in the inverter (i.e. burning of the bridge); it does this until over-current protectors act. This restrictive function is decisive for the calculation of the required inductance of the choke.

The inverter conducts current through thyristors $T_1$ and $T_3$ and then through thyristors $T_2$ and $T_4$, respectively. Thus, rectangular current pulses of alternating polarity are introduced to the heating oscillation circuit *L, C, R*. The thyristors act as switches of these pulses. The rectangular waveform of the impulses is given by the large inductance $L_o$, which tries to conduct constant direct current. The oscillation circuit *L*, *C*, *R* features purely sinusoidal voltage $u_2$. The reason for this is that the capacitor *C* filters higher harmonic rectangular voltage. Also the current $i_L$ in the oscillation circuit is almost sinusoidal. The frequency of the inverter is controlled directly by load, i.e. by the oscillating heating circuit. Alternating switching of thyristor pairs $T_1$, $T_3$ and $T_2$, $T_4$, is provided by the converter regulator. The regulator also contains a device for converter start. Most often, this start is performed in the way that the large capacitor for auxiliary thyristors discharges itself to the oscillating heating circuit. Choked oscillations occur; the regulator responds to the oscillations and it starts to switch thyristors in diagonals of the bridge rhythmically. The required properties of thyristors for the inverter:

1) Short shutdown time tg (for 10 kHz under 10 ms),

2) very fast expansion of the triggering area to the entire working area of the thyristor,

3) high repeatable peak, blocking and cut-off voltage (at least 1200 V),

4) high medium forward current at the respective frequency (300 up to 600 A),

5) high value of limit integral of the square of forward current,

6) high critical increase slope of forward current,

7) high critical increase slope of blocking voltage (100 to 200 $V \cdot ms^{-1}$),

8) a low recovery charge.

The thyristors must be chosen with permanent parameters and identical, so that they can be easily removable in operation in the event of their failure. The comparison of rotation converters with new thyristor converters.

1) The thyristor converters feature about 10% higher energy efficiency.

2) The thyristor converters feature better efficiency curve course depending on the load.

3) The thyristor converters can be easily switched off during breaks at work. Thus, energy withdrawn by the rotation converters at idle operation is saved.

4) High adaptability of the thyristor converters to operational and technological requirements.

5) Operational reliability. In the event of failure, only a certain modular component is replaced.

6) Lower noise levels, lower weight (3.5times lighter).

7) They do not need any robust construction foundations.

8) The thyristor converter works at various frequencies. Fine-tuning of capacitors in the heating circuit is no longer needed. Frequency retuning is carried out through the setting of the oscillating heating circuit. The thyristor frequency converters work with parallel or serial working circuit R, L, C. With the parallel one according to Fig. 4.44.

The disadvantage consists in reverse impact on the power supply network. This disadvantage, however, can be suppressed using suppressors and filters for higher harmonic voltage.

**High-frequency tube generator above 50 kHz**

The wiring diagram for induction heating is shown in Fig. 4.45, where:

1 is a regulating input transformer for the rectifier, 2 is rectifying diodes of the rectifier, 3 is a smoothing capacitor, 4 is a separating choke for alternating high-frequency component on the anode of the power triode 5, 6 are capacitors short-circuiting the high-frequency components of the cathode current, 7 is a heater transformer of the triode 5, 8 is a separating capacitor which transmits only the alternating high-frequency components, 9 is a separating capacitor for direct-current grid bias of the triode 5, which, however, allows flow of alternating high-frequency currents of feedback, 10 is a choke preventing short-circuit of the high-frequency feedback grit voltage by a capacitor 12, 11 is a resistor, on which negative grit bias of the triode 5 occurs, 12 is a filtering capacitor of the grit bias, 13 is a primary winding of the high-frequency transformer, 14 is a secondary winding of the high-frequency transformer, 15 is a compensation capacitor for the heating coil (inductor) 16. Winding inductances 13, 14, 16 form, together with the capacitor 15, an oscillation circuit, which works at the specific high frequency, for example, 360 kHz. The oscillator feedback is made through the branch 13 and conducted to the grit of the triode 5.

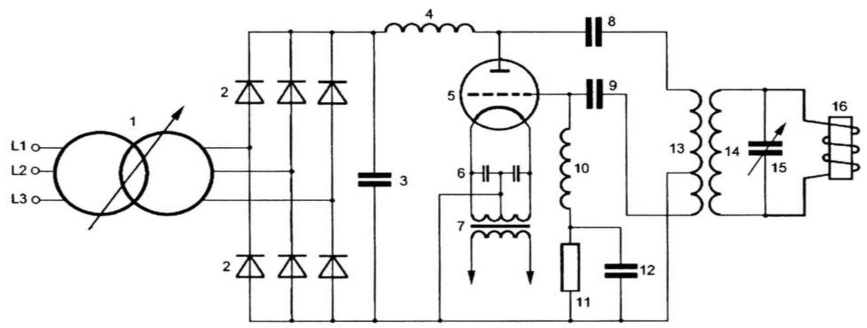


**Fig. 4.45 The connection of the high-frequency tube generator over 50 kHz.**

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# 5. Dielectric Electro-Thermal Equipment and Microwave Heating

## 5.1. Dielectric Electro-Thermal Equipment

Dielectric electro-thermal equipment is designed for heating of non-conductive materials and semiconductors using a high-frequency electromagnetic field. Dielectric electro-thermal equipment is analogical to the induction equipment. Theoretical basics of induction as well as dielectric equipments significantly make use of common findings from electrodynamics resulting from Maxwell equations. Both the kinds of equipment feature a common valuable property, i.e. heat during heating develops directly in the charge.

In dielectric electro-thermal equipment, heat is released using electric polarization phenomena in the real dielectric in the high-frequency electromagnetic field.

### 5.1.1. Physical principle of polarization

Electrically non-conductive substances of various physical states are called dielectrics (insulators). Ideal dielectrics do not contain any free carriers of electric charge, they do not conduct electric current and their conductivity equals to zero. Real dielectrics always feature a low amount of free charges, therefore, they are conductive, although very low. Other charge carriers are bound by internal forces to individual atoms and molecules and under common circumstances, they take positions corresponding to equilibrium states.

The physical principle of dielectric heating is related to the electric polarization of the bound charge carriers in the electromagnetic field. If we insert a dielectric into the outer electric field, the equilibrium state of the charge carriers will be violated. By the effect of field forces, the positive carriers transfer themselves in the direction of the field, the negative ones in the opposite direction. The respective substance, although it contains the same amount of positive and negative charges, sometimes shows a preponderance of a positive or a negative charge. An electric dipole has been created; it is a set of charge carriers of the same size, but the opposite orientation. The electric dipole is characterized by an electric dipole moment *p*

$$\boldsymbol{p} = q\,\boldsymbol{l} \tag{5.1}$$

and it is a measure of the asymmetry of the spatial layout of carriers in the dipole, i.e. a pair of charges $+q$ a $-q$ in the distance of $l$.

Some materials, the so-called polar dielectrics, contain molecules (electric dipoles) arranged in the way that one part of the molecule carries a positive charge, whereas the other part carries a negative charge. These molecules feature the asymmetry of layout of charge carriers even without acting of forces of the external field; therefore, they have permanent (continuous) electric dipole moments and their orientation is random. If they are located in an electric field, their electric dipoles turn to the direction of the field. Only the partial arrangement of dipoles occurs because the molecules collide with each other continuously. The complete arrangement is prevented by a chaotic (thermal) movement. As to non-polar molecules and also in each atom, the centers of all positive charges merges with the centers of all negative charges, therefore the permanent dipole moment of such molecules and atoms is zero.

Regardless of the fact whether dielectrics feature the permanent dipole moments, or not, the atoms and molecules placed in the external electric field obtain induced (temporary) dipole moments. Since the electrons bear a negative charge, they move against the direction of the external electric field intensity vector. Thus a dipole develops; the dipole moment of the dipole features the same direction as the external electric field. Thus asymmetry of layout of charge carriers develops (separation of areas of the positive and negative charges of the molecule); thus the induced dipole moment ceases to exist after removal of this field.

Generally, these phenomena are called electric polarization of dielectrics; it is either orienting (for dipoles) or deformation (for atoms, molecules and ions). Various dielectrics feature various abilities to polarize. This feature is expressed by polarizability $\alpha$ and its unit is $F \cdot m^2$. The polarizability expresses the ratio of the size of the induced dipole moment of the molecule (atom, ion) to the intensity of the polarizing field , $\alpha \sim p/E$. [2], [4], [5], [6]

### 5.1.2. The principle of dielectric heating

Movement of free charge carriers and movement of bound charge carriers (dipoles) in the dielectric which is placed in the electric field is conditioned by its polarity and differs based on the nature of the carriers. The movement of free charge carriers (conductance current) stops after the field ceases to exist, the charge carriers return to their original position. The movement of the bound charge carriers (displacement current) is related to polarization; after the demise of the field, the dipoles return to their original position, however with some delay.

Any movement of free or bound charge carriers of a dielectric due to the external electric field is connected with consumption of the energy of this field and its transformation in the form of heat. The amount of generated heat is expressed by dielectric losses, which are ones of basic parameters of dielectric materials. According to the nature of the physical process of heat generation, the dielectric losses may be either conductive (movement of free charge carriers, they feature a nature of Joule losses), polarizing (they relate to the bound charges and dipoles) and ionizing (the result of impact ionization). The polarizing losses have the greatest importance at the conversion of electrical energy to useful heat in solid dielectrics.

If we insert a dielectric between electrodes of a plate condenser and if we connect electrodes to variable voltage $U$, polarization of the dielectric occurs. The dipoles try to follow the changes of electric field polarity, they try to turn. If the bound charge carriers follow the changes of the alternating electric field without any delay, thus the movement is not related with a transformation of the external field to heat. Should it be this case, polarization dielectric losses equal to zero. But if the internal structure of the dielectric causes delay in the movement of charge carriers (dipoles) with changes to the field intensity, mutual friction occurs; this movement is related to generating of dielectric losses. The reason for the delay is either in the resistance of the environment, or it is caused by its actual weight. In this event, the energy of the field changes to heat and the dielectric heats up. If the polarization dielectric losses are non-zero, the polarization is called relaxation (heat) polarization. Polarization losses are manifested only in the alternating electric field and they increase with growing field frequency only after certain maximum value, which is called the relaxation field frequency.

The degree of dielectric polarization depends on the intensity of the external electric field, into which a dielectric has been inserted. Upon exceeding of a certain intensity of the electric field, a dielectric breakdown may occur. As a result, a conductive path between electrodes in the dielectric occurs; large currents can pass through this path. The dielectric becomes conductive, it loses its insulating properties and it is damaged. The maximum intensity of the electric field, at which no breakdown occurs, is called the "electrical strength of the dielectric". The electrical strength of the dielectric is not constant; it depends on temperature, thickness of the dielectric, its size, shapes of the electrodes, etc., and it is usually measured for dielectric materials in $kV \cdot mm^{-1}$ at 50 Hz.

If a harmoniously varying field with the intensity $\boldsymbol{E}$ and the angular frequency $\omega = 2\pi f$ acts on the dielectric, the field induces current composed of conductivity and displacement components in it. The density phasor of the resulting current is determined by the sum of phasors of both the components, i.e. of the density of the conductive current and the density of the displacement current.

$$\boldsymbol{J}_{\mathrm{c}} = \boldsymbol{J}_{\mathrm{v}} + \boldsymbol{J}_{\mathrm{p}} = \gamma \boldsymbol{E} + \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} = \left(\gamma + j\omega\varepsilon\right)\boldsymbol{E}. \tag{5.2}$$

In the relation, $\gamma$ is conductivity of the dielectric and $\varepsilon$ is a dielectric constant, dielectric permittivity, $\varepsilon = \varepsilon_0 \varepsilon_r$ ($\varepsilon_0$ is vacuum permittivity, $\varepsilon_r$ is relative permittivity of the dielectric). The relative permittivity is an important property of the dielectric and it is a measure of polarizability of bound charge carriers in the dielectric. It expresses the ability of the dielectric to polarize itself. The relative permittivity of linear dielectrics is independent of the field intensity, but it is inversely proportional to the field frequency.

The phasor diagram of currents in the dielectric, which is placed in the electric field with sufficiently high angular speed, is shown in Fig. 5.1, where $I_{\mathrm{v}}$ is conductivity current and $I_{\mathrm{p}}$ is polarizing (displacement) current.

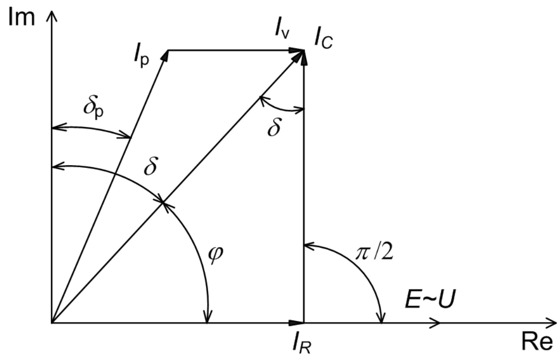


**Fig. 5.1: The phasor diagram of currents in the dielectric.**

Provided that the dielectric subject to heating is homogeneous and the electric field acting in it is homogeneous as well, then the volume density of the heat flux, i.e. the heat flux (the amount of heat generated per time unit) in unit volume, of the dielectric subject to heating $q_z$ (W·m$^{-3}$) can be expressed by the following equation:

$$q_z = \gamma_{\mathrm{ef}} E_{\mathrm{ef}}^2 = \omega \varepsilon \,\mathrm{tg}\delta\, E_{\mathrm{ef}}^2 . \qquad (5.3)$$

The volume density of the heat flux depends on the parameters of the electromagnetic field and material properties of the dielectric. The parameters of the acting electromagnetic field include the intensity of the electric field $E$ and the angular frequency $\omega$. As to the source it can be concluded that the thermal power depends on its voltage and frequency. The material properties of the dielectric are expressed by relative permittivity $\varepsilon_r$ and a loss factor of the dielectric agent tg$\delta$.

The increase in the heat flux volume density $q_z$ by the increase in voltage on capacitor plates is limited by the possible dielectric breakdown. The loss factor of dielectric materials is very low (tg $\delta$ << 1), therefore the thermal power can be increased only by the increase in frequency of the source. Therefore the frequency of dielectric heatings is high, in the orders of MHz, and it can vary between 1 and 300 MHz [3], [4], [7].

## 5.1.3. Equivalent diagram of the plate capacitor

The working element of dielectric equipments is usually a plate capacitor. The principal depicting of a plate condenser with round plates and its equivalent electric diagram and a phasor diagram of currents are shown in Fig. 5.2. In the ideal capacitor, current is ahead of voltage by 90 °. The real capacitor, however, features a lower phase shift due to losses in the capacitor. The real condenser, the plate condenser with losses, can be replaced by a parallel connection of a condenser and a resistor. The capacitor with the capacity $C$ is an ideal capacitor without losses and the resistance $R$ represents losses of the capacitor and its size corresponds to the losses of the real condenser.

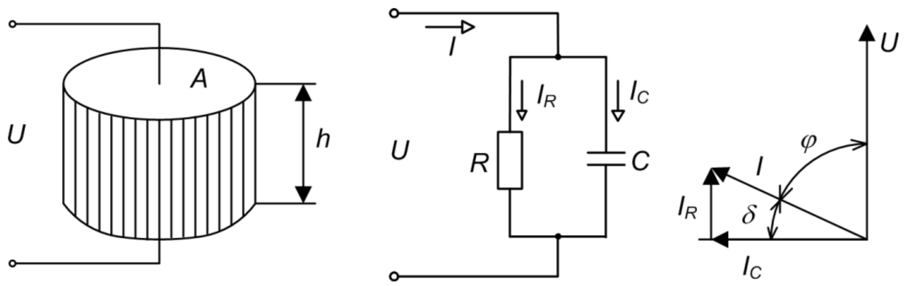


**Fig. 5.2: The plate condenser, the phasor diagram of currents and the equivalent diagram .**

A dielectric with relative permittivity of $\varepsilon_r$ is inserted into the space between the electrodes of the plate capacitor with a thickness of *h* and area of *A,* which is charged with a charge *q*. The relative permittivity $\varepsilon_r$ expresses a drop in the intensity of the force field, i.e. it determines, how many times the action of force *E* drops to a unit charge in the respective dielectric against the action of force $E_0$ to a unit charge in vacuum, $\varepsilon_r = E_0/E$, provided that the density of the electric flow is the same. After inserting of the dielectric, the force action drops to the unit charge $\varepsilon_r$-times, the *q* charge remains unchanged, but voltage *U* drops $\varepsilon_r$-times and the capacity of the *C* condenser rises $\varepsilon_r$ -times. The above-mentioned conclusions on the intensity of the electric field, voltage and capacity of the plate capacitor are applied from the following relations, which are extended by a relation for resistance of the equivalent circuit *R*.

$$E = \frac{E_0}{\varepsilon_r} = \frac{D}{\varepsilon_0 \varepsilon_r}, \; U = Eh = \frac{E_0}{\varepsilon_r} h, \qquad (5.4)$$
$$C = \varepsilon_0 \varepsilon_r \frac{A}{h}, \; R = \frac{1}{\gamma_{\text{ef}}} \frac{h}{A}.$$

In the phasor diagram of currents (Fig. 5.2), the current of the real capacitor *I* is shown; the current is ahead of voltage by the angle $\varphi < 90°$. The angle $\delta$ is known as a loss angle, $\delta = \pi/2 - \varphi$, and it indicates the loss component of the capacitor. It is an angle, by which the current *I* is ahead of the voltage *U* less than it applies for the ideal capacitor. The tangent of this loss angle is an important parameter of the dielectric which is referred to as the loss factor, and it can be expressed from the phasor diagram of currents as the ratio of currents on the resistor $I_R$ and on the capacitor $I_C$.

$$\text{tg}\delta = \frac{I_R}{I_C} = \frac{U/R}{U\omega C} = \frac{1}{R\omega C} = \frac{\gamma_{\text{ef}}}{\omega \varepsilon_0 \varepsilon_r}. \qquad (5.5)$$

In the above-mentioned relation, conductivity $\gamma_{\text{ef}}$ is the total conductivity of the real dielectric and it includes a component of conductivity currents $I_v$ and the component of displacement currents $I_P$ (Fig. 5.1) due to polarization of the bound charge carriers. The following applies to the conductivity of the dielectric.

$$\gamma_{\mathrm{ef}} = \omega\varepsilon_0\varepsilon_r \mathrm{tg}\delta = \frac{\varepsilon_0\varepsilon_r}{RC}. \qquad (5.6)$$

| Material | Relative permittivity $\varepsilon_r$ (-) | Figure of loss $\varepsilon_r \mathrm{tg}\delta \cdot 10^2$ (-) |
|---|---|---|
| paper | 1.5-2.5 | 1.9-5 |
| rubber | 2.5-4.0 | 1.5-16 |
| wood with moisture of 8-10 % | 2-4.5 | 3-45 |
| wood with moisture of 12-18 % | 5-10 | 30-150 |
| neoprene | 4-5.5 | 40-110 |
| porcelain | 4-7 | 2.8-5.5 |
| polystyrene | 2.5-2.6 | 0.02-0.08 |

**Tab. 5.1 Relative permittivity values and figures of loss of some materials**

The dielectric for specific electric ratios is characterized by the figure of loss $\varepsilon_r \cdot \mathrm{tg}\delta$,, i.e. the product of the relative permittivity of the dielectric $\varepsilon_r$ and the figure of loss $\mathrm{tg}\delta$. The loss factor of the dielectric $\mathrm{tg}\delta$ depends indirectly on the frequency of the source and it depends exclusively on its dipole polarization. The loss factor usually grows with the frequency and at the specific frequency (relaxation frequency) it reaches its maximum; with the following increase in frequency it drops. It is due to the fact that the conductivity $\gamma_{\mathrm{ef}}$ grows with frequency (its part depends on polarization), but the relative permittivity also drops $\varepsilon_r$ because of the fact that the polarization charge becomes weaker (dipoles are not enough to turn). In addition, $\varepsilon_r$ and $\mathrm{tg}\delta$ depend on temperature, moisture and other factors. Material properties, relative permittivity values and figures of loss of some base materials are listed in tab. 5.1.

If the dielectric is homogeneous and if the acting electric field is also homogeneous, the relation according to (5.3) applies to the heat flux (power)

$$\Phi = P_{\mathrm{t}} = q_z V = \gamma_{\mathrm{ef}} E_{\mathrm{ef}}^2 A h = \gamma_{\mathrm{ef}} U_{\mathrm{ef}}^2 \frac{A}{h}. \qquad (5.7)$$

If we substitute the conductivity from the relation (5.6), the heat flux will be:

$$\Phi = \omega\varepsilon_0\varepsilon_r \mathrm{tg}\delta U_{\mathrm{ef}}^2 \frac{A}{h} = \frac{\varepsilon_0\varepsilon_r}{RC} U_{\mathrm{ef}}^2 \frac{A}{h}. \qquad (5.8)$$

This heat flux is, actually, active (loss) power of the capacitor. For the reactive power of the capacitor, the following relation applies.

$$Q = \omega C U_{\mathrm{ef}}^2 = \omega \varepsilon_0 \varepsilon_r U_{\mathrm{ef}}^2 \frac{A}{h} = \omega \varepsilon_0 \varepsilon_r E_{\mathrm{ef}}^2 A h. \tag{5.9}$$

The heat flux depends on the parameters of the electromagnetic field and on the material properties and it can be increased, in particular, by a higher frequency of the source with regard to possible breakdown of the dielectric [1], [7], [8].

### 5.1.4. Heterogeneous dielectrics

In practical application of the dielectric heating, material composed of multiple dielectric materials is often heated up (so-called heterogeneous dielectrics). If we assume that we have two different dielectrics with various properties (relative permittivity, loss factor), then two situations can occur. The dielectrics are connected crosswise or lengthwise (Fig. 5.3).

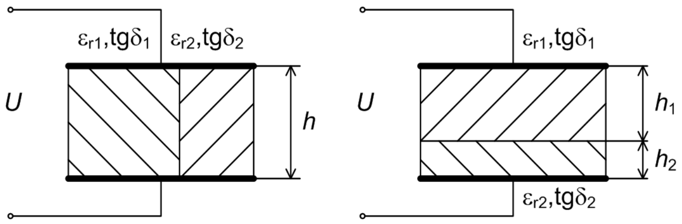


**Fig. 5.3: The crosswise and lengthwise connection of dielectrics**

The crosswise and lengthwise connections are actually parallel and series connection of dielectrics and in the equivalent wiring diagram, the dielectrics are depicted by the parallel combination of the capacitor and resistor (Fig. 5.4).

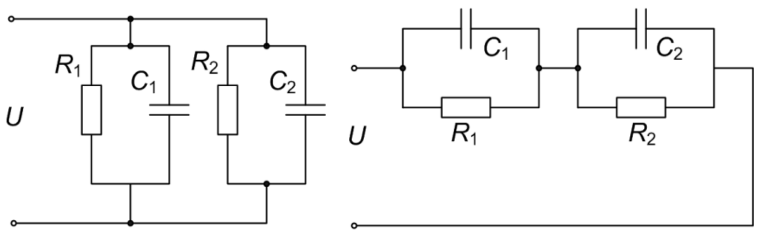


**Fig. 5.4: The equivalent wiring diagram of heterogeneous dielectrics.**

For the crosswise (parallel) connected dielectrics, the dielectrics feature various surfaces, but their thickness values are identical ($A_1 \neq A_2$, $h_1 = h_2$). In both the dielectrics, there is identical intensity of the electric field $E = U/h$. However, conductivity of each dielectric is different. Volume density values of the heat flux (heat fluxes in the volume unit) are according to (5.3) defined as follows:

$$q_{z1} = \gamma_{ef,1} E_{ef}^2 = \omega\varepsilon_0\varepsilon_{r1} \mathrm{tg}\delta_1 E_{ef}^2,$$
$$q_{z2} = \gamma_{ef,2} E_{ef}^2 = \omega\varepsilon_0\varepsilon_{r2} \mathrm{tg}\delta_2 E_{ef}^2. \quad (5.10)$$

The ratio of the volume density values of heat fluxes is given for this connection of dielectrics by the ratio of the figures of loss, i.e. the volume density of heat fluxes are divided in the heated dielectrics in the ratio of figures of loss.

$$\frac{q_{z1}}{q_{z2}} = \frac{\varepsilon_{r1}\mathrm{tg}\delta_1}{\varepsilon_{r2}\mathrm{tg}\delta_2}. \quad (5.11)$$

The total capacity of crosswise connected dielectrics is given by the sum of partial capacities, and the total resistance is given by parallel connection of two resistors.

A different situation occurs in the lengthwise (serial) connection of dielectrics. The dielectrics feature the same surface($A_1$ = $A_2$), but the thickness values of each dielectric layer are different ($h_1 \neq h_2$), therefore intensities of the electric field of both the dielectrics are different. The ratio of volume densities of heat fluxes is then:

$$\frac{q_{z1}}{q_{z2}} = \frac{\varepsilon_{r1}\mathrm{tg}\delta_1}{\varepsilon_{r2}\mathrm{tg}\delta_2}\frac{E_{ef,1}^2}{E_{ef,2}^2} = \frac{\varepsilon_{r1}\mathrm{tg}\delta_1}{\varepsilon_{r2}\mathrm{tg}\delta_2}\frac{U_{ef,1}^2}{U_{ef,2}^2}\frac{h_2^2}{h_1^2}. \quad (5.12)$$

From the above-mentioned relation we can see that the volume densities of the heat flux are divided in the ratio of figures of losses, in the ratio of voltage squares of individual dielectrics and the reverse ratio of squares and their thickness values.

We will express intensities of the electric field for each dielectric. The voltage on the capacitor is given by the relation:

$$U_{ef} = U_{ef,1} + U_{ef,2} = h_1 E_{ef,1} + h_2 E_{ef,2}. \quad (5.13)$$

Due to the fact that the longitudinal connections of dielectrics equal to electric inductions ($D_1$ = $D_2$), it can be said that:

$$D_1 = D_2 = \varepsilon_0\varepsilon_{r1}E_{ef,1} = \varepsilon_0\varepsilon_{r2}E_{ef,2}, \quad (5.14)$$

and then:

$$U_{ef} = h_1 E_{ef,1} + h_2\frac{\varepsilon_{r1}}{\varepsilon_{r2}}E_{ef,1}. \quad (5.15)$$

From there we can express the electric field intensity for each dielectric which depends on the voltage value, the thickness of dielectrics and permittivity of each dielectric.

$$E_{\mathrm{ef},1} = \frac{U_{\mathrm{ef}}}{h_1 + \frac{\varepsilon_{\mathrm{r}1}}{\varepsilon_{\mathrm{r}2}} h_2}, \quad E_{\mathrm{ef},2} = \frac{U_{\mathrm{ef}}}{h_2 + \frac{\varepsilon_{\mathrm{r}2}}{\varepsilon_{\mathrm{r}1}} h_1}. \tag{5.16}$$

The ratio of volume density values of heat fluxes for longitudinal connection of dielectrics is then expressed only by dependence on material properties, relative permittivity and the loss factor.

$$\frac{q_{z1}}{q_{z2}} = \frac{\varepsilon_{\mathrm{r}1} \mathrm{tg}\delta_1}{\varepsilon_{\mathrm{r}2} \mathrm{tg}\delta_2} \frac{\left( h_2 + \frac{\varepsilon_{\mathrm{r}2}}{\varepsilon_{\mathrm{r}1}} h_1 \right)^2}{\left( h_1 + \frac{\varepsilon_{\mathrm{r}1}}{\varepsilon_{\mathrm{r}2}} h_2 \right)^2} = \frac{\varepsilon_{r2} \mathrm{tg}\delta_1}{\varepsilon_{\mathrm{r}1} \mathrm{tg}\delta_2}. \tag{5.17}$$

## 5.1.5. Application of dielectric heating

Dielectric heating is applied mainly in heat processing of dielectric materials. This way of heating achieves uniform and precise heat-soaking of the product, thus increasing its quality. Dielectric heating is a technology with a high-frequency heating way, which is equipped with automatic control system. Thus its operation is easier. The advantage also consists in the fact that much thicker layers than with other types of heating can be heated up. Repairs to equipment for dielectric heating are less time consuming compared to repairs to conventional equipment. Less production capacity is required for them. However, it must be noted that the investment costs for dielectric equipment are relatively high.

The main applications include:

- plywood manufacturing
- drying of wood, paper and textiles,
- pre-heating of plastics, thermoset processing,
- welding of plastics,
- heating in the food industry.

In production or pressing of plywood, beech veneers are coated with artificial resin and they are inserted into the press in the layer of up to 40 cm high. An electrode of high-frequency voltage $U_{\mathrm{ef}}$ shall be inserted into the medium layer. Press jaws are the other electrodes which are grounded. The heating power is set with regard to the time which is necessary for the heating cycle. The required power can be achieved by changing supply voltage or changing the frequency. The power of the equipment depends on the thickness of the material which has been inserted between the electrodes. The electrodes of the working capacitor are intended not only as the energy supply and for shaping of the electric field, but they also have the function of maintaining of the joined parts in the required position and/or transfer the required pressure of the press onto the joints. Dielectric devices for plywood manufacturing can feature the power of up to 1600 kW.

Drying is a technological process, during which moisture is reduced or fluid in the material is completely eliminated, e.g. with textiles, paper products, timber products, etc. With dielectric drying, moisture from the inside is uniformly reduced in the entire volume of the charge. On the other hand, with common drying, materials are dried gradually from the top layers towards the inside of the material. The design is similar to plywood manufacturing, the material is inserted between the electrodes on the transport belts. The frequency used is usually about 10 MHz. Powers of paper driers vary between 200-1000 kW and their efficiency is about 70 %. Wood dielectric dryers are operated in the periodical or continuous mode. In particular, it is given by the fact that wood products show tendencies to cracking, therefore the power concentration must limited or power is supplied to wood in impulses. Drying of 1 $m^3$ of wood requires 150-500 kWh.

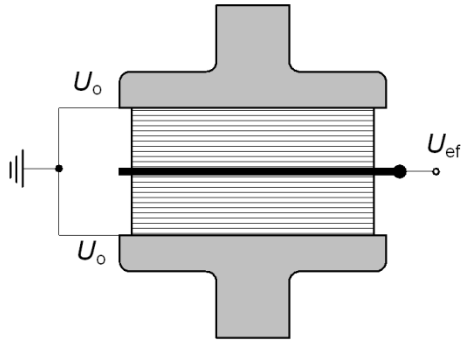


**Fig. 5.5: A diagram of equipment for plywood manufacturing.**

For processing of thermosets, dielectric heating can be used with an advantage as these materials are dielectric. Thermoset powder is pre-pressed into the shape of tablets which are then inserted into the form in the shape of a capacitor (Fig. 5.6). Preheating time takes about 1 min., the actual curing (polymerization) occurs after a longer period of time. Polymerization temperature ranges about 100 °C and it depends on the type of the filler. In the dielectric equipment, the pre-heated tablets are inserted into the pressing die. Both the parts are electrically heated and the temperature rises to about 140 °C, when polymerization occurs. If the thermoset is pre-pressed and dielectrically pre-heated, the total time for manufacture of a pressed piece is double or triple. Thus the required pressure for pressing is decreased and the quality of the pressed piece is higher. Energetically this solution is a significant saving of electrical energy.

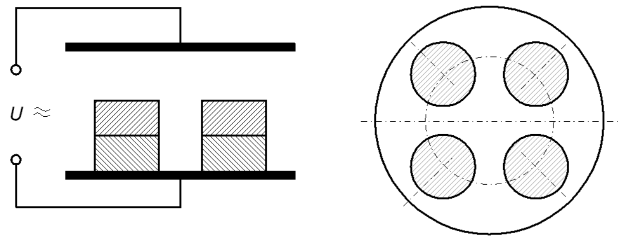


**Fig. 5.6: Pre-heating of thermoset tablets.**

Nowadays, dielectric (high-frequency) welding is wide-spread technology in various industries; in particular, it is used in the automotive industry, both in the flat and 3D variants. It is used for connection of various plastics, e.g. PVC, polyamide, etc. Dielectric welding of plastics creates a homogeneous weld with the maximum strength. Its great advantage consists in welding speed.

During dielectric welding of plastic films (Fig. 5.7), thermoplastic films are inserted between two metal electrodes with the required shape of joint; the electrodes are connected to high-frequency voltage. Heat develops in the material between the electrodes until the phase transformation of plastics. The outer part of the material is cooled by the touch of electrodes but the internal part of the material reaches the temperature about 300 °C in a few seconds. This temperature is sufficient for the films to melt. Material sintering occurs and the required joint is created. This working procedure of welding is stepwise, working in an intermittent periodic mode. In the course of one welding operation, the entire or a part of the required joint is made.

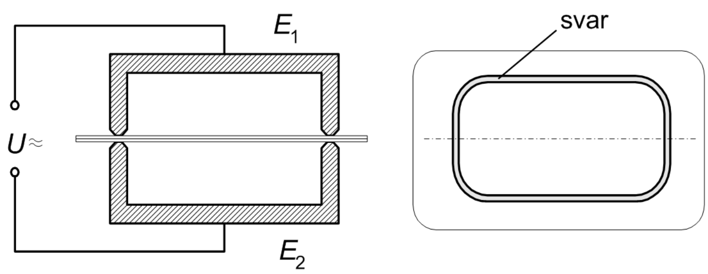


**Fig. 5.7: Dielectric welding of plastic films**

Another type of dielectric welding is described in Fig. 5.8 and it is suitable for continuous operation. Welding of such a type is carried out continuously in a way that a three-metal roller fitted in the handle is rolled along the seam of the material subject to welding (an electrode in the shape of a roller); the roller can be manually held. A flexible cable supplies high-frequency voltage from the generator with power of 100 W and frequency of 100 ÷ 300 MHz to the handle. The other electrode 2 can also have a cylindrical shape or a shape of a moving belt. In practice, the described simple equipment is getting the shape similar to a sewing machine (high-frequency sewing). Instead of a needle, a roller is placed. Using a foot pedal, the roller is lowered onto the material and simultaneously high-frequency energy is introduced. The connected material is moved at an appropriate speed, thus creating the required weld.

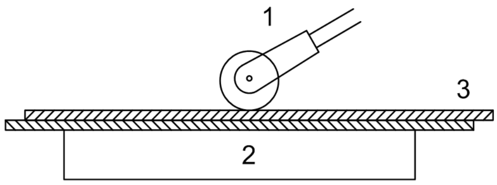


**Fig. 5.8: Dielectric welding**

Some complications with welding may occur, if an air gap (Fig. 5.9) occurs between dielectric materials and/or between the electrode and the material, e.g. due to imperfect pressing of both materials or due to manufacturing defect and/or damages to the electrodes. According to relations derived in chapter 5.1.4, it can be said, that the voltage $U_{ef}$ supplied to the electrodes is divided in an indirect proportion of permittivities. According to (5.16), the following can be said as to the intensity of the electric field of material $E$ and air $E_{vz}$

$$E_{vz} = U_{ef} \frac{\varepsilon_r}{\varepsilon_{vz} h + \varepsilon_r h_{vz}}, \quad E = U_{ef} \frac{\varepsilon_{vz}}{\varepsilon_{vz} h + \varepsilon_r h_{vz}}, \tag{5.18}$$

where $h$, $\varepsilon_r$ is the thickness and permittivity of the dielectric, $h_{vz}$, $\varepsilon_{vz}$ is the thickness and permittivity of the air gap. Due to the fact that the relative permittivity of air is approximately equal to one ($\varepsilon_{vz}$ = 1), the intensity of the electric field, i.e. also the voltage stress, in the air gap will be several times higher than the actual dielectric material. With respect to the fact that the electrical strength of the air is lower than the electrical strength of dielectric materials, e.g. 2 kV·mm$^{-1}$ for dry air and 30-50 kV·mm$^{-1}$ for polystyrene ($\varepsilon_r$ = 4-7), breakdown of the air gap is most likely to occur. Subsequently, breakdown of the actual material due to heat occurs, while an arc can be formed. Electrical strength of various materials commonly used for dielectric welding is mentioned in Tab. 5.2.

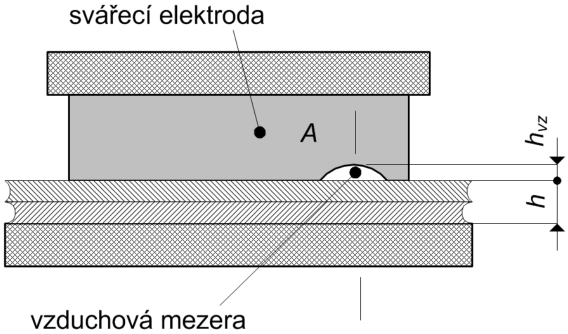


**Fig. 5.9: Air gap during welding**

Another danger of breakdown of materials subject to welding can be caused by the increase in the intensity of the electric field on sharp edges, mainly at the edges of electrodes, on the blades and tips. The same danger arises if there are any impurities in the material subject to welding.

Dielectric heating is applied in the food industry for defrosting of foodstuff, for bread baking and also for tinning of foodstuff and its pasteurization and sterilization. The disadvantage of dielectric bread baking consists in the fact, that there will be no crust on its surface. Therefore the dielectric furnace is combined with an infrared furnace at the stage of final baking. For such combined furnaces, the baking time is reduced by half compared to the common way of baking. [7], [9], [10]

| **Material** | $E_p$ **(kV·mm$^{-1}$)** |
|---|---|
| plasticized PVC | 20-30 |
| non-plasticized PVC | 40-50 |
| polyamide | 25-40 |
| polyethylene | 50-60 |
| herculite | 30 |
| Teflon | 40-80 |
| silicone | 25-40 |
| polystyrene | 30-50 |
| dry air | max. 2 |

**Tab. 5.2: Electrical strength of basic materials**

## 5.2. Microwave dielectric equipment [1]

With the progress of high-frequency technology, it was possible to start using frequency bands over 1 GHz even for high-frequency heating. Microwave heat allows such heatings which were not possible before dielectric heating. The used radiation is in the field of decimeter and centimeter wavelengths and it allows heatings of substances with an actually hundred times lower figure of loss $\varepsilon_r$tgδ compared to substances heated dielectrically.

### 5.2.1. The principle of microwave heating

Microwave heating is a special type of dielectric heating. Similarly to dielectric heating, it is electric heating, during which heat is developed inside the dielectric (semiconductor) due to polarization effects (see chapt. 5.1.1) The energy of high-frequency field is transferred to the material by means of microwaves and the same relation (5.3) applies for the volume density of heat flux (heat flux in a volume unit) as for dielectric heating. After quantification of constants, it is often mentioned in the following form:

$$q_z = 55{,}6 \cdot 10^{-12} f\, \varepsilon_r \,\mathrm{tg}\delta\, E_{\mathrm{ef}}^2. \tag{5.19}$$

It is evident from the above-mentioned relation that the source frequency is an important parameter, and the dielectric figure of loss εrtgδ depends on the frequency.

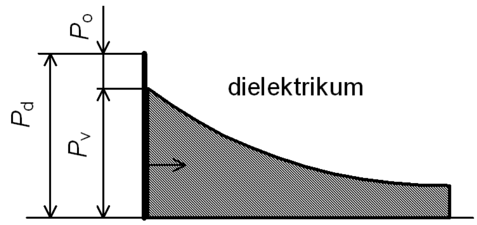


**Fig. 5.10: Power absorption during microwave heating**

Due to very high frequencies, high power values are reached without any risks of electric breakdowns of the air around the dielectric material, which would occur with the application of dielectric heating to 30 MHz. A part of incident radiation $P_\mathrm{d}$ is reflected $P_o$ from the dielectric material, the rest $P_\mathrm{v}$ enters the material and its amplitude drops exponentially, as shown in Fig. 5.10. The relation describing the electric field intensity course can be used for the calculation of energy in the specific depth of material:

$$E_x = E_\mathrm{o} e^{-x/a}, \tag{5.20}$$

where $E_0$ is the intensity of the electric field on the surface, *x* is the distance from the surface, *a* is a damping constant (absorption coefficient) of the material. As an analogy to induction heating, the penetration depth is introduced, i.e. the distance from the surface when the dampening of electric field intensity to the value of 1/e, i.e. approx. to 37 % of surface intensity, occurs. The penetration depth according to [8] is given by the relation:

$$a = \frac{\lambda_0}{2\pi}\sqrt{\frac{2}{\varepsilon_r\left(\sqrt{1+\mathrm{tg}^2\delta}-1\right)}}, \tag{5.21}$$

where $\lambda_0$ is a wavelength in vacuum, εr is relative permittivity and tgδ is a dielectric loss factor at working frequency.

Lay-out diagram of microwave heating is shown in Fig. 5.11. Microwaves are emitted from the microwave radiation source (magnetron) through an aerial and directed by a waveguide to the application space (cavity resonator) with minimum losses. The material subject to heating is placed on a pad, usually made of glass or ceramics in order to prevent its heating. The cavity resonator mostly features a quadratic shape; the shape and the design depend on the way of application. Internal walls of the cavity resonator are made of conductive material, mostly of aluminium or stainless steel. The cross-section of the waveguide is usually round or rectangular; its dimensions are given by the wavelength of the transmitted waves. In order to provide for optimum transfer of waves, transfer without losses, the internal diameter of the waveguide must be always larger that the wavelength. Usually, there is a natural atmosphere in the waveguide.

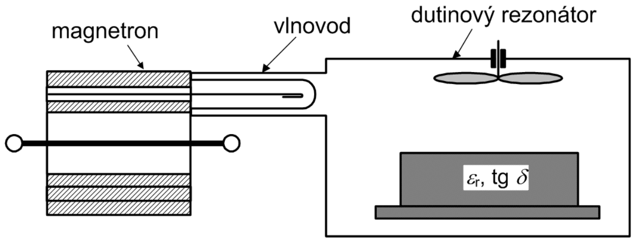


**Fig. 5.11: Schematic arrangement of microwave heating**

Each microwave equipment comprises three parts:

- source part (generator of microwave radiation, magnetron),
- transferring part (waveguide),
- working part (resonance heating space, cavity resonator).

In the cavity resonator, standing electromagnetic wave motion is created. This standing wave motion with solid nodes and antinode vibration loops can cause uneven heat-soaking of the heated material. In the areas where the intensity of the electromagnetic wave motion is zero, no heat develops. Therefore the shift of nodes and antinode vibration loops of electromagnetic wave motion against the heated material must be provided for, thus ensuring uniform heating. This can be done in two ways. The pad, on which there is the heated material, moves (e.g. a turning plate), or a turning metal segment is placed into the resonator; the segment changes ratios in the resonator cavity periodically, thus providing for the shift of nodes and antinode vibration loops. Mostly, both the ways are used simultaneously.

The design of the access opening to the application space also plays a significant role in microwave heating. The design of the access opening must be made in the way that maximum of the electric field intensity rests on the separation section. Only in this case the flowing current of the separation part equals to zero. The electric field intensity node is created by a short connection in the distance of $\lambda/4$ from the separation part, as shown in Fig. 5.12.

Each microwave equipment must be perfectly designed not to radiate energy due to safety and hygienic reasons as well as due to prevention of radio interferences [1], [7]

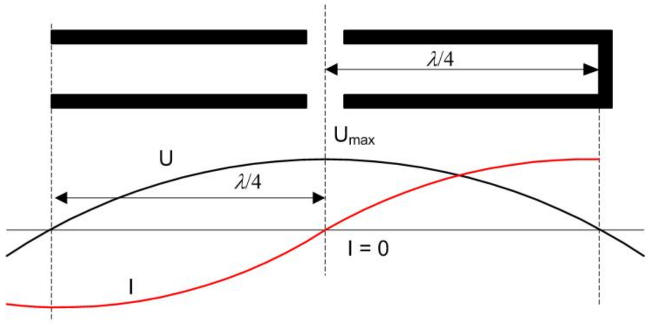


**Fig. 5.12: Design of the access opening to the working zone.**

## 5.2.2. Sources of microwave radiation

Differences between dielectric and microwave heatings are caused mainly by different frequencies. The frequency range of microwaves is set by limiting frequencies of ∈ <0.3; 300> GHz. For such high frequencies, a working capacitor cannot be used. The main working element of microwave devices is a magnetron.

A magnetron is the most common source of microwave radiation which was designed in the 1940s for military purposes in radio technology. It is a source of high power and it works with high efficiency.

Actually, a magnetron is a special power electron tube. Common electron tubes cannot be used for such high frequencies. Basically, it is a vacuum diode which is able to generate electromagnetic radiation in the area of microwaves, i.e. it can change energy from a direct current or alternating source to the high-frequency energy. The electron tube does not feature any external oscillation circuits connected as it is common with common oscillators. The oscillating circuit is formed by the actual structure of resonators, whose dimensions determine the resonance frequency of these waves. Since resonators are firmly made, the frequency cannot be changed arbitrarily.

A magnetron (Fig. 5.13) consists of a massive copper anode. The anode is hollow and a thermionic (hot) cathode is placed in its axis. The anode electrode, however, is not a simple cylinder, but it is fitted with an even number of circular openings (cavities) on its internal surface. These cavities (circular resonators) are connected to the cathode space through narrow slots. To provide for symmetry, the high-frequency field in the slots connecting the peripheral resonators with the space cathode-anode, is the same everywhere. The entire magnetron is inserted between the poles of a powerful permanent magnet or an electromagnet excited by AC current. The excited high-frequency energy is taken away through a loop inserted into one of the cavities. The loop is ended with an aerial, so that the high-frequency energy can be transported to the application space with minimum losses.

The principle of magnetron operation is as follows: The cathode is directly heated with transformer branches and electrons are released from its surface. The electrons are accelerated by the connected electric field towards the anode. Ferrite magnets, which surround the magnetron, create the magnetic field, whose induction lines are perpendicular to the electron orbit and they affect the movement of the electrons. The induction direction of the magnetic field *B* is the same as the magnetron axis and they are perpendicular to the intensity of the electric field *E.* The electron trajectories curve and the larger the magnetic field is, the higher is the curve. The electrons do not move in the magnetic field directly from the center to the periphery, but they curve to the left. This means that they do not impact in the middle of the closest metal segment, but to its left part.

The cavities together with the slots behave in the same way as miniature oscillating LC circuits. Actually, the slots are capacitors and the conductive rest of the cavity connecting the edges of the slot is a miniature coil. Electrons charge one edge of the slot (one plate of the capacitor), so current around the cavity (coil) starts flowing to the other edge of the slot immediately. This current invokes the magnetic field, which, in the second half of the cycle, induces the opposite flux through the cavity. The slots and cavities form the oscillation circuit with its own frequency. Microwave radiation causes discharging of oscillation circuits, therefore they must be continuously charged by the electrons from the cathode.

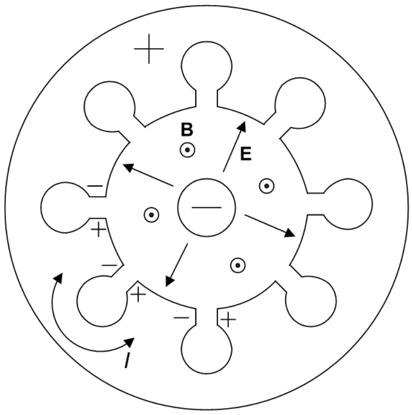


**Fig. 5.13 Principal arrangement of a magnetron**

The magnetron cathode is usually heated directly from an alternating source. The direct voltage on the anode ($10^0$-$10^1$ kV) is obtained from the thyristor rectifier fed by an HV transformer. Magnetron efficiency is 60-80 %. Magnetron power ranges between 0.8 and 6 kW. The technical service life of magnetrons may be 6000-8000 operating hours. If needed, multiple magnetrons can work in parallel in one working area. From the economic point of view, the most suitable microwave source is a magnetron with a power of 1.5 kW. Therefore, 100 and more modules with magnetrons 1.5 kW are parallel connected in large equipments.

## 5.2.3. Application of microwave heating

The advantage of microwave heating consists in the fact that heat develops inside the material and it is possible to reach higher temperature inside the product than on its surface. It is similar with dielectric heating. As a result of the fact that the material is heated throughout its volume, high uniformity of heat-soaking and precise temperature control can be achieved. Heating of a part of material above the limit temperature can also be prevented. During heating of some materials, in particular those ones containing water, a significant change to relative permittivity against temperature must be taken into account.

The main benefits of microwave heating include heating speed and dosing of supplied heat energy. Individual thermal heating can thus be carried out with a precisely defined amount of energy. Products can be processed in packaging materials, because the action of electromagnetic waves does not depend on thermal conductivity. Therefore the external insulating layers do not affect them. Microwave heating allows heating of materials of various physical states and heating can be performed in various environments, e.g. in vacuum, in natural and protective atmosphere and at various pressure values.

Nowadays, microwave heating is widely applied in various industries and technologies. It is used, for example, in the food, pharmaceuticals, chemical, lumber and paper industries, in agriculture and for manufacturing of ceramics and other materials.

The widest application of microwave heating is mainly in the food industry. It includes not only cooking and baking of meals, heating up and defrosting of ready-made dishes, pasteurization of various kinds of durable bakery, sterilization, etc., but also destruction of insect pests in cereal products, dried apples and other products.

In the chemical industry, microwave heating is mainly used for rubber cure and plastic processing, for acceleration of chemical reactions, in agriculture for treatment of agricultural products. Microwave technologies are also applied in manufacturing of ceramics, for example, for drying, burning, curing and finishing of surfaces. Microwave drying is applied not only in the textile, paper and lumber industries, but also in civil engineering, foundry and other industries. [11], [13]

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[10] http://www.odbornecasopisy.cz/index.php?id_document=26181

[11] Elektro - odborný časopis pro energetiku. http://www.odbornecasopisy.cz/ index.php?id_document=26310.

[12] http://fyzweb.cuni.cz/dilna/krouzky/mikrov/podr1.htm

[13] http://www.romill.cz/cz/mikrovlny

# 6. Arc furnaces, mains connection

## 6.1. The physical principle of developing of the electric arc by ionization of gases

The ionization of elementary particles, atoms and molecules is such a process, during which three kinds of split particles develop. They include electrons, positive ions and negative ions.

If the ionization process occurs gradually, then there are also excited atoms and molecules in the gaseous column; these are particles, in which internal energy is increased without any change to the electrical charge.

The basic types of elementary particle ionization are briefly explained in the following sections.

**Contact ionization of atoms and molecules**

The contact (surface, thermal) ionization of atoms and molecules occurs on the boundary of the gaseous sphere at the point of transition to the solid material which surrounds it (for examples electrodes).

**Photoionization**

This kind of ionization occurs at the absorption of atoms by radiation. It is known that each quantum of radiation has energy:

$$w_v = h \cdot v \tag{6.1}$$

where $w_v$ is quantum energy radiation, $v$ is radiation frequency and $h$ is Planck's constant.

For ionization of an elementary particle, ionization work must be known.

$$w_i = e_0 \cdot U_i \tag{6.2}$$

where $e_0$ is an electron charge and $U_i$ is ionization potential.

The following applies:

$$w_\upsilon \geq w_i \tag{6.3}$$

If the radiation quantum energy is higher than or equal to the required ionization work, then the atom ionizes itself. This condition is usually met only for ultraviolet radiation.

Most gases or metal vapors cannot be ionized by one-shot absorption of energy quantum, but it is excited only gradually, so the photoionization occurs only after several processes. In practice, this kind of ionization occurs mainly in the core of the arc column at high temperatures.

**Elementary particle ionization at mutual collision**

In arc charges, it is the most common way of ionization. For this method, we further distinguish:

- Ionization at a collision of gas particles with electrons
- Ionization at a collision of gas particles with ions

**Ionization at a collision of gas particles with electrons**

Ionization work can be expressed by the relation (6.2). In the electric field, the electron gains the kinetic energy.

$$w_k = \frac{1}{2} \cdot m_e \cdot v^2 \qquad 6.4)$$

In the relation (6.4), m is the weight of the electron and $v$ is its speed. The electron, however, ionizes the atom or molecule only if its kinetic energy at the moment of a collision with the atom is as high as the ionization work or higher:

$$w_k \geq w_i \qquad (6.5)$$

Upon substituting to the preceding condition, it applies that:

$$\frac{1}{2} \cdot m_e \cdot v^2 \geq e_0 \cdot U_i \qquad (6.6)$$

In this case it is also possible that ionization occurs by the gradual excitation of atoms in steps, until the accumulated kinetic energy is sufficient to perform the required ionization work.

**Thermal ionization of gases**

This type of ionization is important mainly for development of a plasmatic state in the arc. If the arc is in a closed volume, where we exclude the effects of external factors, such as the electric field, radiation - then these ionization processes occur in this gas.

a) Cleavage of neutral particles at mutual collisions
b) Photoionization of particles resulting from heat radiation of the wall surrounding the gas volume
c) Ionization at collisions of neutral molecules and electrons, which are represented in the gas as a result of the first two mentioned ionizations

Theoretically, all these processes must happen at any temperature. In practice, they start to manifest themselves only at gas temperatures of 2000 - 3000 K.

**Ionization at a collision of gas particles with ions**

The ion mass is approximately 2000-times higher than the weight of electrons, while the electric charges are equal. The electric field gives both electrons and ions the same energy under the condition of identically large differences of this field. Due to the large mass, ions feature lower speed values.

In some cases, ions can also reach sufficient kinetic energy, so that the energy can initiate ionization of a neutral particle at a mutual collision. Mostly the energy is so small that this kind of ionization can be neglected.

## 6.2. Electric characteristics of the arc

**Development of the DC arc in the electric field**

Development of the electric arc will be explained on a simple case of an electrical circuit fed by a DC voltage source *E* containing a regulating resistor *R* and a variable arc resistor $R_o$ (Fig. 6.1).

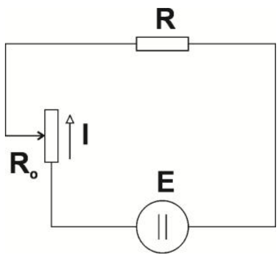


**Fig. 6.1: An example of an electrical circuit fed by a DC source**

The arc develops between two electrodes. The cathode is connected to the negative pole of the source, the anode is connected to the positive pole of the source. If we get the two electrodes closer to each other so that they touch, then the current *I* determined by the quantities of the circuit *E*, *R,* will flow through the circuit. These values shall be set, so that the current flowing through the circuit is greater than 0.5 A.

If we take the electrodes away, then a conductive path starts to exist between them at the interruption of their mutual contact. The path occurs due to ionization of the environment between the two electrodes. Conductive elements between the electrodes are ionized vapors of the material of both the electrodes and the air. An electric arc occurs.

The circuit current decreases with increasing resistance of the $R_o$ arc. The resistance of the electric arc $R_o$ is characterized by great non-linearity and it depends on the character of the arc and it changes quickly in the range from zero to infinity.

The arc charge consists of the ionized column, through which current flows, and of surrounding gases (the halo) at high temperature. The length of the arc then surrounds the electrodes, cathode and anode (fig. 6.2).

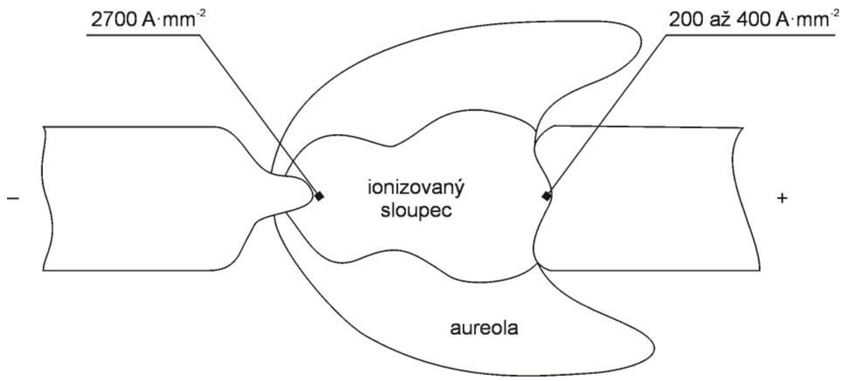


**Fig. 6.2: Electric arc**

During the subsequent burning of the arc, the cathode forms itself to the shape of a cone, whereas the anode has a hollow in its central part. The cathode area of the conductive charge is immediately adjacent to the cathode. The length of this part is tiny, about $10^{-5}$ cm, and it does not depend on the length of the arc. In the cathode area, the ionization processes to elementary particles occur.

In the middle of the area, there is the so-called **cathode spot.** Current density in the cathode spot (2700 $A{\cdot}mm^{-2}$) is significantly higher than in the surrounding areas of the cathode surface. The cathode area is then followed by the column, which creates the longest part of the conductive path between electrodes. It consists of the ionized column containing ionized particles which allow passing of the current between electrodes. In this ionized column, the basic part of electric energy is transformed into heat energy.

This ionized column is followed by the anode part of the arc. Its part is also tiny and it does not depend on the arc length. According to research done in a laboratory, current densities of the cathode spot between 2700 $A{\cdot}mm^{-2}$ and 2900 $A{\cdot}mm^{-2}$ and current densities of the anode spot between 200 $A{\cdot}mm^{-2}$ and 400 $A{\cdot}mm^{-2}$ were set.[4]

A significant area of the arc is just the area of the cathode, from which electrons are transported to the anode by action of thermoelectric emission.

### Distribution of potential along the arc

In fig. 6.3, there are dependency curves of voltage loss on the length of the arc. The largest voltage loss falls within the area of the anode (CD), the lower voltage loss then falls within the area of the cathode (AB). Anode and cathode losses of potential do not depend on the arc length or the value of the total voltage of the source. The sum of both the potentials equals to the potential of ionization of arc gas or vapors. This implies that the difference in the potentials of the conductive column is approximately equal to the difference in the potential of the applied voltage and ionization potential.

In fig. 6.3 there are three curves relating to different working pressure in the arc. You can see that increase in pressure raises the total voltage level of the arc.

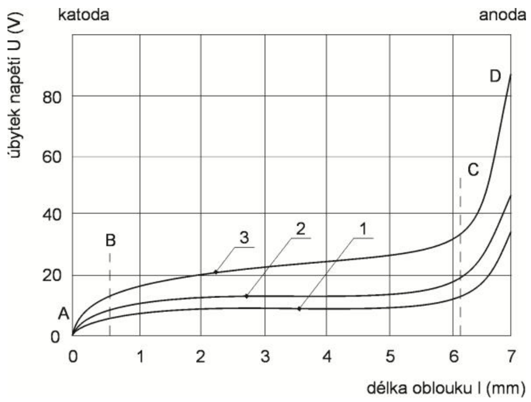


**Fig. 6.3: Distribution of potential along the arc**

**Characteristics of a DC arc**

The mutual relations between basic properties of electrical discharge are called the arc characteristics. The basic characteristic of the arc is a **volt-ampere characteristic.**

The best-known empirical relation expressing electrical properties of the DC current is Ayrton formulas:

$$U = a + b + l + \frac{c}{I} + \frac{d \cdot l}{I} \qquad (6.7)$$

where (V), *b* (V.cm$^{-1}$), *c* (W), *d* (W.cm$^{-1}$) are constants dependent on the material of the electrodes, *l* is the arc length, U is the voltage on the arc and *I* is current intensity.

According to Ayrton's relation (fig. 6.4), two volt-ampere characteristics of the DC arc are designed. We can see that upon an increase in the current, they get closer to the steady voltage value asymptotically.

The curve No. 1 applies to the arc length of 2 cm, the curve No. 2 applies to the arc length of 6 cm.

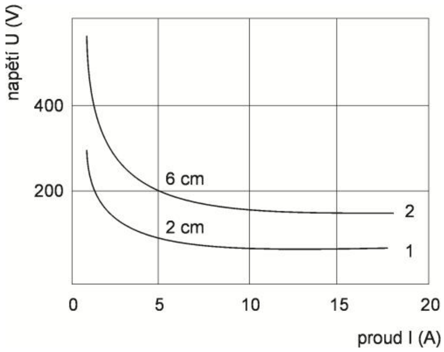


**Fig. 6.4: Volt-ampere characteristics of a DC arc**

The dependence of the voltage on the arc length is shown in Fig. 6.5 and 6.6

In fig. 6.5, there is a linear dependence of the voltage *U* of the arc on its length for lengths between 2 and 10 cm. Current values for the curve 1 - between 930 and 1150 A, for the curve 2 - between 680 and 880 A, and for the curve 3 - between 410 and 590 A.

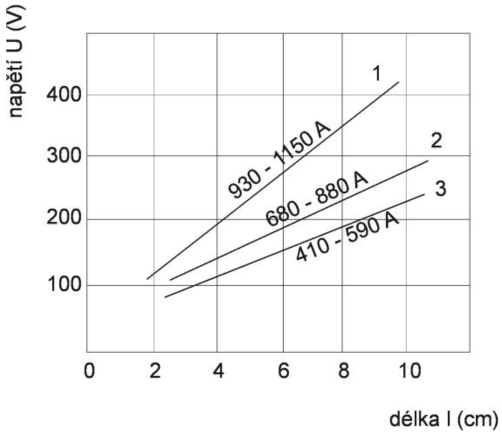


**Fig. 6.5: Linear dependence of voltage on the arc length.**

In fig. 6.6 there is a non-linear dependence of voltage on the arc length, which manifests itself with arcs of smaller lengths up to 2 cm at a voltage of 20 to 100 V.

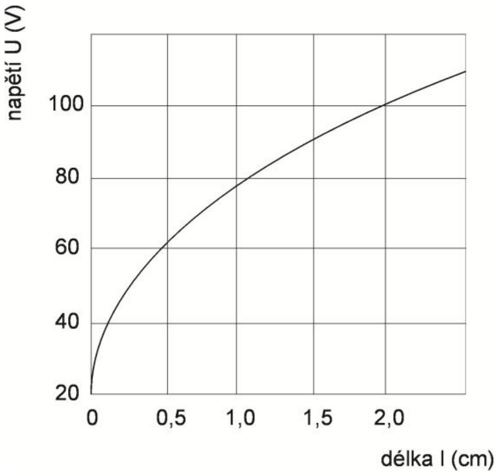


**Fig. 6.6: Non-linear dependence of voltage on the arc length**

**Stabilization of the arc by a resistor**

The development of the arc between two electrodes is explained with the circuit fed from a direct-current source. The circuit contains a solid resistor *R* and a variable resistor of the arc $R_o$.

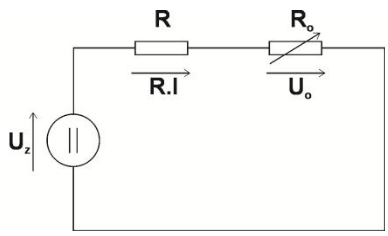


**Fig. 6.7: An example of an electrical circuit fed by a DC source**

The development of the arc is explained by separating the electrodes from each other and forming of the conductive ionized path. For stable burning of the arc, Sisojan gives the value *I* = 0.5 A. This current value is related to the ballast resistor *R* in the arc circuit. [4]

The following applies to source voltage at constant current in the arc:

$$U_z = U_O + R \cdot I \tag{6.8}$$

According to Ayrton's relation (fig. 6.8) the characteristic of the arc is a hyperbole.

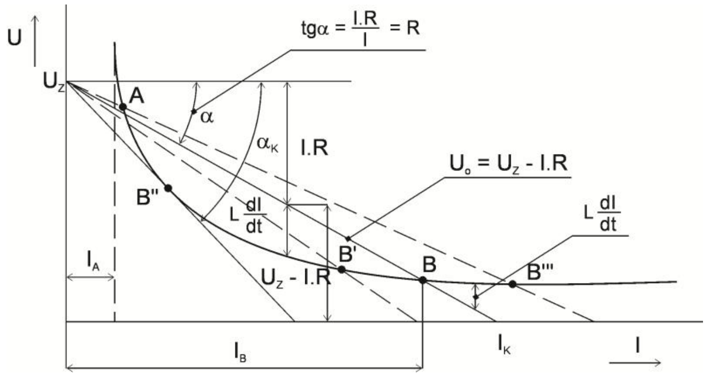


**Fig. 6.8: Arc characteristic**

## 6.3. Characteristic of the alternating arc

For the deriving of the characteristics of the alternating arc, we use an equivalent diagram of the electrical circuit with the arc according to Fig. 6.9.

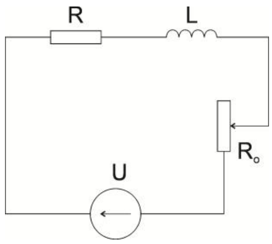


**Fig. 6.9: The equivalent diagram of the electrical circuit with the arc**

In the arc, elements of an alternating source U*,* active resistors of the circuit *R* and $R_o$ and inductance of the circuit *L* are included.

If we increase the source voltage at the constant distance of the electrodes, then, at a certain voltage value, a breakdown occurs, and electric current will start to flow between the electrodes through the arc.

Upon the periodical, sinusoidal change to voltage, forced breaks occur during the arc burning; they occur when the voltage on the arc falls under the value $U_{min}$.

In fig. 6.10, a course of voltage and current in the circuit containing only the ohmic resistance without inductance is shown.

As soon as the transformer voltage $U_{TR}$ reaches the ignition value of the arc $U_{za}$, current starts to flow through the circuit; it will flow until the transformer voltage falls down under the extinction voltage $U_{zh}$. The extinction voltage is usually a bit lower than the ignition voltage. This process is repeated at each half-wave. The arc voltage features the saddle caused by negative characteristics of the arc.

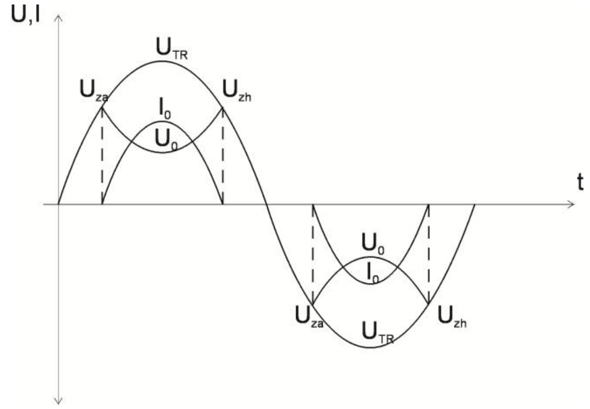


**Fig. 6.10 The course of voltage and current in the circuit.**

**Stabilization of the alternating arc by a phase shift.**

If we connect inductance in series to the arc, not only the phase shift occurs between voltage and current, but also the extension of the burning of the arc due to the inductance of the choke.

It is possible to prove in the calculation that a minimum phase shift $\cos\varphi = 0.85$ is required for continuous arc burning. In fig. 6.11, there is an idealized course of voltage and current at $\cos\varphi = 0.85$. The arc voltage is rectangular. This is typical of the arcs with very high currents, where practically no changes to voltage depending on the current occur.

**Stabilization of the alternating arc through the increase in voltage on the transformer**

The stabilization of the alternating arc through the increase in voltage in the transformer is shown in fig. 6.11 by a dashed line.

For industrial electrical arc furnaces, the change to the electrode occurs in addition to the change to current. If a carbon cathode forms the electrode in the first half-period, then the material subject to melting takes over this role in the other half-period. Compared to the carbon electrode, the material features different electrical and thermal properties. This is manifested, in particular, at the beginning of the melt if the charge is relatively cold, and the cathode spot cools down quickly in the first half-period due to good thermal conductivity of metal. As a result, higher current flows through the circuit in the half-period, when the electrode (cathode) is formed by the carbon electrode, then in the other half-period, when the polarity is reversed.

The cold charge of the furnace requires higher voltage at lower arc currents due to worse emission ability. Unevenness of the emission causes strong fluctuations. Moreover, sometimes break of the arc occurs. This must be eliminated by short-circuit of the circuit and subsequent separation of the electrodes.

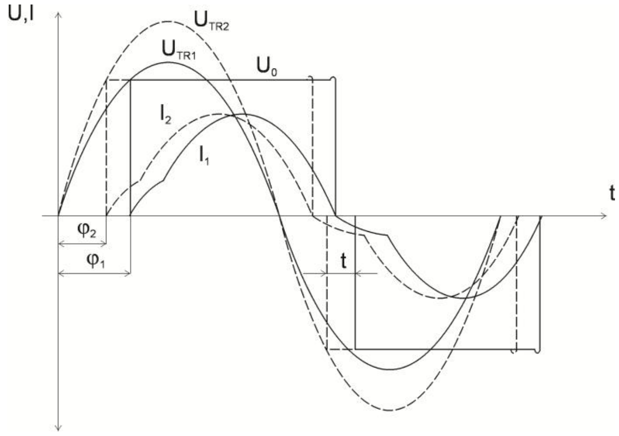


**Fig. 6.11: Stabilization of the alternating arc**

## 6.4. Theoretical fundamentals of electrical arc furnaces

The current circuit of electrical arc furnaces consists of a constant ohmic and inductive resistance determined by the properties of the conductive material and the geometrical shape of supplies and surrounding components of the electrical arc furnace. These components affect, in particular, the value of reactive losses, thus also the loss of voltage on electrode clips.

Furthermore, the circuit resistance is represented in the current circuit; it can be replaced by a variable ohmic resistor (fig. 6.12).

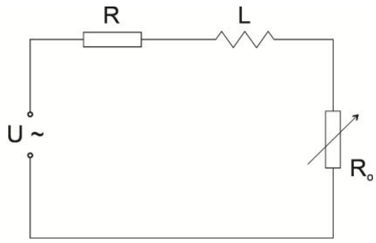


**Fig. 6.12: An example of the electrical circuit**

The fictional resistance of the circuit $R_O$ is a variable value. This value can be obtained if we divide the arc voltage by the current of the furnace circuit. It can be changed from zero, when the electrodes are short-circuited, up to infinity, when the circuit is broken and the arc does not burn. Conditions in the furnace can be well shown using a vector diagram in fig. 6.13

From the diagram in fig. 6.13, **short-circuit impedance** results. Short-circuit impedance determines the short-circuit current of the furnace. The operating point moves along the BX straight line depending on the size of the arc resistance. The furnace resistance then determines the immediate value of the furnace impedance, where $Z_m$ is impedance for maximum power of the furnace (fig. 6.13).

$$Z = \sqrt{(R + R_o)^2 + (\omega \cdot L)^2} \tag{6.9}$$

In the relation (6.9), $R_o$ is a variable value of the arc resistance. The power factor of the furnace can be determined from the vector diagram.

It applies to the value of $\cos\varphi_K$ (short-circuit - $R_o = 0$):

$$\cos\varphi_K = \frac{R}{\sqrt{R^2 + (\omega \cdot L)^2}} \tag{6.10}$$

The following applies to any operating point marked X:

$$\cos\varphi = \frac{R + R_o}{\sqrt{(R + R_o)^2 + (\omega \cdot L)^2}} \tag{6.11}$$

The arc current is inversely proportional to the variable value of the *Z* impedance.

$$I = \frac{U}{Z} = \frac{U}{\sqrt{(R + R_o)^2 + (\omega \cdot L)^2}} \tag{6.12}$$

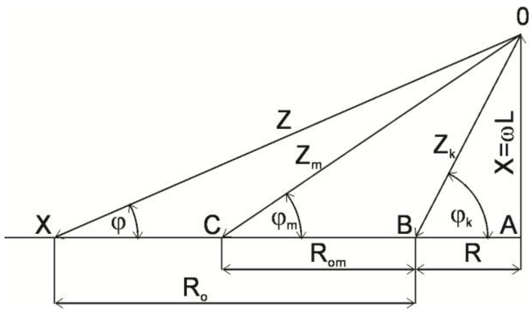


**Fig. 6.13: Vector diagram**

We assume that with operational short-circuits, magnetic materials of the circuit (choke, transformer coils, conductors, structure) work in the direct part of magnetizing characteristics. Individual reactances *X* do not thus depend on the passing current. It is the same with the constant part of the active resistors *R* (except for the arc resistor), as they do not depend on the passing current. Under this assumption, the end points of current vectors for various values of the arc resistance $R_o$ will move around the circle.

In fig. 6.14, there is a circle with marked points A´, B´, C´, X´, O´. Each of this point is equivalent to the inverted value of impedance *Z*, which changes depending on the value of arc resistance.

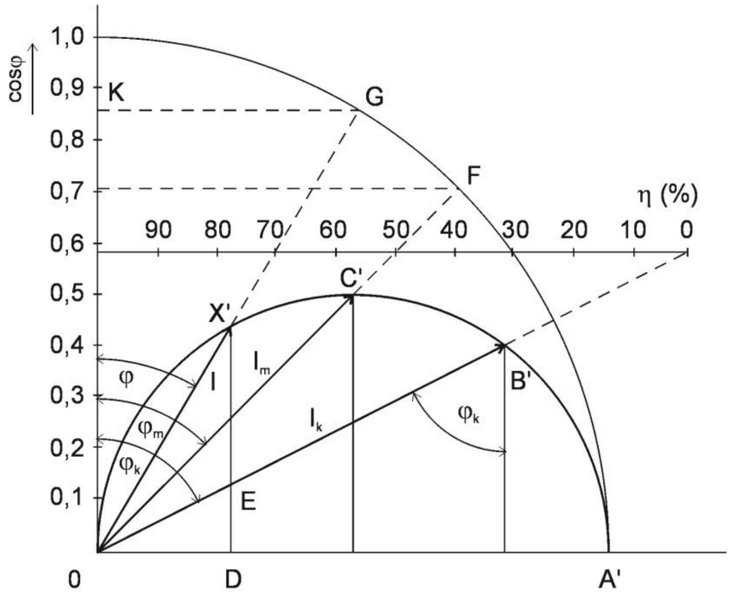


**Fig. 6.14: Circular diagram**

The point A´ on the circle (fig. 6.14) corresponds to the point A in fig. 6.13). The line segment 0B = $Z_K$ (fig. 6.13) corresponds to the line segment 0B′, expressing short-circuit current. The line segment 0C (fig. 6.13) corresponds to the line segment 0C′ (fig. 6.14), it is the point for maximum power of the furnace. The line segment 0X (fig. 6.13), i.e. any current of the arc, corresponds to the line segment 0X´in the circular diagram.

If the arc resistance $R_o$ converges to the infinity, we will get to the point 0´ in the circular diagram upon breaking of the arc. If we plot the voltage vector *U* perpendicularly to the diameter of the circle, 0A´, then the angles $\varphi_K$, and $\varphi$ are phase shifts between voltage and the immediate current of the furnace. We can plot a scale for values of the furnace power factor cos$\varphi$ on the vertical axis.

The intersection of the extended vector of current with the circle A′FG, whose radius is a diameter of the circular diagram, shows the cos$\varphi$ on the vertical line (the y axis), with which the furnace works at the specific current.

The efficiency $\eta$ for individual currents of the furnace are obtained in the form of an intersection of extended vector of these currents with the line section of efficiency. The efficiency line section can be created by extension of the short-circuit current vector $I_K$ and uniform distribution of a perpendicular from a point of this vector $I_K$ on the vertical axis *y*.

Decrease in current adversely affects the stability of the electric arc. The criterion for assessment of this stability is the following ratio:

$$\frac{I}{I_K} = \frac{\text{proud pece pracovní}}{\text{proud pece nakrátko}} \qquad (6.13)$$

This ratio $I / I_K$ is increased by the rise in the current, but at the expense of electric efficiency of the furnace.

Reversing effects on the network are more dampened at a smaller difference, i.e. if the current vector on the circular diagram gets closer to the point B′ (fig. 6.14), which corresponds to the short-circuit current $I_K$.

Three basic fields can be distinguished on the circular diagram:

- the area of unstable operation - the section 0'X' – long arc,
- the area of optimum operation - the section X'C',
- the area of guaranteed stable operation - the section C'B' – short arc.

**The maximum power input of the arc furnace**

The power factor of the arc furnace depends on both the solid component of the active resistance $R$ and reactance $X$ and on the variable value (arc resistance) $R_o$.

For the long arc (large resistance, small current), see the circular diagram in fig. 6.14, i.e. with high resistances of the arc, the power factor is high and it approaches to one. Should it be the case, the entire change to the arc length does not have any practical effect on the power factor. The situation is different if the size of the $R$ resistance is close to the size of the $X$ reactance. Electrodes are short-circuited. Then each change to the arc resistance $R_o$ also induces a change to the power factor, thus a change to the transfer of the energy to the arc.

According to fig. 6.13, the following applies:

$$\mathrm{tg}\varphi = \frac{\omega \cdot L}{R + R_o} \qquad (6.14)$$

Out of which:

$$R_o = \frac{\omega \cdot L}{\mathrm{tg}\varphi} - R \qquad (6.15)$$

The arc power

$$P_o = R_o \cdot I^2 = R_o \cdot \left(\frac{U}{Z}\right)^2 \qquad (6.16)$$

If we substitute for the *Z* impedance (from the relation 6.17).

$$\sin\varphi = \frac{\omega \cdot L}{Z} \qquad (6.17)$$

then we will obtain the following for the power:

$$P_o = R_o \cdot \left(\frac{U}{\omega \cdot L}\right)^2 \cdot \sin^2\varphi \qquad (6.18)$$

We supply from the (6.15) for $R_o$ and upon this adjustment we get:

$$P_o = \frac{U^2}{\omega \cdot L} \cdot \sin\varphi \cdot \cos\varphi - \frac{U^2 \cdot R}{(\omega \cdot L)^2} \cdot \sin^2\varphi = \\ = \frac{U^2}{2 \cdot \omega \cdot L} \cdot \sin 2\varphi - \frac{R}{\omega \cdot L} \cdot \frac{U^2}{\omega \cdot L} \cdot \sin^2\varphi \qquad (6.19)$$

The first part of the relation (6.19), i.e. the input to the circuit of the electrical arc furnace:

$$\frac{U^2}{2 \cdot \omega \cdot L} \cdot \sin 2\varphi = \frac{U^2}{\omega \cdot L} \cdot \sin\varphi \cdot \cos\varphi = \\ = \frac{U^2}{\omega \cdot L} \cdot \frac{\omega \cdot L}{Z} \cos\varphi = U \cdot I \cdot \cos\varphi \qquad (6.20)$$

This resulting relation expresses the power input to the entire circuit, including overall losses. The maximum of this energy can be obtained at the maximum of the function sin2*φ;* if sin2*φ* = 1, i.e. 2*φ* = 90°, i.e. *φ* = 45°. The greatest power in the circuit is obtained if the phase shift at the constant voltage *U* will be *φ* = 45°. So, the inductive resistance of the *X* circuit will equal to the resistance of $R + R_o$.

$$R + R_o = \omega \cdot L \qquad (6.21)$$

The power factor for maximum power results from that.

$$\cos\varphi = \frac{1}{\sqrt{2}} = 0{,}707 \qquad (6.22)$$

The max. power in the entire circuit of the electrical arc furnace equals to:

$$P_{o\max} = \frac{U^2}{2 \cdot \omega \cdot L} \qquad (6.23)$$

So, it depends only on the source voltage and the inductance of supply leads $\omega L$.

The furnace power can be increased at constant voltage only by decreasing in the inductance of supply leads. The inductance of supply leads must be, however, minimum ($\cos\varphi = 0,85$) to maintain the stable arc and it cannot be decreased arbitrarily.

A certain solution helping to increase the power of the electric furnace is an equipment of series capacity on the primary side of electrical arc furnace supplying. This measure allows stabilization of the voltage level for the electric arc and it decreases the $\omega L$ component if there is sufficient reserve in the reactance in relation to the arc stability.

**Maximum power on the arc**

The maximum power in the circuit of the electric arc is not identical with the maximum power on the arc. The power on the arc is lowered by losses in supply leads and losses of energy emitted to the surroundings by the electromagnetic field. Maximum power on the arc can be obtained by derivation of the relation:

$$\frac{\mathrm{d}P_o}{\mathrm{d}\varphi} = 0 \tag{6.24}$$

$$P_o = \frac{U^2}{\omega \cdot L} \cdot \sin\varphi \cdot \cos\varphi - \frac{U^2 \cdot R}{\omega \cdot L} \cdot \sin^2\varphi \quad \text{(ztráty celkové)} \tag{6.25}$$

$$\frac{\mathrm{d}P_o}{\mathrm{d}\varphi} = \frac{U^2}{\omega \cdot L} \cdot (\cos^2\varphi - \sin^2\varphi) - \frac{2 \cdot U^2 \cdot R}{(\omega \cdot L)^2} \cdot \sin\varphi \cdot \cos\varphi \tag{6.26}$$

After adjustments:

$$\frac{\mathrm{d}P_o}{\mathrm{d}\varphi} = \frac{U^2}{\omega \cdot L} \cdot (\cos 2\varphi - \frac{R}{\omega \cdot L} \cdot \sin 2\varphi) \tag{6.27}$$

We will apply the condition for maximum:

$$\frac{\mathrm{d}P_o}{\mathrm{d}\varphi} = 0 \tag{6.28}$$

It follows from the above mentioned that:

$$\cos 2\varphi_m = \frac{R}{\omega \cdot L} \cdot \sin 2\varphi_m \tag{6.29}$$

Or:

$$\mathrm{tg}2\varphi_m = \frac{\omega \cdot L}{R} = \mathrm{tg}2\varphi_k \quad (6.30)$$

It follows from fig. 6.13 and 6.14:

$$\frac{\omega \cdot L}{R} = \mathrm{tg}2\varphi_k \quad (6.31)$$

In this relation, $\varphi_k$ is the angle of the circuit of the short-circuit arc furnace. Thus:

$$\varphi_k = 2\varphi_m \quad (6.32)$$

The relation (6.32) is created on the basis of comparison of the two preceding relations.

The maximum power on the arc can be obtained at the phase shift with the value of $\varphi_k$ = 2. Thus also the power on the arc depends on ohmic and inductive resistance of the supply lead and it is determined for the respective construction of the electrical arc furnace as follows:

$$P_{o\max} = \frac{U^2}{2 \cdot \omega \cdot L} \cdot \sin 2\varphi_m - \frac{R \cdot U^2}{(\omega \cdot L)^2} \cdot \sin^2 \varphi_m \quad (6.33)$$

$\sin 2\varphi_m = \sin\varphi_k$ according to the relation (6.32)

$$\sin 2\varphi_m = \frac{\omega \cdot L}{\sqrt{R^2 + (\omega \cdot L)^2}} \quad (6.34)$$

$$\sin \varphi_m = \sin \frac{\varphi_k}{2} = \sqrt{\frac{1}{2} \cdot (1 - \cos \varphi_k)} = \frac{1}{\sqrt{2}} \cdot \sqrt{1 - \frac{R}{\sqrt{R^2 + (\omega \cdot L)^2}}} \quad (6.35)$$

Upon substitution by relation (6.34) for $\sin 2\varphi_m$ to the initial relation for maximum power on the arc $P_{omax}$, i.e. relation (6.33), the following is obtained:

$$P_{o\max} = \frac{U^2}{2 \cdot \sqrt{R^2 + (\omega \cdot L)^2}} - \frac{R}{2 \cdot \sqrt{R^2 + (\omega \cdot L)^2}} \cdot \frac{U^2}{\sqrt{R^2 + (\omega \cdot L)^2} + R} \quad (6.36)$$

The second part of the expression shows the losses in the circuit of the electric arc:

$$\Delta P_{\text{ztrátové}} = R \cdot I_{\max}^2 = \frac{R}{2 \cdot \sqrt{R^2 + (\omega \cdot L)^2}} \cdot \frac{U^2}{\sqrt{R^2 + (\omega \cdot L)^2} + R} \quad (6.37)$$

The first part of the expression (6.36) is the total power of the circuit at maximum power on the arc:

$$P_{\text{celkový}} = \frac{U^2}{2 \cdot \sqrt{R^2 + (\omega \cdot L)^2}} < \frac{U^2}{2 \cdot \omega \cdot L} \quad (6.38)$$

This power is lower than the maximum possible power in the circuit, which is determined by the relation 6.23.

The power factor at the maximum power on the arc is:

$$\cos \varphi_{\max} = \cos \frac{\varphi_k}{2} = \sqrt{\frac{1}{2} \cdot (1 + \cos \varphi_k)} = \frac{1}{\sqrt{2}} \cdot \sqrt{1 + \frac{R}{\sqrt{R^2 + (\omega \cdot L)^2}}} \quad (6.39)$$

This maximum power factor is a function of resistance $R$ and the inductance of supply leads $L$. The practical value of this power factor for real furnaces varies between 0.78 and 0.8 according to dimensional ratios of parameters $R$ and X of supply leads and losses due to the energy transfer to the neighboring equipment.

## 6.5. Theoretical relations for a three-phase arc furnace

In order to understand theoretical dependencies applicable to a three-phase electrical arc furnace, let us have a look at fig. 6.15, where there is an ideal wiring diagram of a three-phase electrical arc furnace.

In our deliberations, we will assess only the part of supply network from secondary terminals of a transformer up to melt, including the arc. These values can be easily converted to the primary side as well.

The real circuits of a three-phase furnace were replaced by equivalent resistors and chokes. Each current circuit of the furnace contains its own ohmic resistor of supply leads $R_1$, $R_2$, $R_3$ and equivalent resistors of arcs $R_{o1}$, $R_{o2}$, $R_{o3}$. The actual inductance of the circuits is replaced by chokes with equivalent inductance $L_1$, $L_2$, $L_3$.

The influence of currents flowing through each circuit on both the neighboring circuits is expressed by equivalent mutual inductance $L_{13}$, $L_{12}$, $L_{23}$. The following relation applies to a planar arrangement of conductors.

$$L_{12} = L_{23} > L_{13} \tag{6.40}$$

With a certain simplification, we can consider ohmic resistances $R_1$, $R_2$ and $R_3$ identically large; also inductances $L_1$, $L_2$, $L_3$ are identically high. The values of mutual inductances, however, are different. Generally speaking, $L_{12}$, $L_{23}$ (acc. to fig. 6.15) are identically high, but still larger than $L_{13}$. For the arc resistors it also applies that $R_{o1} \neq R_{o2} \neq R_{o3}$. It is related to the planar arrangement of supply leads of short path.

When the furnace is ideally symmetrical, the voltage in each of the circuits $U_1$, $U_2$, $U_3$ is the same and it is determined by the terminal voltage of the transformer. In practice, however, the condition of the ideally symmetrical furnace cannot be met, mainly during charge melting-down, when currents flowing through individual circuits are different and they constantly change due to the change to the arc length.

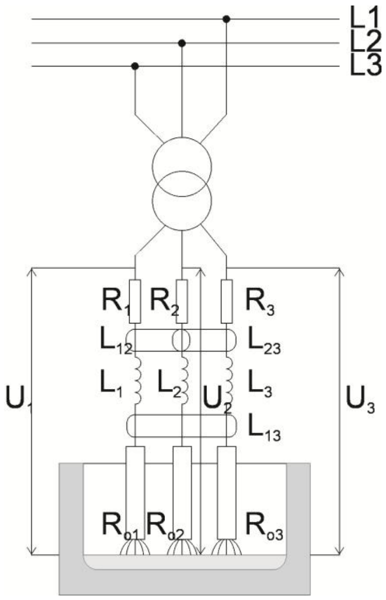


**Fig. 6.15: Wiring diagram of a three-phase arc furnace**

**Simplified equations of voltage for a three-phase planar arrangements of a short network**

General theoretical relations for a three-phase electrical arc furnace, whose diagram is in fig. 6.15, can be described in the following way:

$$\underline{U}_1 = (R_1 + R_{o1}) \cdot \underline{I}_1 + j\omega L_1 \cdot \underline{I}_1 + j\omega L_{12} \cdot \underline{I}_2 + j\omega L_{13} \cdot \underline{I}_3 \tag{6.41}$$

$$\underline{U}_2 = (R_2 + R_{o2}) \cdot \underline{I}_2 + j\omega L_2 \cdot \underline{I}_2 + j\omega L_{21} \cdot \underline{I}_1 + j\omega L_{23} \cdot \underline{I}_3 \tag{6.42}$$

$$\underline{U}_3 = (R_3 + R_{o3}) \cdot \underline{I}_3 + j\omega L_3 \cdot \underline{I}_3 + j\omega L_{31} \cdot \underline{I}_1 + j\omega L_{32} \cdot \underline{I}_2 \qquad (6.43)$$

As we have already mentioned, the following applies to equivalent mutual inductances:

$$L_{12} = L_{21} = L_{23} = L_{32} = M \qquad (6.44)$$

$$L_{13} = L_{31} = N \qquad (6.45)$$

while M > N

The following applies to the currents in the individual phases:

$$\underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0 \qquad (6.46)$$

It is a delta connection. The currents must be compensated.

Using the equations for mutual inductances (6.44) and (6.45), we can write equations for voltage in the following form:

$$\begin{aligned}\underline{U}_1 &= (R_1 + R_{o1}) \cdot \underline{I}_1 + j\omega L_1 \cdot \underline{I}_1 + j\omega M \cdot \underline{I}_2 - j\omega N \cdot \underline{I}_1 - j\omega N \cdot \underline{I}_2 = \\ &= [(R_1 + R_{o1}) + j\omega(L_1 - N)] \cdot \underline{I}_1 + j\omega(M - N) \cdot \underline{I}_2 = \underline{Z}_1 \cdot \underline{I}_1 + \underline{Z} \cdot \underline{I}_2\end{aligned} \qquad (6.47)$$

$$\begin{aligned}\underline{U}_2 &= (R_2 + R_{o2}) \cdot \underline{I}_2 + j\omega L_2 \cdot \underline{I}_2 + j\omega M \cdot \underline{I}_1 - j\omega M \cdot \underline{I}_1 - j\omega M \cdot \underline{I}_2 = \\ &= [(R_2 + R_{o2}) + j\omega(L_2 - M)] \cdot \underline{I}_2 + j\omega(M - M) \cdot \underline{I}_1 = \underline{Z}_2 \cdot \underline{I}_2\end{aligned} \qquad (6.48)$$

$$\begin{aligned}\underline{U}_3 &= (R_3 + R_{o3}) \cdot \underline{I}_3 + j\omega L_3 \cdot \underline{I}_3 - j\omega N \cdot \underline{I}_2 - j\omega N \cdot \underline{I}_3 + j\omega M \cdot \underline{I}_2 = \\ &= [(R_3 + R_{o3}) + j\omega(L_3 - N)] \cdot \underline{I}_3 + j\omega(M - N) \cdot \underline{I}_2 = \underline{Z}_3 \cdot \underline{I}_3 + \underline{Z} \cdot \underline{I}_2\end{aligned} \qquad (6.49)$$

Impedances $Z_1$, $Z_2$, $Z_3$ a $Z$ were substituted for the following relations:

$$\begin{aligned}\underline{Z}_1 &= (R_1 + R_{o1}) + j\omega(L_1 - N) \\ \underline{Z}_2 &= (R_2 + R_{o2}) + j\omega(L_2 - M) \\ \underline{Z}_3 &= (R_3 + R_{o3}) + j\omega(L_3 - N) \\ \underline{Z} &= j\omega(M - N)\end{aligned} \qquad (6.50)$$

Certain simplifications are allowed for electrical arc furnaces with steady arc burning. So it applies that:

$$\begin{aligned} R_1 &= R_2 = R_3 = R \\ R_{o1} &= R_{o2} = R_{o3} = R_o \\ L_1 &= L_2 = L_3 = L \\ \underline{I}_1 &= \underline{I}_3 \end{aligned} \qquad (6.51)$$

Upon substitution of these assumptions into the equations for voltage (6.47), (6.48), (6.49), we can get an equation for voltage:

$$\underline{U}_1 = \underline{U}_3 \qquad (6.52)$$

And impedance:

$$\underline{Z}_1 = \underline{Z}_3 \qquad (6.53)$$

This means that phases 1 and 3 are equivalent. The difference is only in the middle phase.

**Development of live and dead phases**

Asymmetry in circuits of the electrical arc furnace will be explained on the basis of relations (6.47), (6.48) and (6.49) for voltage, which, after some simplifications, allow an insight into the basic courses. Upon multiplying of phase voltages by the respective currents, we obtain apparent power in each phase. These powers are practically divided into two components:

The first component in the actual apparent power of the phase:

$$\underline{Z}_1 \cdot \underline{I}_1^2 \,, \quad \underline{Z}_2 \cdot \underline{I}_2^2 \,, \quad \underline{Z}_3 \cdot \underline{I}_3^2 \qquad (6.54)$$

The other component for the first and the third phases then determines the effect of currents of neighboring *phases*, reduced to the effect of the middle phase.

For the phase one, this power is given by the product of $Z.I_1.I_2$ and for the third phase by the product of $Z.I_2.I_3$. This means that the effect of the neighboring phase was reduced to the effect of the middle phase 2.

It is known that the transfer of electric energy from one phase to the other through the magnetic field occurs in magnetically bound circuits of alternating current. In our example, the energy is given by the vector-preceding phase to the following phase. It is a common phenomenon in three-phase systems which are always bound by the magnetic field. In symmetrical three-phase circuits, this effect is usually annulled. In the planar arrangement, the effect of the middle phase on the first extreme phase must be respected (fig. 6.16).

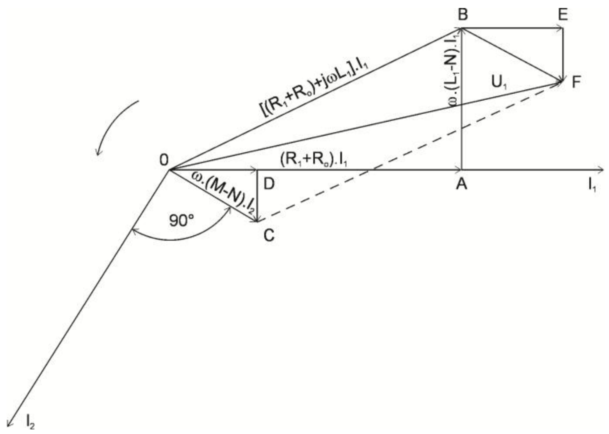


**Fig. 6.16: Phasor diagram - development of a dead phase**

The voltage vector $\underline{U}$ is composed of a sum of the ohmic component marked on the segment line 0A and given by the circuit ohmic resistance, and the inductive component on the segment line AB given by the inductive resistance minus the effect of the middle phase effect. This part of the voltage induced by the current $\underline{I}_2$, which influences the voltage of the first phase, is determined by the vector on the segment line 0C perpendicular to the direction of the current $\underline{I}_2$. Upon decomposing of this voltage into the direction of the current $\underline{I}_1$ and perpendicularly to the direction of the current $\underline{I}_1$, one can see that the effect of the middle phase current is such as it would increase the ohmic resistance of the first phase by the value of 0D (equivalent to BE). For the time being, the inductive resistance decreases by the value proportional to the DC segment line (equivalent to EF). Upon gradual vector summation of all these partial voltages, we can obtain the resulting vector of voltage $\underline{U}_1$ on the segment line 0F. This vector of voltage $\underline{U}_1$ is larger than the original vector determined by the following relation:

$$[(R_1 + R_{o1}) + j\omega L_1] \cdot \underline{I}_1 \approx \text{ segment line 0B} \qquad (6.55)$$

The vector diagram shows that the first part gives a part of its energy to the middle phase, therefore it behaves in such a way as it would have bigger apparent resistance in the circuit than the middle phase. Since the terminal voltage of the transformer is the same for all three phases, lower current flows through this circuit. The power on the arc fed from the first phase is lower. This phase is called the "**dead phase**". The situation with the third phase is different (fig. 6.17):

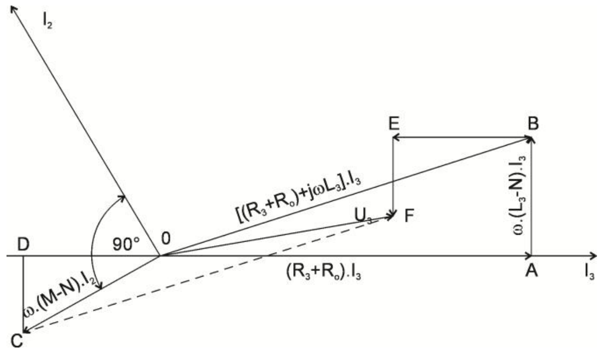


**Fig. 6.17: Phasor diagram - the formation of the live (wild) phase**

As we have already derived above, the voltage of this phase is determined by the summation of the ohmic and inductive components and the component affected by the current of the middle phase. In this situation, the middle phase is a "giver", as the vector of the $I_3$ current is delayed after the vector of the current $I_2$ . The middle phase gives almost all its energy it has received from the first phase. The effect of the middle phase on the third phase manifests itself by an apparent decrease in the inductive resistance, which induces an increase in current as well as power in this circuit.

In fig. 6.17, the vector of the voltage affecting the middle phase $I_2.j\omega.(M-N)$, proportional to the segment line 0C, is decomposed again into the active and reactive components of the 0D and DC voltages and it decreases the original voltage vector to the voltage $U_3$, proportional to the segment line 0F.

$$[(R_3 + R_o) + j\omega L_3] \cdot \underline{I}_3 \approx \text{ segment line 0B} \qquad (6.56)$$

This phase with a higher current load is called the "**live (wild) phase**".

It arises from the above-mentioned that while dealing with an arc furnace, the middle phase works normally, one of the extreme phases is loaded less; it is the dead phase. The other extreme phase is loaded more as to current; it is the live phase. Which of these extreme phases takes over the role of the dead or the live phase depends only on the connection of the three-phase system. By reversing the sense of rotation, their roles swaps.

## 6.6. The equipment of three-phase furnaces

Electrical equipment is significantly involved in acquisition costs of electrical arc furnaces and it has a great effect on their operation. Each part of electrical equipment of arc furnaces can be divided, preferably according to their function:

- Heavy-current electric circuit
- The circuit of automatic regulation of electrode movement

- Measuring instruments, safeguards and signaling
- Control computer

The most important group is formed by the heavy-current circuit. It represents the bulk of the value of electric equipment of the furnace and it significantly influences furnace operations. Another substantial part is the circuit of automatic controlling of electrode movement. The task of the power circuit is to conduct electrical energy to the working space of the furnace and to transfer it into heat. The power circuit of the three-phase arc furnace is schematically shown in fig. 6.18.

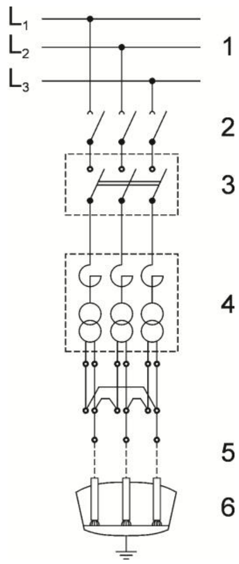


**Fig. 6.18: The power circuit of a three-phase arc furnace**

The description of the power circuit of a three-phase electric furnace (fig. 6.18):

1 - Supply network; 2 - Isolator; 3 - High-voltage power switch; 4 - Furnace transformer and chokes; 5 - Short network; 6 - Electrodes

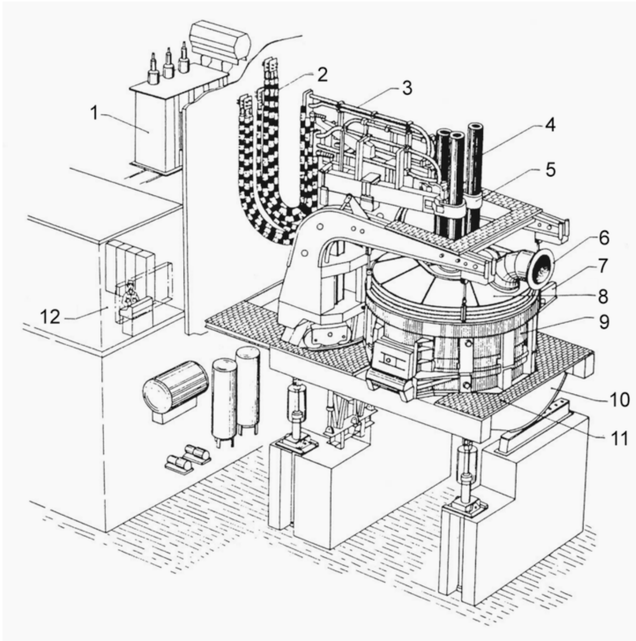


**Fig. 6.19: Three-phase electrical arc furnace**

Description of a three-phase electrical arc furnace, fig. 6.19.

1 - Furnace transformer; 2 - Water-cooled flexible cables; 3 - Water-cooled horizontal tubular conductors; 4 - Electrodes; 5 - Electrode holders; 6 - Discharge of gases from the furnace space; 7 - Outlet of steel at tilting; 8 - Water-cooled furnace lid; 9 - Furnace space; 10 - Furnace tilting mechanism; 11 - Assembly platform 12 - Control workstation. [3]

## 6.7. High-voltage supply network

In terms of energetics, electrical arc furnaces are ones of the largest consumers of electric energy concentrated to one point of electric distribution network of a metallurgical plant. The HV supply network of electrical arc furnaces is loaded irregularly by fluctuating current peaks which moves from zero values at the break of the arc up to the triple of the rated current at the short-circuit of electrodes with melt. These irregular fluctuations of currents cause voltage fluctuations on the respective impedance of the supply network, which adversely affects other electrical equipment supplied from the same system. Moreover, this fluctuation causes decrease in electric power given by the arc to the melt.

In particular, x-ray equipment, TV sets and data transfer systems are very sensitive to these variable fluctuations of voltage. Therefore the requirements for the HV supply network of electrical arc furnaces are precisely defined and given mainly by the power of the furnace transformer and the way of melting.

An important factor for assessing of a HV power network of electrical arc furnaces is short-circuit power in the place of the connection of furnaces to the energy system. Above all, it is necessary to separate the system of supplying of electrical arc furnaces from appliances, in particular those which are sensitive to changes in voltage.

In some cases, however, we cannot separate them. This situation occurs, in particular, in smaller steelworks supplied with high voltage 22 kV and lower, where even other consumers are connected to common bus-bars. In these situations, calculations must be done whether the short-circuit power at the place of connection of the electrical arc furnace guarantees maintaining of network disruptive effects within acceptable limits.

### The causes of disruptive effects of arc furnaces in the HV supply network

The power system supplying plants operating electrical arc furnaces is often exposed to the disturbing effects of irregularly fluctuating voltage. The origin of this irregularly fluctuating voltage must be searched for in the irregular current load of electrical arc furnaces. This load causes voltage drops on network impedances; they lead to voltage fluctuations and to the reverse effect of electrical arc furnaces on the power system. While examining changes in load of the power supply system by electric steel furnaces, it was proved that current fluctuations which occur during melting-down are basically overlapping of two types of current pulses. [14]

**The first type of changes to current load** occurs with the frequency of 0.5 times to one per second, mainly during the first 30 minutes of melt. Short-circuit current at the ignition of the arc between the electrode and the charge is as high as the steady middle current of the furnace. If the charge is cold, the short-circuit situation repeats continuously. Furthermore, the short-circuit current can occur upon an accidental contact of a piece of charge with an electrode, e.g. by the additional collapse of the charge.

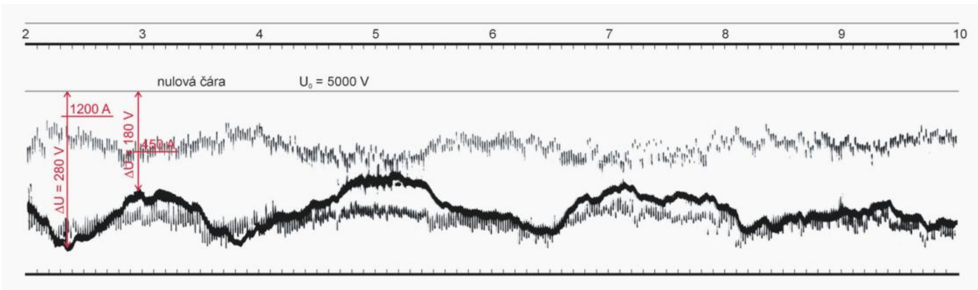


**Fig. 6.20: Oscillographic courses of voltage and current - the first type of changes**

**The second type of changes to current load** occurs twice to twenty times per second. The current intensity fluctuates within ±15 to ± 50 % of its nominal value.

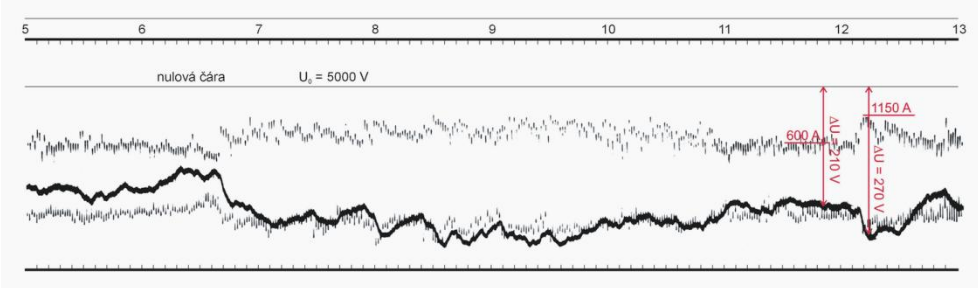


**Fig. 6.21: Oscillographic courses of voltage and current - the second type of changes.**

It seems that these fluctuations of current are not subject to clear laws as they change continuously both in their amplitudes and their frequencies. The result of these irregular current changes is thus voltage fluctuation which manifests itself in the primary circuits of electrical arc furnaces. In the research into voltage fluctuations in primary HV circuits in electrical arc steel furnaces, the courses of voltage and current were put into oscillographs at melting-down of 6-ton electric arc furnace fed by a furnace transformer 5 MVA, 5 kV/nn (fig. 6.20 and fig. 6.21).

The record of the current was taken by a current loop of the oscillograph 12LS-RFT-DDR from the current converter on the primary side of the furnace transformer. Its record with a frequency of 50 Hz is shown in the middle part of each figure. The voltage record was taken from the voltage converter by a voltage loop of the same oscillograph. The input voltage signal was adjusted in a special device with Zener diodes. This apparatus allowed us to adjust voltage in the circuit subject to measurement for recording in the way that we could monitor the voltage envelope curve directly; the values of the voltage change in the direct connection with changes to the current.

In fig. 6.20, there is a record of the variable value of the current (50 Hz) and a record corresponding to the respective voltage value (the thick line). At higher current load, the loss of voltage will be higher with the distance from the zero line. For the marked current I = 1200 A, this loss of voltage is U = 280 V. Changes to the curve of voltage might be related to the mentioned changes of the first type.

In fig. 6.21, there is a record of the variable value of the current (50 Hz) and a record corresponding to the respective voltage value (the thick line). In the left part of the record, there is smaller current load which increases gradually, so the voltage loss U = 270 V at current I = 1150 A occurs in the right part of the record. The changes to the voltage curve in the middle part of the figure might be related to the changes of the second type, which significantly interferes with the power supply system. The changes in the current of this type lie at the zone of insensitivity of automatic regulation of the electrode shift, so that they are intangible with traditional equipment of the automatic regulation of the electric arc furnace.

### The causes of formation of the second type of disruptive changes

The set of disruptive effects of irregularly fluctuating voltage, which results mainly from current changes of the second type, is called the “flicker effect”. On the basis of the research into voltage fluctuations in HV supply mains of electrical arc furnaces it can be concluded, that the disruptive effect of voltage oscillation is observed at tolerances of 0.5 % of the rated voltage value and at occurrence frequency 6-8 vibrations per a second. This disruptive effect causes blinking of light bulbs, it has an adverse effect on x-ray equipment, TV sets and computers fed from the same system.

**The first cause of fluctuations**

It is based on the theory of the arc - the effect of different ionization of the electric arc zone is recorded. That implies that the electric arc intensity changes suddenly from one half-wave to the other without an identifiable change to the length of the electric arc in the respective photos (fig. 6.22).

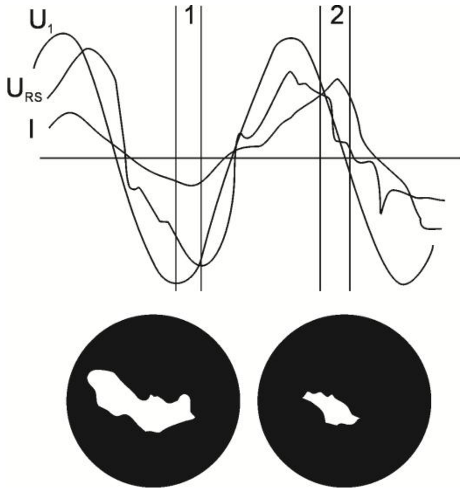


**Fig. 6.22 The influence of different ionization zone of the electric arc.**

The transfer of current in the electric arc happens with electrons and ions, which are formed by ionization. The concentration of these current carriers in the electric arc column is subject to changes. The charge consists of parts of various metallurgical composition. The electric arc burns on the piece, which happens to be located under the graphite electrode. Due to the high temperatures of the foot point, this piece is liquefied and it partly evaporates. Each part of the piece features various melting points and boiling points, so one or the other material always evaporates according to the closest ambient temperature. Due to strong local overheating, the evaporation of the steel additive can be explosive. Rising vapors with high temperature of the arc column ionize, thus increasing the concentration of the carriers in the electric arc, which can now transport higher current. The influence of charge composition on ionization conditions of the electric arc manifests itself mainly at the beginning of the melt and it is lower with increasing temperature of the metal.

**The second cause of the fluctuations**

It has been found out from film shots that the electric arc skips from one charge piece to the other at different time intervals. At the same time, the arc still burns on its protruding tips or edges. The electric arc current intensity always changes in the synchronized oscillographic record when the electric arc skipped to the other piece. If the length of the electric arc is shorter, the current is higher, and vice versa. Measurements show that the jumps of the electric arc from one charge piece to the other happen in the interval of five to max. twenty periods; i.e. 0.1 to 0.4 sec. The jumping of the electric arc happens when the tip or the edge of the charge piece rounds due to the high temperature of the electric arc. These processes are shown in fig. 6.23 and fig. 6.24

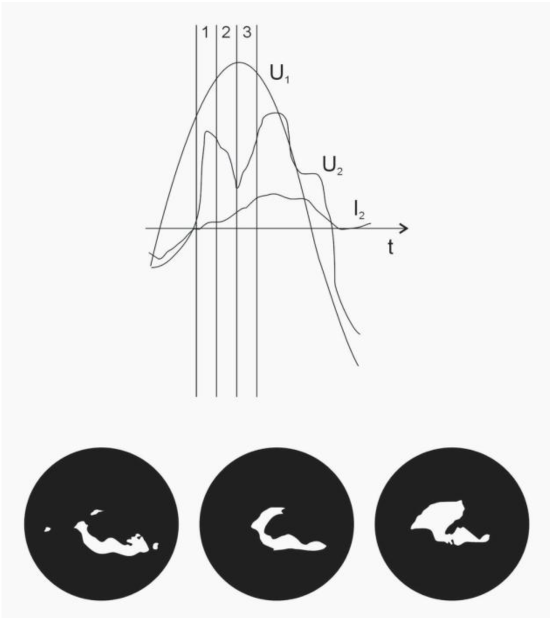


**Fig. 6.23: Jumps of the electric arc form various pieces of charge to the others.**

In fig. 6.23, the jumps of the electric arc from individual pieces of charge to the others within one period are evident from the photo shot. In fig. 6.24 it is already evident that the tip of the piece from fig. 6.23 has been melted away.

These movements of the electric arc are the main cause of current fluctuations of the electric arc. Changes to current amplitude induced by the above-mentioned reasons are the causes for light flickering (flicker effect) due to their occurrence frequency of 2 to 10 times per a second.

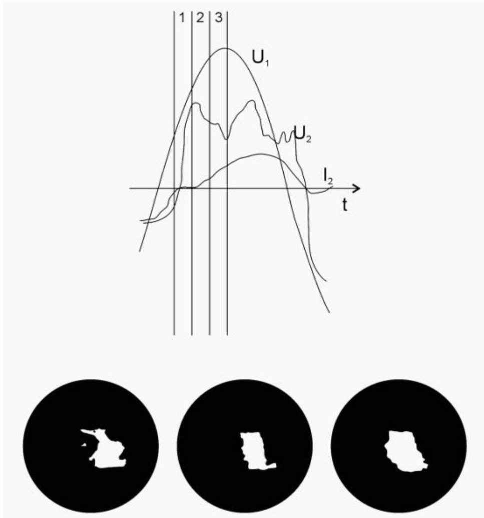


**Fig. 6.24: The effect of the arc length after rounding of charge tips**

**The third cause of fluctuations**

The third cause is a loop movement of the arc, during which the arc is extended. The current amplitude decreases during this movement In photos as well as in the oscillograph, the loop movement of the electric arc is shown in fig. 6.25.

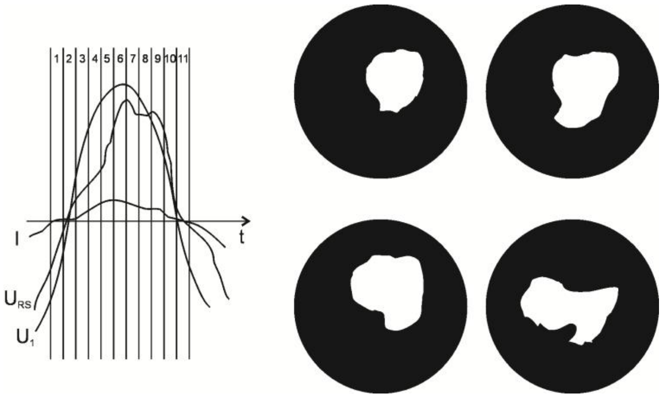


**Fig. 6.25 The effect of the loop movement of the arc due to the action of forces of the electromagnetic field.**

In fig. 6.25, there is an oscillographic shot of the electric arc current *I* and the voltage of the same phase on the first and the second sides of the furnace transformer $U_1$ and $U_2$ and photo shots of the electric arc shot at sections 4, 6, 8 and 10. The arc deflection is evident from the fig. 6.25 and it is a consequence of the increased magnetic field of the neighboring phase. This increased magnetic field of the neighboring phase arises from the flow of higher current of asymmetrically loaded electrodes. Each electrode works independently according to conditions occurring in the immediate vicinity of burning of its arc. The current value can reach up to the short-circuit value at the beginning of melting-down, whereas the neighboring electrode works with the mean value of the current. This operational state lasts until the piece which is the cause of the short-circuit on the electrode, is burnt, or until the control picks the electrode up. All the three described reasons for current fluctuations overlap arbitrarily (they superimpose) and they lead to such current fluctuations which cannot be precisely defined as to the size of the amplitude and their frequency. All the described effects have the strongest effect at the beginning of melting down and they weaken with higher melting-down of the charge.

The effect of higher harmonics is evident in fig. 6.22 of the oscillographic record of current and voltage in the secondary circuit of the steel furnace. The curves are deformed mainly by the 3rd and 5th harmonics. The electric arc furnace is a large generator of higher harmonics to power supply networks.

**Voltage fluctuations and the current asymmetry in the primary circuits of steel arc furnaces**

The irregular fluctuations of the current during charge melting-down cause voltage drops on network impedances; they lead to voltage fluctuations as the reverse reaction of the arc furnace to the power network (fig. 6.26).

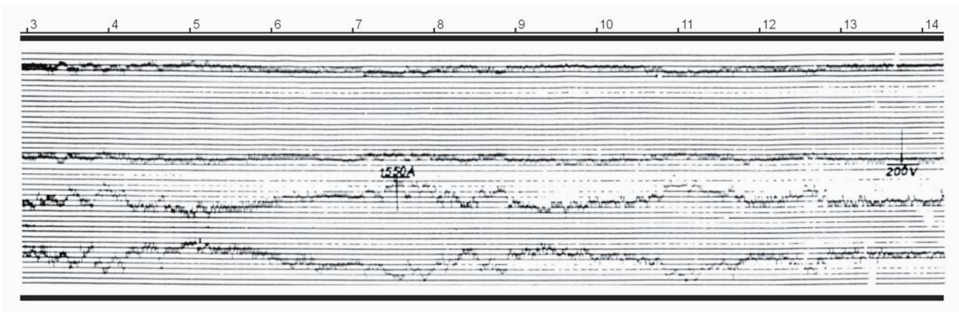


**Fig. 6.26: Time dependence of current I and voltage U of the LV middle phase - the secondary side of a furnace transformer 5 MVA.**

These voltage fluctuations were oscillographically taken with the electric arc steel furnace in Vítkovice, Ostrava, Czech Republic. Voltage fluctuations in HV circuits are shown in fig. 6.27.

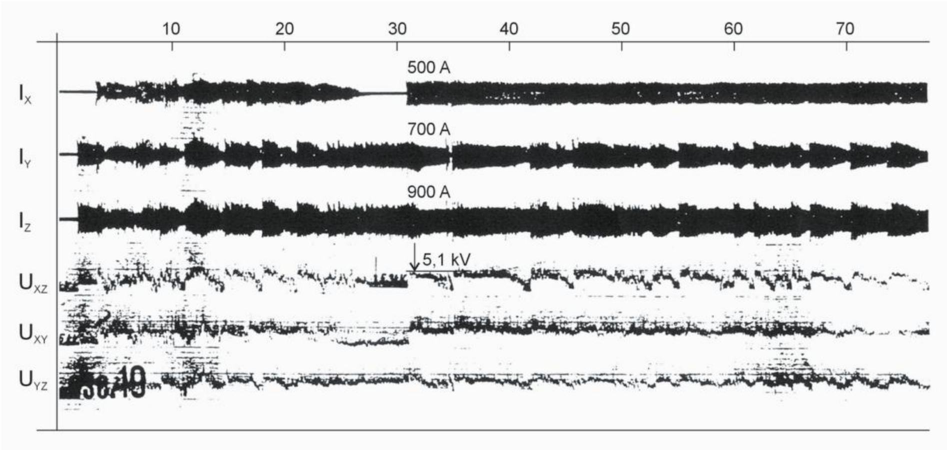


**Fig. 6.27: Time dependence of currents and voltages on the three phases measured on the secondary side of a transformer 10 MVA, 22/5 kV.**

In the upper part, we can see a record of asymmetrically fluctuating current of all three circuits of the arc furnace on the primary side of the furnace transformer 5 MVA, 5/0.4 kV. In the bottom part of the figure, there is a record of relevant HV voltages which fluctuate in the rhythm of current impacts on the power network. Upon short-circuit of an electrode with charge, the power switch performs shut-down (fig. 6.28).

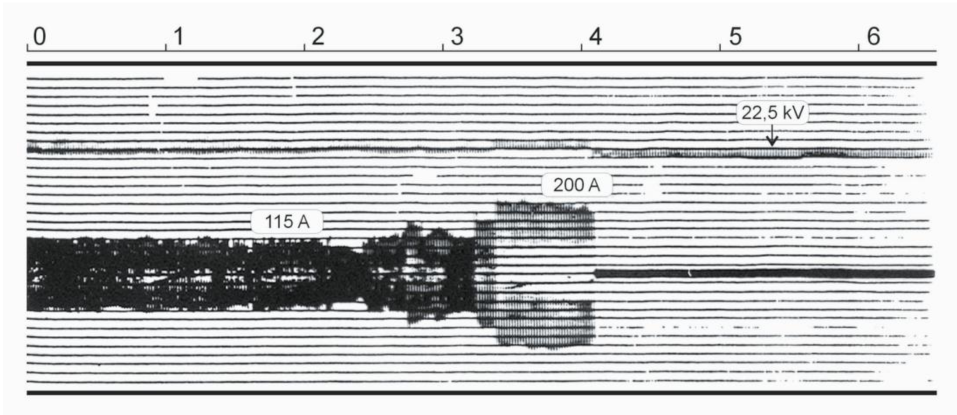


**Fig. 6.28: Time dependency of current I and voltage U of the live phase on the primary side (22 kV) of a furnace transformer 40 MVA The switch cuts off upon exceeding of the maximum current**

Asymmetrical load of each circuit in the electric arc steel furnace results mainly from often short-circuits of the electrode and charge and from unstable burning of the arc due to various influencing factors. This asymmetry is transferred even into the power system. Current asymmetry in the supply switching station on 5 kV bus-bars was measured with the electric arc steel furnace under consideration. The measurement was performed with separated feeding of the actual electric arc furnace. Asymmetry was found out using a method of symmetrical components (positive-sequence component and negative component) which were monitored with filters of positive-sequence components and registered by registration instruments during melting-down. The course of melting-down was accompanied by a current impact at a frequency of 15 to 20 impacts per minute. While evaluation was being performed, the value of the positive-sequence and the negative components of the current ($I_1$ and $I_2$) was found out planimetrically, out of which the value of average asymmetry was derived based on the following relation:

$$\rho = \frac{I_1}{I_2} \cdot 100 \qquad (6.57)$$

The asymmetry fluctuated in the rhythm of current impacts around this mean value. Therefore also an average bandwidth of the amplitude $\rho_i$ (%) was found out planimetrically. Six intervals of about 15 minutes were evaluated from the record. An average asymmetry $\rho$ = 45.6 % was evaluated based on results of measurement of positive-sequence and negative components. It can be concluded from the above-mentioned analysis that electric arc furnaces adversely affect a power system due to irregularly variable voltage fluctuations, great current asymmetry and, from the view of energetics, a bad power factor cos$\varphi$ = 0.7. [8]

**The possibilities of reduction of interfering effects of electric arc furnaces on the HV power system**

Reduction of short-circuit and high currents. Since the most significant losses occur at the highest currents, there is an effort to reduce these peaks. This can be reached by connection of a choke into a series with a furnace transformer. Due to the stability of the alternating arc, it is necessary to connect inductance. Short-circuit and high currents occur mainly during charge melting-down. After charge melting-down the front-end choke is eliminated. It is also possible to have a multi-stage choke.

Increase in short-circuit power on the connection spot. Short-circuit power is a quantity of primary significance for the size of voltage fluctuations in the network. For the current electric arc furnaces, the required size of the short-circuit power on the connection spot can be found out. Thus the network can be adjusted. Increase in short-circuit power on the connection spot is carried out as follows:

- Network amplification, which means doubling of supply leads, enlargement of a transformer, then connection of a new generator and similar adjustments. On the one hand, this solution is expensive, but, on the other hand, it is used nowadays despite this fact, mainly because the current arc furnaces were often additionally connected to the current supplies, which must be reconstructed today as they are no longer suitable.
- Connection of a synchronous compensator in a suitable place in the network.

- Serial compensation
- Parallel compensation

**Indirect compensation**

This method of compensation maintains withdrawing of constant reactive energy on the maximum value. Voltage fluctuations do not occur; however, the power factor is permanently very bad, therefore a parallel static capacitor with such a value to compensate the whole reactive input is connected.

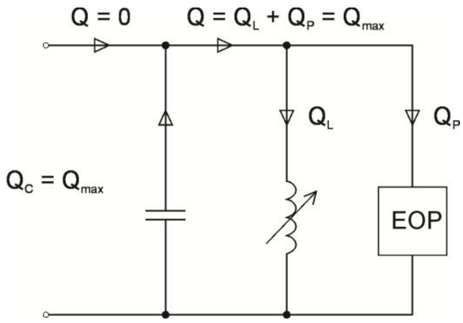


**Fig. 6.29: Wiring diagram of indirect compensation**

For indirect compensation, controlled static rectifier can be used as a consumer of the variable inductive power. 6.30). The function consists in the fact that the sum of consumptions of the furnace and the rectifier is constant and it is compensated by the static condenser. This equipment has already been theoretically elaborated and tested on a model. From the designing point of view, this compensator is simple and probably it is the most cost-effective solution which suits thanks to its dynamic properties.

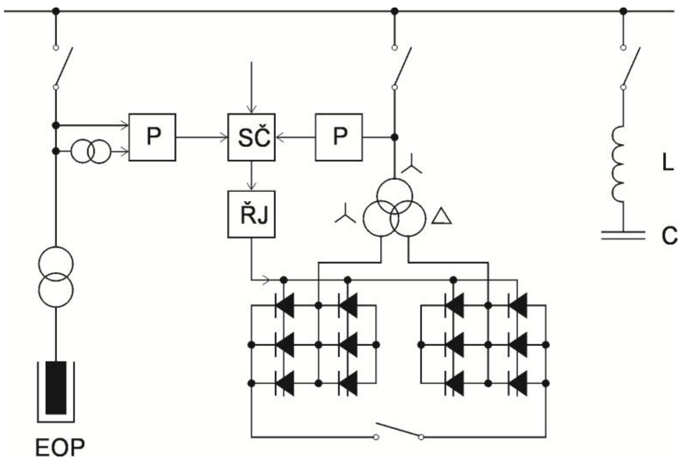


**Fig. 6.30: Diagram of a compensation equipment with a controlled static rectifier**

In addition to common disadvantages of all equipments working on the principle of indirect compensation, this concept is characteristic by the fact that the current withdrawn from the mains features a twelve-pulse course, by means of which the rectifier becomes a harmonic source. Furthermore, it should be noted that this concept compensates only the positive component. Another possibility is the use of only one choke controlled by thyristors for each phase whose current of basic harmonic is changed by anti-parallel-connected thyristors (fig. 6.31).

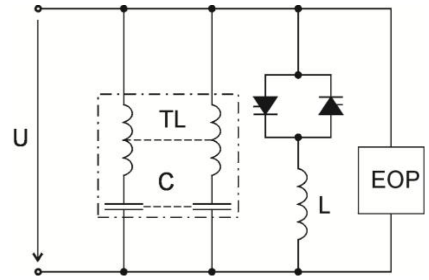


**Fig. 6.31: The diagram of a compensation equipment with anti-parallel-connected thyristors**

This connection is the source of higher harmonics. Therefore, it is appropriate to divide the parallel-connected capacitor battery into several parts and each part of the capacitor battery complete with a serial choke on the resonance circuit tuned to some of the harmonics which arise, and thus filter them out.

**Direct compensation**

Using the direct compensation, as it is indicated in Fig. 6.32, production of inductive energy in the compensator in the synchronism with the consumption of the electric arc furnace can be changed. Basically, it is parallel compensation.

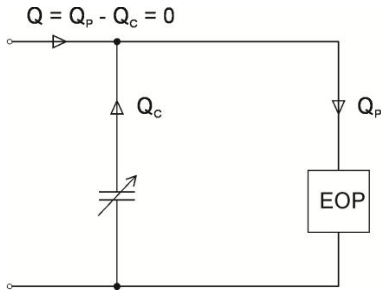


**Fig. 6.32: Wiring diagram of direct compensation**

The connection of a solid capacitor battery is not suitable because at the time when the power input of the arc furnace drops under the installed size of the battery, the furnace would work with a compensator altogether with the capacity power factor, which is not desirable. The solid capacitor battery can be used only for compensation of idle current of the furnace transformer or for compensation of the so-called permanent component of consumption $Q_c$. The thyristors of a switched capacitor battery seem to be perfect (fig. 6.33). This direct way of gradual compensation has been known for a long time. However, there was a problem with the implementation of the switch which would suit both with its speed and durability at switched powers.

A suitable switching element is a pair of anti-parallel-connected thyristors. Such equipment does not have any movable components, therefore it features long durability. No harmonics form with suitably selected switching mode. The equipment features completely silent operation and basically it does not require any maintenance. The equipment can be implemented both as symmetrical or even for compensation of asymmetrical load. Transport delay reaches a maximum of 10 ms, similarly to some of the preceding equipments. [8]

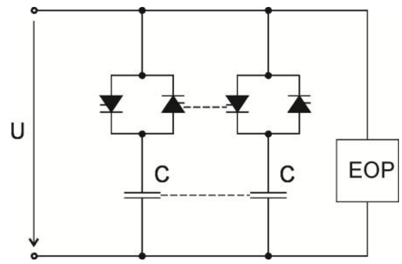


**Fig. 6.33: A wiring diagram of direct compensation with a capacitor battery switched by thyristors**

**Serial-parallel compensation of an electric arc furnace**

Serial compensation is a solution mainly to increase in and stabilization of voltage in mains; while with parallel compensation, we obtain improvement in the power factor by reduction in the apparent current by the respective component of reactive current supplied by the capacitor battery directly at the consumption spot.

Fluctuate drops in voltage in supply mains and significant consumption of reactive energy in equipments needed for operation of arc furnaces (such as chokes, transformers, etc.) occur in electric arc furnaces. For this reason, inclusion of combined, serial-parallel compensation is justified. However, there is another reason for the mutual interaction.

In fig. 6.34, there is a vector diagram of the voltage and current for operation of an electric arc furnace.

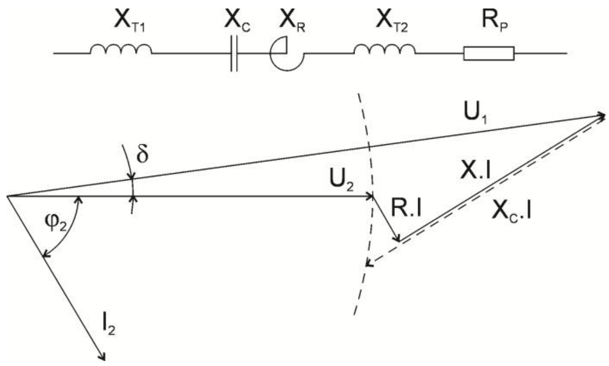


**Fig. 6.34: A vector diagram of the voltage and current for operation of an electric arc furnace.**

The voltage vector $X_c.I$, thus the voltage value which had to be compensated by the serial included capacitor to meet the condition $\Delta U$ = 0 is plotted using a dashed line.

If we include a parallel capacitor $C_1$ in the upper part of the fig. 6.34, the one compensating consumption of reactive energy of the choke $X_R$, then the vector diagram of voltage and currents will feature the following changes (fig. 6.35).

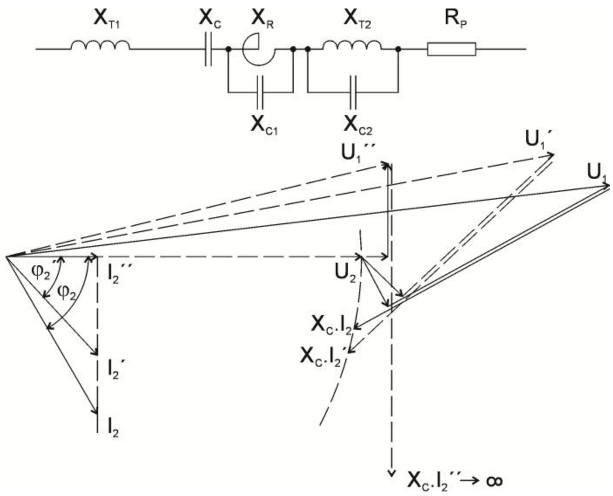


**Fig. 6.35: A vector diagram of serial-parallel compensation during operation of an electric arc furnace**

The apparent power $I_2$ lowers its value to $I_2´$ and the angle $\varphi_2$ also lowers its value to $\varphi_2´$. The vector diagram of voltage losses corresponding to active resistance of the furnace circuit and its reactance is shown using the full line (fig. 6.35). Contrary to the original condition at current $I_2$, turning of the vector diagram of voltage losses occurs and both the components of voltage losses drop. Despite this fact, the vector $X_c$. $I_2´$ corresponding by the size of boosting voltage on the serial capacitor, is higher than in the previous situation (fig. 6.34), if we keep maintaining the condition $\Delta U = 0$ in mind. This phenomenon is caused by the decrease in the angle $\varphi_2$ and overall turning of the vector diagram of voltage losses. Therefore the vector $X_c$. $I_2´$ intersects the circle marking the geometrical place of points with the voltage size $U_2$ in the far point.

Theoretically, if we compensate the power factor of the furnace circuit to cos$\varphi$ = 0, then, at the zero angle, the vector of the reactive voltage loss will be perpendicular to voltage $U_2$. Both the components of reactive voltage and active loss of voltage $\Delta U$ will have the smallest value in this case. Despite this fact, the vector $X_c$. $I_2´´$ does not intersect the circle marking the geometrical spot of points with voltage $U_2$, so it converges to infinity.

$$X_c \cdot I_2'' \to \infty \qquad (6.58)$$

In the event when $\varphi_2´´ = 0$, the condition $\Delta U = 0$ cannot be met. It can thus be concluded that the active component of voltage loss significantly affects the size of compensation power of the serial capacitor by its value; it is the same with the angle $\varphi_2$ between the current and voltage of the furnace circuit. Thus we can help the overall effect of serial serial capacity of voltage by parallel compensation of the power factor provided that the active component of voltage loss is much smaller that the component of the reactive voltage loss. Then it is enough to compensate the reactive voltage loss according to the selected level of compensation and the small active voltage loss shall be left uncompensated.

At very low active voltage loss of the compensated network with electric arc furnaces, the simultaneous parallel compensation of the power factor is very beneficial and it provides significant savings of costs of compensation power of serial condensers.

In conclusion, it can be said that the serial compensation is more efficient with a larger angle between voltage and current vectors, i.e. the power factor is worse. [11]

**Isolator**

It is intended only for disconnecting of the whole electric equipment of the arc furnace from high-voltage supplies during repairs to the equipment or during inspections of it. Otherwise, it does not have any direct effect on electric arc furnaces, therefore we will not deal with it in a more detailed way.

**High-voltage power switch**

The task of the high-voltage power switch is to switch and disconnect the power circuit at the beginning and at the end of the melt, upon dangerous overloading of the transformer and in emergency situations. Power switches are highly stressed. They often perform 60 to 70 switches within 24 hours. Their continuous and flawless operation is the prerequisite of the function of the whole furnace equipment. For example, failure to disconnect of longer short-circuit between the electrode and the melt can result in serious damages to the electrical equipment or destruction of the electrical equipment due to the effects of short-circuit currents. The most frequently used types are compressed-air vacuum $SF_6$ isolators.

**Furnace transformers and chokes**

Heat input to the furnace, thus the furnace power, is limited by the power of the furnace transformer. It manifests itself particularly at the stage of charge melting-down when the duration of this process depends mainly on the amount of supplied heat. The selection of furnace transformer power shall be made based on the size of furnace charge and the selected operating mode (fig. 6.36).

The furnace transformers of electric arc furnaces significantly differ from normal power transformers because they work with highly variable load at frequent short-circuits. Their specific feature is relatively low secondary voltage and high secondary current. Since the requirements for the amount of energy supplied to the furnace during the melt vary significantly, the transformer must allow regulation of the secondary voltage within wide limits.

Voltage control on the secondary side would be very difficult due to the high values of secondary currents (10 to 60 kA) for common furnaces with medium power. Therefore, it is done by changing the turn number of primary winding. Switching of the primary winding in delta-connection to star-connection allows doubling of the number of voltage grades. For most of our current arc furnaces, transformers with the switching of voltage grades with the disabled power switch are used; more modern furnaces also with load.

The selected melting mode with ultra high productivity (UHP) features a short arc. The ratio of furnace transformer powers for the UHP mode and normal mode is 2.1 to 2.4 (fig. 6.36.)

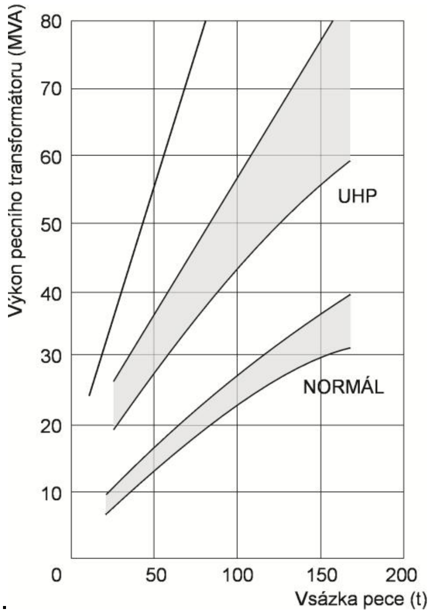


**Fig. 6.36: The ratio of powers of furnace transformers for the UHP mode and normal mode**

### Chokes

Chokes which are incorporated into the supply mains between the power switch and the furnace transformer limit the value of short-circuit currents upon contact of the electrodes with the melt. They are located in a common container with the transformer and they can have several reactance grades. A choke is usually included at the delta-connection of the transformer primary, thus at higher voltage grades. Upon switching to star-connection, the choke is disabled. The ideal state would be in the case of continuous control of choke reactance.

The downside of chokes of electric arc furnaces is deterioration of overall efficiency of arc furnaces. It is dealt with the lower grade of reactance or with a totally disabled choke mainly in the second part of melting-down, i.e. in the period when the arc is shorter, more stable and when short-circuits are less frequent. Work with lower choke reactance allows an increase in the power of the electric arc, thus accelerating of melting-down with a simultaneous decrease in power consumption. However, a possibility of controlling of both choke reactance grades switching, and its complete disabling are required to do so.

### Short path

A short path is a power line from the outputs of the secondary winding of furnace transformer to the workspace of the furnace. The designs of the short paths may differ for various furnaces. However, we can always divide the short path into the following sections:

- belt part
- flexible ropes
- guides of electrode holder arms
- electrode holders
- electrodes and connectors

High currents flow through the short path, so although the resistances in the short path are negligible, in the order of $10^{-3}$ Ω, losses in the power line are significant. At current of $10^4$ A, the voltage loss on the resistor $10^{-3}$ Ω is 10 V. 100 kWh of energy is lost in one hour. Short-path reactance has even more significant voltage loss. This does not cause immediate energy losses, but it significantly decreases voltage on the actual electric arc, thus decreasing the power supplied to the workspace of the furnace.

Each phase reactance is affected not only by the length of its conductors, but also by its mutual position and position of steel structures of the surrounding halls and cranes. Uneven distribution of powers in each phase adversely affects furnace work and durability of furnace linings. The most suitable measure to reduce asymmetry is the implementation of bifilar power line of the short path. 6.37).

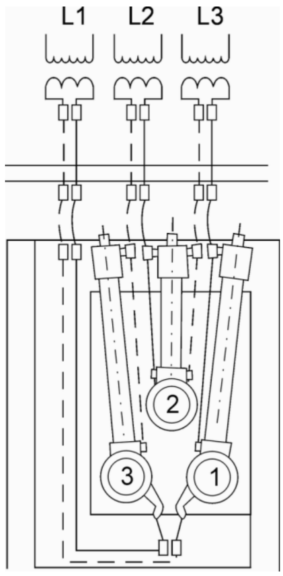


**Fig. 6.37: Bifilar implementation of the short path**

The condition for performance of the bifiliary connection consists in suitable tapping points of the secondary winding of the furnace transformer reachable outside the actual transformer. Perfect balancing of inductance can be done, however, only with the bifiliary connection up to the electrodes and with the connection of the tapping points of the transformer secondary winding at the electrodes. This wiring is shown in Fig. 6.37.

In smaller ore-thermal and steel furnaces with output up to 15 MVA, the short path is made using flat steel with supply through flexible supply cables in parallel bundle arrangement.

For larger arc furnaces, the short path is made of copper pipes cooled by water flow.

**Electrodes**

Electrodes, closing a part of the electric equipment, are very important parts of electrical circuits of electric arc furnaces. Heat losses in electrodes as well as the price of consumed electrodes are  significant parts of operating costs.

Main requirements for electrodes include:

- Good electric conductivity
- High mechanical strength
- High oxidation temperature
- Low content of ash and sulfur

In practice, three kinds of electrodes are used for arc heating:

- Carbon electrodes
- Graphite electrodes
- Söderberg electrodes

Carbon electrodes are made from a mixture of anthracite, coke, natural graphite and resin upon firing at 1200 °C - 1300 °C

## 6.8. Arc furnace control by computers

Control computers are more and more penetrating into various fields of technical practice. With the gradual improvement of computer properties, the way of their use is changing, as well. This decade, computers have begun to be applied even to direct control of production processes. This is allowed by utilizing of integrated circuit technology, development of improved computer peripherals for contact with the external environment and improvement of a logical structure of computers. The computer which features the required properties for direct control of production processes in real time, is called a **control computer.** The essence of determination of optimal operating mode of an electric furnace consists in the fact that the behavior of the controlled system is calculated for various combinations of voltage grades and arc currents at the following time interval and the expected value of a multi-objective function is quantified. The mode, for which the minimum of this function is found, is then entered as the optimal one.

Actual inductive and resistance changes to current and voltage will be calculated in the control computer. These values shall be deducted from the measured voltage values $U_{OR}$, $U_{OS}$ and $U_{OT}$ on the transformer and then we obtain the current arc voltage. Arc voltage is determined by the following equations (6.59). [8]

$$\begin{aligned} U_{LR} &= U_{OR} - U_{XR} - U_{RR} \\ U_{LS} &= U_{OS} - U_{XS} - U_{RS} \\ U_{LT} &= U_{OT} - U_{XT} - U_{RT} \end{aligned} \tag{6.59}$$

**BBC method with a microprocessor**

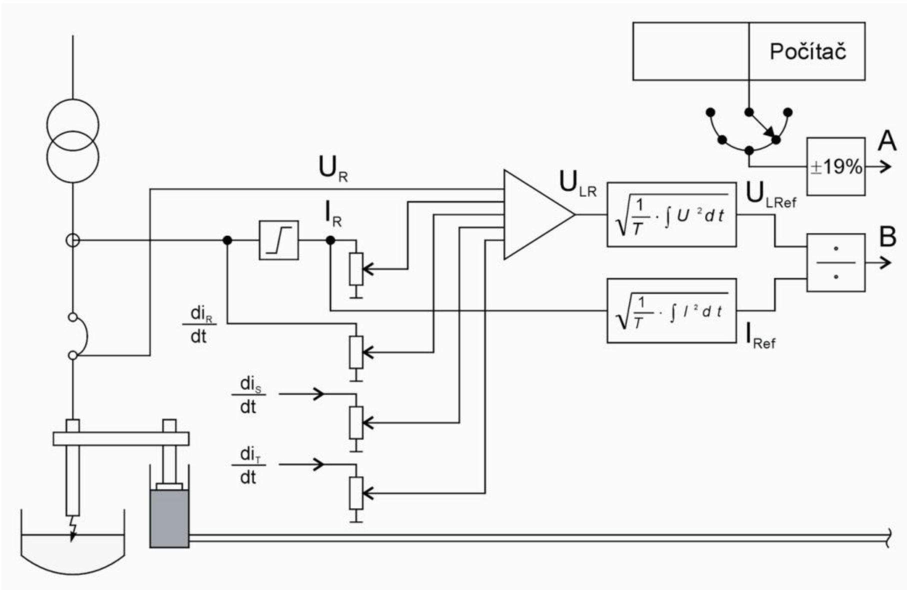


**Fig. 6.38: Arc furnace computer control scheme**

Inductive voltage losses:

$$\begin{aligned} U_{XR} &= -\frac{\mathrm{d}i_R}{\mathrm{d}t}\cdot\frac{\mathrm{A}+\mathrm{C}}{2}+\frac{\mathrm{d}i_S}{\mathrm{d}t}\cdot\left(\mathrm{A}-\frac{\mathrm{B}}{2}\right)+\frac{\mathrm{d}i_T}{\mathrm{d}t}\cdot\left(\mathrm{C}-\frac{\mathrm{B}}{2}\right) \\ U_{XS} &= \frac{\mathrm{d}i_R}{\mathrm{d}t}\cdot\left(\mathrm{A}-\frac{\mathrm{C}}{2}\right)-\frac{\mathrm{d}i_S}{\mathrm{d}t}\cdot\frac{\mathrm{A}+\mathrm{B}}{2}+\frac{\mathrm{d}i_T}{\mathrm{d}t}\cdot\left(\mathrm{B}-\frac{\mathrm{C}}{2}\right) \\ U_{XT} &= \frac{\mathrm{d}i_R}{\mathrm{d}t}\cdot\left(\mathrm{C}-\frac{\mathrm{A}}{2}\right)+\frac{\mathrm{d}i_S}{\mathrm{d}t}\cdot\left(\mathrm{B}-\frac{\mathrm{A}}{2}\right)-\frac{\mathrm{d}i_T}{\mathrm{d}t}\cdot\frac{\mathrm{C}+\mathrm{B}}{2} \end{aligned} \tag{6.60}$$

The constants A, B and C depend on the geometrical arrangement of electro-conductive path of currents. Resistance voltage losses

$$U_{RR} = i_R \cdot R_R$$
$$U_{RS} = i_S \cdot R_S \qquad (6.61)$$
$$U_{RT} = i_T \cdot R_T$$

The constants $R_R$, $R_S$, $R_T$, as well as A, B and C are fixed-set for the respective furnace on the digital potentiometer. The electrode impedance regulator is thus controlled using the current voltage on the arc.

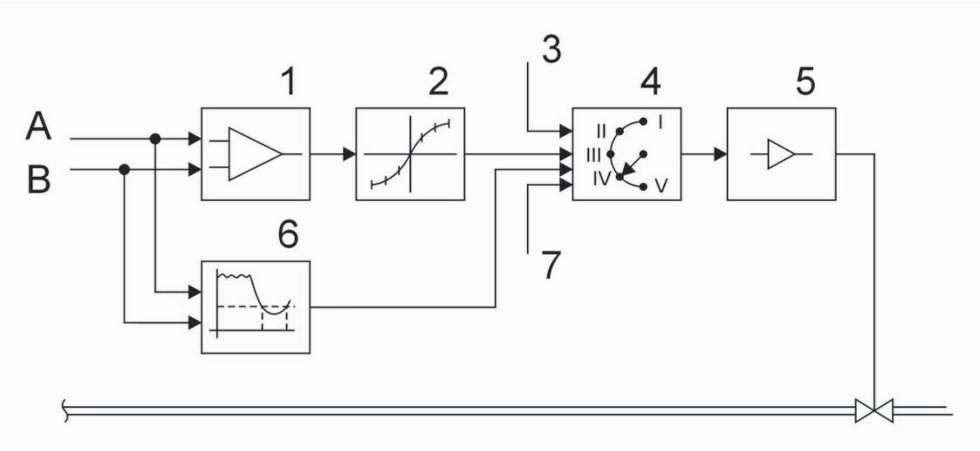


**Fig. 6.39: Automatic control system**

**Key to fig. 6.39:**

1 - The regulator performs correction in current settings or a change to the voltage grade between the current state in the furnace and the entered mode in the block 2; it allows a tolerance of ±19 % of current *I* against the nominal current; 2 - The function showing the optimum entered operating mode; 3 - Manual control; 4 - Priority logics (Electrode manual control 3, Automatic movement at short-circuit, Deceleration of electrode movement, Automatic control); 5 - Output amplifier; 6- Short connection logics; 7 - Locking at electrode break.

## 6.9. Automatic control of electrode position

The automatic control of electrode position is an important part of electrical device of electric arc furnaces, because it substantially affects their operation and values of technical and economic indicators. The task of the automatic control of electrode position consists in maintaining a constant amount of energy supplied to the working area of the furnace independently of continuously changing conditions for a certain period of time.

We require the regulator to be able to perform the control sufficiently fast. It must be set to the optimum condition after a deviation as soon as possible. Since in practice we do not have a regulator with an immediate response, due to inertia of masses of loaded parts and due to delay in the actual regulator system we have to take into account that there is a certain response period, i.e. period which elapses from releasing the command to the resetting at the moment when the electrode actually takes its new position.

At this time, however, a new condition can occur in the furnace due to very fast changes during arc burning. The changes occur, in particular, at jumping of the arc foot from individual charge pieces to the others due to arc burning on the tips and edges as well as due to different ionization of the arc burning zone and loop movement of the arc at changes of current load in neighboring circuits. The automatic control of electrode movement is not able to respond to the changes of this type immediately. Thus, it works in a certain zone of insensitivity. It responds only to current and voltage changes lying outside this zone. Insensitivity is a defined area, in which a change to the controlled quantity does not invoke a change to electrode movement.

For electric arc furnace regulators, the insensitivity usually relates to changes in current intensity and it is expressed in percents according to the expression.

$$h = \frac{I_n - I_d}{I_n + I_d} \cdot 100 \qquad (6.62)$$

where $I_d$ is current intensity, at which the electrode starts moving downwards; $I_n$ is the same for the upward movement.

By reduction of insensitivity zone, reduction in controlling stability might occur. Therefore, mainly at the time of charge melting-down, we are forced to work with the higher insensitivity of a regulator with a view of increasing in stability.

The most widespread control system is current-voltage or differential regulation, during which the regulator tries to maintain the set ratio of voltage and current at a constant level. The advantage of the current-voltage control is the ability to ignite the arc automatically and the fact that the distortion of a quantity controlled in one phase does not affect regulators of other phases. [10]

## 6.10. Steel melting arc furnaces

Steel arc furnaces feature a tub lined with basic fettling (lining). The charge content amounts up to 100 tons of steel. Electric power inputs amount to tens of MW. The optimum operating mode of steel electric arc furnaces depends on the whole range of technological factors, furnace structure, quality of electrodes, composition of the fettling, etc.

The main influencing factor, however, is the properly selected electric mode of the furnace. This mode can be regulated either by a change to voltage supplied to furnace electrodes and/or by changing the arc length - hence current.

The first way is commonly used during individual stages of melting in an electric arc furnace and it has direct connection to the metallurgical process.

The other way is determined by activities of automatic control of electrode movement which maintains constant optimum power input of electric energy to the melt at the respective voltage grade.

**Working characteristics**

For proper understanding of optimum operation of the electric arc furnace, it is necessary to know dependences of main electric quantities of furnace circuits, i.e. power, power factor, efficiency, on independently varying current.

The electric characteristics of an electric arc furnace can be set either by calculation or graphically using a circular diagram of an electric arc furnace, or by measuring on specific equipments of electric arc furnaces.

Using a simple equivalent scheme, we can express a power circuit of electric equipment in an electric arc furnace; see fig. 6.40. Individual elements of electric devices of the furnace are connected single-phase in series. Upon gradual simplification, we obtain a transformed equivalent scheme for resistances $R_N$, $X_N$, and $R_o$. Of course, we base our assumptions on some simplifications which cannot be neglected in practice.

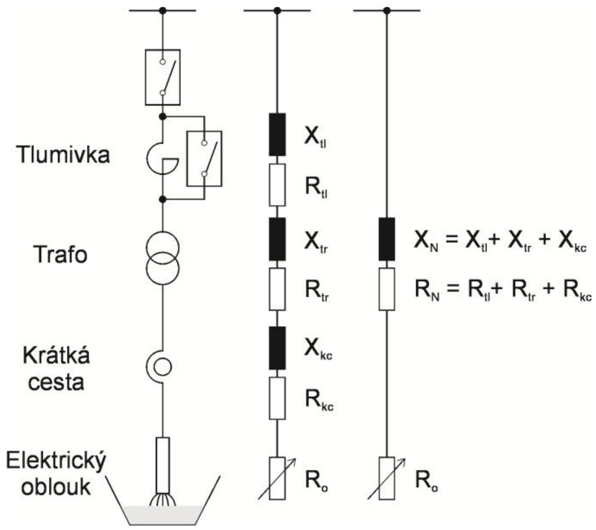


**Fig. 6.40: An equivalent scheme of electric equipment of the arc furnace**

The mains voltage is symmetrical and independent of load. The impedances of individual phases (except for the resistance of the electric arc) are the same and independent of the current. Electric arc resistance is linear. Transformer current at idle operation equals to zero. The values of equivalent resistances $X_N$ and $R_N$ are generally measured within short-circuit measuring.

The actual electric characteristics can be then formed using circular diagrams. An example of electric characteristics can be seen in fig. 6.41

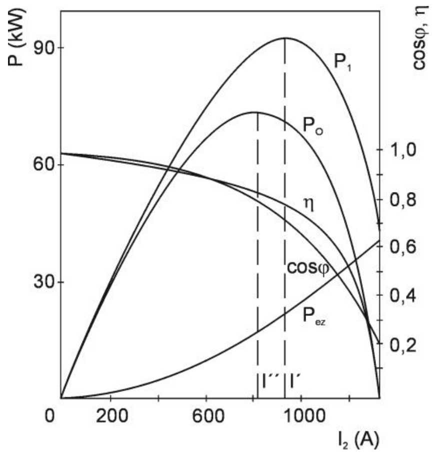


**Fig. 6.41: Theoretical operating characteristics**

The current corresponding to maximum primary power *I´* as well as maximum current on the arc *I´´* can be indicated on the characteristics. Sometimes, this current is mistakenly considered the optimum current corresponding to the maximum melting speed. However, determination of the optimum current value based only on the theoretical characteristics is a very reduced approach.

In particular, the mentioned simplifying assumptions cannot be accepted in practice. Further, it is necessary to take heat losses (in particular at the end of melting) into account; the losses are bigger than electric losses, so they affect both melting speed and specific consumption of electric energy. These heat losses can be considered constant and basically independent of electric quantities. The power compensating these heat losses is deducted from the power of the electric arc. The useful power thus equals to:

$$P_{už} = P_1 - P_{ez} - P_{tz} \qquad (6.63)$$

where $P_{tz}$ is power compensating heat losses; $P_{ez}$ compensates electric losses.

Energy efficiency of melting can be expressed by the following relation:

$$\eta_{en} = \frac{P_{už}}{P_1} \cdot 100 \qquad (6.64)$$

The specific consumption of electric energy *w* equals to:

$$w = \frac{W_{už}}{\eta_{en}} \qquad (6.65)$$

where $W_{už}$ is energy consumed for melting of one ton without considering any losses.

The equation for the melting speed is determined by the ratio of the total useful power consumed only for melting of charge to the power input of useful energy.

$$G = \frac{P_{u\check{z}}}{W_{u\check{z}}} \tag{6.66}$$

So, it is evident that upon considering of heat losses, we can express energy efficiency, specific energy consumption and melting speed as the function of current. In fig. 6.42, there are working characteristics of a 10-ton electric arc furnace for the voltage grade of 220 V.

From the chart, one can see the current value $I''$, corresponding to maximum energy efficiency $h_{en}$ and minimum specific consumption of energy $w_1$, $w_2$. It also applies that $I'' < I^*$ . This means that the mode of minimum consumption is reached at lower power input than the mode of maximum melting speed.

It can thus be said that the area of operating modes for the respective values of heat losses is limited by the above-mentioned characteristic values of currents $I''$ and $I^*$.

Based on theoretical working characteristics, it is not rational to work outside this area. The effect of higher heat losses on individual dependent variables can also be considered.

It follows from graphical dependencies that higher heat losses result in the reduction in melting speed, increase in specific consumption and the shift of its minimum to higher current intensities. The position of maximum melting speed, however, is not affected by the amount of heat losses. In fig. 6.43, the characteristics on the basis of measuring in the real 10-ton electric arc in Vítkovice Steelworks are indicated graphically.

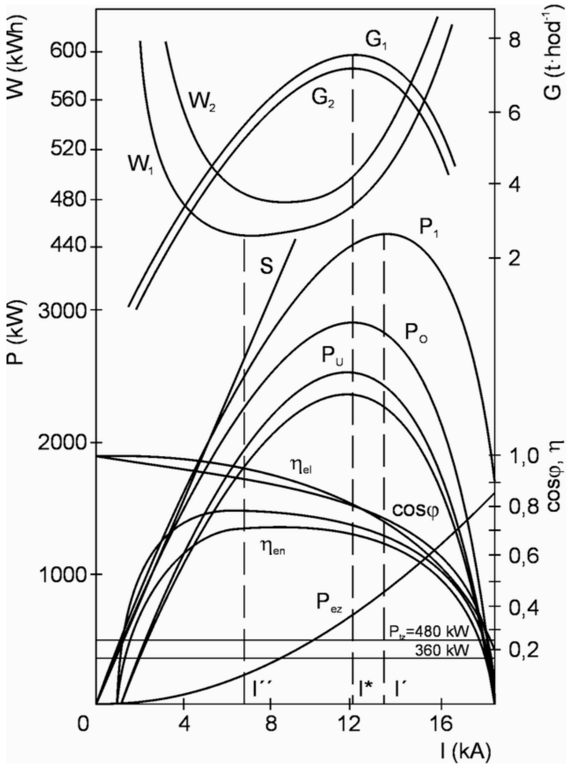


**Fig. 6.42: The working characteristics of a ten-ton electric arc furnace for the voltage grade of 220 V.**

In fig. 6.43, the characteristics are measured for each phase separately. There are certain differences between individual phases which manifest themselves mainly at higher current values. It is due to the influence of above-stated simplifying assumptions when each circuit of the specific phase works independently and the influence of live and dead phases is also applied [10].

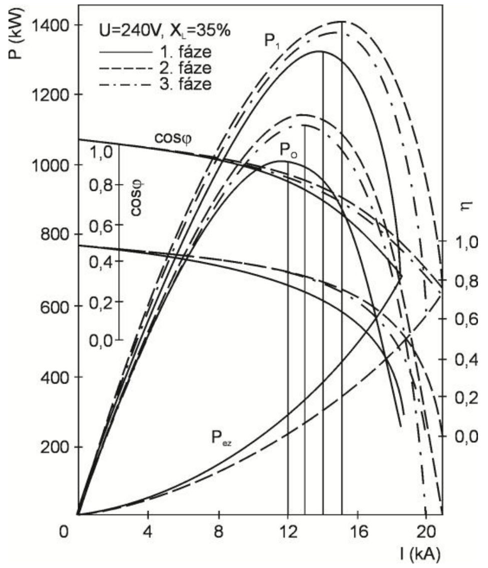


**Fig. 6.43: Working characteristics measured in a real ten-ton arc furnace**

# 6.11. DC electric arc furnaces

DC supplying of electric arc furnaces is a new technology in the field of application of the arc heat (fig. 6.44).

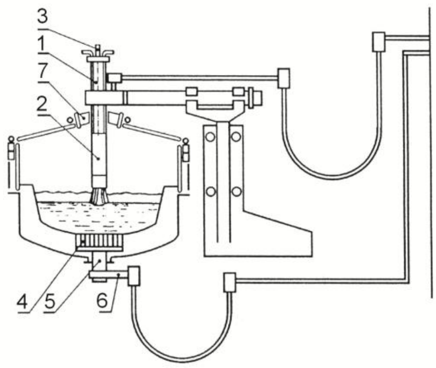


**Fig. 6.44: The scheme of DC arc furnace**

Description of DC arc furnace (Fig. 6.44):

1 - cooling of the electrode; 2 - graphite part of the electrode; 3 - supply of Ar or $N_2$; 4 - conductive bottom (anode); 5 - bottom electrode; 6 - DC supply circuit;

7 - ceramic electrode cap

A rectifier is a place between the transformer and the arc furnace. The electro-conductive path as well as the actual furnace differ from the traditional way of AC power supply. The power source consists of a regulating transformer and the actual transformer with a fixed transmission to low voltage. Furthermore, a fully-controlled six-pulse rectifier with bridge connection is included; it ensures good utilization of the transformer power and it complies with the dynamic requirements of arc furnaces.

In the DC part of the furnace power circuit, there is a choke for DC current. This choke limits stressing of thyristors by operating short-circuits and it helps to stabilize the arc.

The equipment for power factor compensation and filtration of higher harmonics is not required for DC furnaces. It is used only when the local power supply system features an insufficient short-circuit power at the place of the connection of a DC arc furnace (Fig. 6.45). [10]

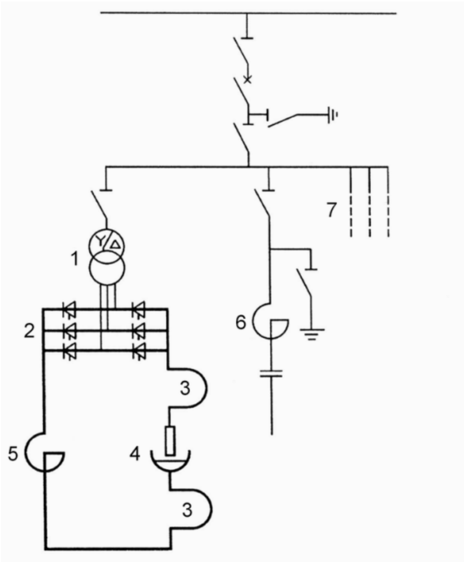


**Fig. 6.45: A wiring diagram of a DC arc furnace**

Description of DC arc furnace wiring (Fig. 6.45):

1 - regulating transformer; 2 - six-pulse bridge rectifier; 3 - short supply path; 4 - arc furnace; 5 - choke; 6 and 7 - filtration compensation equipment.

The actual tub of the DC arc furnace must have a conductive bottom and a special structure for the outlet of circuit current through the system of bottom electrodes. The main advantage of DC arc furnaces consists in substantial reduction in consumption of graphite electrodes which sometimes amounts up to 50 %. Furnaces with one electrode are designed up to 30 tons of charge. Then they show smaller wear and tear of linings in the furnace due to uniform distance of the electrode form furnace walls. Another advantage is the elimination of interferences influencing power mains, mainly dynamic voltage deviations. The noise level of operation is reduced from 110 dB to 90 dB compared to AC supply. A DC arc furnace is operated with a long arc, thus with the highest possible voltage grade in the feeder transformer. The long arc requires the use of foamed slag. The electrically conductive bottom of a DC furnace requires operation with fluid charge immediately at the start of melting. Therefore a part of liquid steel is left in the furnace; it allows connection of the circuit of the electro-conductive path. The advantage is the possibility of refurbishment of the current AC arc furnaces to DC arc furnaces while maintaining the original power of the transformer. [10]

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# 7. Electro-thermal plasma, laser, electron and infra-red equipments

In the second half of the 20th century, there were increasing requirements not only for quality and an amount of manufactured products, bus also for reduction of energy demands in production of melting processes and various heat treatment processes.

Melting processes in arc and induction furnaces during production of steels and their alloys have a lot of advantages in comparison with fuel furnaces. These furnaces, however, do not comply with the requirements for uniform crystallization, homogeneous distribution of various additives, etc., while producing high-quality alloy steels. Neither vacuum arc furnaces found widespread application in metallurgy; therefore other melting principles complying with demanding requirements for product quality were being searched for.

In the following chapters, we present electro-thermal equipments for plasma, electron, laser and infra-red heating. We will describe basic principles, electro-thermal equipments for individual types of heating as well as various technical applications, especially from the field of electro-thermal technology.

## 7.1. Plasma electro-thermal equipment

Plasma in its innumerable forms and technological applications is everywhere around us. Various types of plasma and its unique features (it is conducive, it responds to electric and magnetic fields, it can be an efficient radiation source) condition its use in various applications which are important to our lives and the world around us.

### 7.1.1. Plasma

The properties of plasma differ from the properties of gases and liquids; therefore it is called the fourth physical state of mass.

If we supply energy to the substance, the substance gradually transforms itself from the solid state to the liquid state and further to the gaseous state. With further increase in the kinetic energy of substance particles (molecules, atoms), the disintegration of molecules into smaller molecules or atoms occurs. This process is called molecule dissociation. Two situations can occur during mutual collisions of atoms based on their kinetic energy. At lower values of their kinetic energy, changes to their electronic envelope and atoms pass to the excite state (the state with a higher energy level). The excited state of particles is not stable and these particles pass automatically back into the basic condition. The energy which caused their excited state - the excitation energy - releases quanta of energy - photon - by radiation. At high values of the kinetic energy of atoms, electrons are released during collisions from the atom envelope and atom ionization occurs. Electrically charged particles thus occur in the gas - they are free electrons and positive ions. Upon electron capture by a neutral atom, a negative ion is produced. Neutral particles and ions are collectively referred to as heavy particles since their weight is much greater than the weight of electrons. The substance in the plasma state thus contains heavy particles, free electrons and photons. A typical manifestation of plasma is an ionized state and intensive radiation.

The changes to energy of individual particles, or occurrence and disappearance of colliding particles are called elementary processes whose existence is a prerequisite for development, maintenance and extinguishing of plasma of electric charges. The acceleration of electrons and ions by the electric field is considered to be the simplest elementary process. The arranged motion in the opposite or the same direction of the electric fields overlaps the random thermal motion of electrons and ions. The movement caused by the electric field is called the drift and it has negligible influence on the thermal motion of particles. The speed of the drift motion is much lower than the speed of electrons and ions between individual collisions.

Due to the presence of charged particles in plasma and their mutual electric interaction, the particles influence themselves at a greater distance than in the electrically neutral gas. This mutual electric interaction is one of the conditions for plasma existence; it is usually referred to as the collective behavior of plasma.

Another condition for plasma existence is the condition of quasi-neutrality, i.e. the plasma seems to be electrically neutral outwardly. The numbers of charged particles in the plasma can be arbitrary, but the total positive and negative charges must feature the same length. The number of neutral particles does not matter. The layout of charges in plasma, however, is not uniform. Electrons form clusters around positive ions; they are called the electron clouds which electrically shield the charge of positive ions. Areas, the so-called Debye spheres, inside of which the condition of electric neutrality is not met, occur. The mean size of these areas is called the Debye length $\lambda_D$.

After the introduction of the Debye length, we can define plasma as ionized gas (a set of charged and neutral particles) in the volume with a length size $L$ which is much longer than the Debye length ($\lambda_D << L$) a and meets the condition of quasi-neutrality. The mechanism of the Debye shielding applies only if there are enough $N_D$ particles in the charge cloud. If there is only one or two particles, the term of the Debye shielding cannot be valid. Therefore the condition $N_D >> 1$ must apply.

The third condition must also be met, so that we can call the ionized gas plasma. This condition relates to collisions between particles in the ionized gas, i.e. the inequality $\omega\tau > 1$ must be valid. In this inequality, $\omega$ is frequency of typical plasma oscillations and $\tau$ is the mean period of time between collisions with neutral atoms.

Since plasma contains free charged particles, it is, unlike gases, conductive. Through charged particles in plasma, we can affect plasma by the electromagnetic field and the plasma can receive energy from the electric and magnetic fields, or it can give the energy.

### Types of plasma

Plasma is the most common form of mass in the whole universe, both by its weight and its volume. We meet plasma at every step. It is all around us. Most of the universe is actually plasma. It is said that 99% of the universe is plasma. Above all, stars, nebulae and interstellar space are plasma. Stars, the Sun included, are actually big plasma balls. The amount of uncharged particles in plasma changes significantly; from 95 % in the low ionosphere up to less than 1 % in the solar wind in the continuous flow of plasma from the Sun.

There are a huge amount of plasmatic systems and the range of the scale for temperature and concentration (density) of particles is really wide. In the Earth condition, the temperature of about 0 K was measured for artificially formed plasma (for crystalline “non-neutral” plasma = purely electron plasma or purely ion plasma), and, conversely, magnetic thermonuclear plasma can reach the values in the order of $10^8$ K. For the cosmic plasma, temperatures from $10^2$ K (aurora) to $10^7$ K (sun core) can be considered. The concentration of particles varies in a wide range. In terms of the Earth, it varies in the range of $(10^7 – 10^{32})$ $m^{-3}$, for the cosmic plasma in the range of $(10^0 – 10^{30})$ $m^{-3}$. Plasma occurs in a wide range of pressure values. Although the basic parameters of plasma can vary within a wide range, the characteristics of plasma can be very similar.

Interactions between particles occur in plasma; interactions between charged and neutral particles in plasma are important to determine its behavior. The kinds of atoms (molecules) in the plasma, the ratio of ionized particles to neutral and energy particles result in a wide range of plasma types, their properties and behavior. Neutral particles predominate in weakly ionized plasma, so charged particles collide much more with neutral particles than with one another. The percentage of noncharged particles in fully ionized plasma can be lower than 1 %.

There are several criteria, according to which various types of plasma can be distinguished. The most general division of plasma is the division into natural (nature) plasma, e.g. (space plasma, interstellar gas, Earth ionosphere, etc.), and artificially formed plasma, created mainly in an electrical discharge. In addition to the division of plasma into the natural and artificial ones, we can also come across distinguishing between equilibrium and non-equilibrium plasma. The equilibrium plasma is usually referred to as the thermal (isothermal) plasma and the non-equilibrium plasma is known as non-isothermal (“thermal plasma” and “non-thermal plasma”).

Upon supplying of plasma energy, increase in particle kinetic energy occurs. The energy can be supplied by heat (thermal plasma), the electric field (typical non-equilibrium plasma occurs) or the electromagnetic field (RF plasma).

The thermal plasma (e.g. high-pressure electric arc) is plasma which is in the state of local thermodynamic equilibrium (i.e. the temperature of all particles is the same, the temperature of electrons is the same as the temperature of heavy particles $T_e = T_t$) or it is close to this state. The thermal plasma reaches temperatures up to $10^4$ K with electron concentrations of $(10^{21}$-$10^{26})$ $m^{-3}$. Non-equilibrium (non-thermal) plasma is the plasma with a large deviation from the kinetic equilibrium, i.e. $T_e >> T_t$. It is often referred to as “cold” plasma because of the low temperature of heavy particles. Non-equilibrium plasma systems usually work at low pressure values, $p <$ 10 kPa.

Another division of plasma is based on plasma temperature. Then plasma is often referred to as low-temperature or high-temperature plasma. The low-temperature plasma reaches temperature of max. $10^5$ K and no existence of nuclear reactions (such as nuclear fission into protons, neutrons, etc.) is assumed. The high-temperature plasma means the thermonuclear plasma which is usually formed by gases of light elements (such as hydrogen, deuterium, tritium, helium, lithium), with temperatures between $10^7$ and $10^8$ K, at which thermonuclear reactions can be assumed.

The low-temperature thermal plasma is significant for electro-thermal applications. [1]-[4]

### 7.1.2. Plasma heating application

The possibility of the combination of many extremely high parameters achievable by other thermal sources suits to technical utilization of thermal applications, individually:

- extremely high temperatures (compared to other thermal sources)
- extremely high temperature gradients in a small space,
- high heating dynamics,
- extremely high velocities of plasma flow even at sub-sonic flow,
- high plasma enthalpy,
- high kinetics of heating or subsequent cooling.

The possibility of generation of plasma with virtually arbitrary composition by a suitable selection of working gas, or controlled mixture of gases which can be also even very pure (compared with combustion products of conventional burners) is also important.

Thermal plasma created by the electric arc is used in multiple technological applications, e.g.at thermal modification of surface properties, spraying of materials in the form of powder or vapors, at the synthesis of ceramic materials with specified properties or with destruction of harmful substances and wastes.

The use of plasma for various technologies gave rise to new technologies collectively known as plasma technologies. The plasma technologies are among the most dynamically developing fields of technology and they play an important role in the development of new materials and new production processes which are more efficient and less harmful to the environment. Compared to other heat sources, the plasma technologies use extremely high temperatures and high velocities of the plasma flow (even at sub-sound flow).

The possibilities of use of plasma technologies are wide [5]-[9], form conventional, modern up to state-of-the-art technology. Individual applications are listed in the following summary:

- Conventional plasma technologies:

- plasma melting, plasma spraying, plasma (dry) etching, plasma cutting, plasma welding
- Modern plasma technologies:
- changes to material properties, development and production of new materials with surprising properties, disposal of persistent and toxic substances, processing of hazardous wastes, reprocessing and recycling of waste.
- Top plasma technologies:
- the field of nanotechnologies, vacuum plasma technologies.

Plasma furnaces in electric metallurgy are used mainly for melting or remelting of refractory metals for the purpose of increase in the purity of the final product. Plasma furnaces working with arc (electrode) plasmatrons which follow the design of arc furnaces are widely used. The heat source in plasma melting furnaces is low-temperature plasma with high temperature and speed, which is generated in the plasmatron. The plasmatron replaces the function of electrodes, which carburize charge, working gas and create the neutral protective atmosphere. Plasma furnaces have multiple advantages; in addition to high temperature and speed of plasma, it is also a high concentration of heat power of plasma, simple control and high purity of plasma, and therefore plasma processes.

Plasma spraying is a method which allows application of all kinds of materials from pure metals up to refractory metals (e.g. ceramics, cements) thanks to the high temperature of plasma. The high temperature and speed of impacting particles result in formation of coating with a very smooth and dense structure and with high adhesion to the base material. The electric arc between the water-cooled cathode (mainly the tungsten one) and the cylindrical anode (e.g. the copper one) simultaneously forming the plasma burner nozzle, burns. The gas (usually argon or other inert gas with several percents of gas increasing enthalpy of plasma, e.g. $H_2$, He, $N_2$) is supplied axially to the burner; plasma with high temperature (up to 20000 K) and enthalpy rises at its other end. The applied material in the form of powder is supplied into it using carrier gas. The temperature and velocity of the plasma flow depend mainly on the design of the burner, energy power input and the gas used. There are three basic types of plasma spraying: atmosphere plasma spraying (APS), vacuum plasma spraying (VPS) and spraying with high-frequency plasma (RF plasma).

Plasma can be used not only for application of layers onto the material surface, but also for removal of materials from the surface. In this case, it is plasma (dry) etching. This technology allows creation of smoother structure that can be obtained by conventional wet etching. Plasma etching plays a significant role in the production of chips in electronics;  without this technology, we would hardly have efficient computers and memory chips.

Another plasma technology, the technology of plasma cutting, has many advantages, it is mainly a possibility of use of this technology for cutting of all conductive materials, further high quality of the cut, high cutting speed, very good automation and a possibility of cutting by plasma under water. The disadvantage only consists in a wider kerf and limited thickness of material. While using plasma for plasma cutting, energy density in the plasma beam can reach the value of up to $2 \cdot 10^6$ W/cm$^2$. For cutting process, the pilot arc between the nozzle and the cathode is ignited by high voltage at first. This energetically weaker pilot arc partly prepares the path between the plasma burner and the object subject to machining by ionization. The power of the main arc is automatically increased by touch of the pilot arc with the machined object. This touch is called momentary incision or momentary punch. Thanks to its high temperature, plasma expands and flows at ultrasonic speed in the direction of the machined object (anode). The plasma temperature at leak from the nozzle can reach up to 30000 K and the speed up to the speed of sound. The nozzle is cooled down by circulating water and at some processes, the material subject to cutting is even placed under water. Water protects the nozzle from melting, it prevents noise and annoying smoke. The kind of gas flowing into the nozzle and which is subsequently heated up by the arc differs according to application; the most often used gases include argon, nitrogen, hydrogen, oxygen and their mixtures. By recombination of atoms and molecules outside the plasma nozzle, the received energy releases and the thermal effect of plasma on the machined object is increased.

Plasma welding allows welding of materials of various properties and dimensions. Currents flowing through the arc then range from 0.1 A for material welding thinner than 0.1 mm up to currents of 200 A, which can weld materials with a thickness of up to 15 mm. Again, it is an electric arc which is formed by supplying of voltage between the tungsten electrode and the material subject to welding. The nozzle is cooper and again, it is cooled down by circulating water. The plasma flow does not cut at this moment, but it forms a kind of shielding envelope. A mixture of argon and hydrogen is mainly used as shielding gas.

Plasma is an excellent means for surface treatment of materials. On the one hand, it is possible to treat the surface at the molecular level, so it will be able to form a bond with other substances easily. On the other hand, it is possible to cover the surface with other layers upon suitable choice of working gas. Thus, new materials with surprising materials are obtained or some of its properties change or improve. The ability of plasma to influence surface properties results from high energy of electrons, which are able to break chemical bonds. In addition to surface treatment, plasma can be used even for cleaning or disposal of persistent and toxic substances or decomposition of harmful substances into environmentally safe products. To do this, various types of discharges in gases and liquids or the machine called plasmatron are used. The operating principle of this equipment consists in the fact that plasma leaves the output nozzle at high speed; thanks to high temperature and reactivity of plasma, waste is decomposed into simple molecules or atoms.

Nowadays, plasma is starting to be applied also to the so-called nanotechnologies. In general, nanotechnology means a field of research and development, which deals with targeted forming and using of material structures in the scale of several nanometers at least in one dimension (0.1-100 nm)- The constructional elements of nanotechnology are molecules or even the actual atoms. Nanotechnologies are used in various scientific and technological fields, whose aim is precise controlling of each atom and molecule, so that an object (such as a chip, which is a thousand times smaller than structures so far made by common technology) or a structure featuring new properties (electrical, optical, physical and other) is produced. Applications of nanotechnologies are very broad and they are already used in many areas, for example, in electronics, mechanical engineering, textile industry, chemical industry, health care, electro technical industry and others.

### 7.1.3. Plasmatrons

Plasma technologies use a device as a source of thermal plasma; the device is referred to as a plasmatron or a plasma burner or a generator of thermal plasma. [5] [6]

Specific types of plasmatrons are distinguished according to different views. The determining criterion is the way of plasma generation, according to which we can distinguish arc plasmatrons (plasmatrons with electrodes) and induction and dielectric plasmatrons (plasmatrons without electrodes). The induction plasmatrons create and maintain plasma by the method of induction heating and generally, they are supplied from high-frequency sources with working frequencies from 1 to 30 MHz. Dielectric plasmatrons are new types of plasmatrons, for which the principle is analogous to dielectric equipment. We will deal with the arc plasmatrons in a more detailed way.

Plasma generation in arc plasmatrons (hereafter referred to as plasmatrons) is similar to the generation of an arc discharge (arc plasma) in arc furnaces. The difference consists in the plasma-formatting environment. In plasmatrons, plasma is generated and maintained in the environment of plasma-formatting gases and within the wide range of pressure values; in arc furnaces using elementary processes in electrode vapors and in gases from the surrounding, usually at the atmospheric pressure. In addition, the plasma flow shape, the way of energy releasing and heating of flowing working gas depend on the plasmatron design.

Thus, a plasmatron is a source of thermal plasma and it is based on heating of a working substance (gas or liquid) by an electric arc. Plasmatrons must comply with the following requirements:

- temperature of plasma must be thousands to tens of thousands degrees (up to 50000 K),
- plasma properties must comply with the required application,
- controllable heating process (a possibility of changing plasma parameters),
- sufficiently long continuous operation (in the order of hours up to hundreds of hours),
- sufficiently high efficiency.

To meet the above-mentioned requirements, these equipments with the electric arc feature advantageous properties:

- a possibility of implementation of high-temperature plasma flow with high power (and high content of enthalpy),
- use of the standard sources of electric power (rotation assembly - engine-DC generator, controlled thyristor rectifiers, controlled semiconductor converters),
- gas heating even at high pressure values (up to 10 MPa),
- a possibility of continuous controlling of plasma parameters during equipment operation.

Other criteria of plasmatron division include, for example: the kind of power supply, technological effect, working medium or a way of arc stabilization in the plasmatron. Division according to the above-mentioned criteria is shown in the following list:

According to the type of power supply:

- with direct current (prevailing),
- with alternating current (usually three-phase, special).

According to technological action:

- with an independent arc (a separate anode);
- with a dependent arc (the anode is machined material).

According to working (stabilization) medium:

- gas,
- liquid.

According to the way of stabilization of the arc column shape:

- with intensively cooled wall (Maecker burner),
- with intensive axial blowing of the arc,
- with whirling stabilization of the arc,
- with the liquid wall.

A typical example of design layout of the plasmatron with a DC supply and the independent arc is shown in Fig. 7.1. and it consists of the following parts: a - cathode and the cathode area, b - plasmatron channel (arc channel), c- anode with the typical anode foot, d - stilling chamber,
1, 3 - cooling water inlet and outlet, 2 - working gas supply

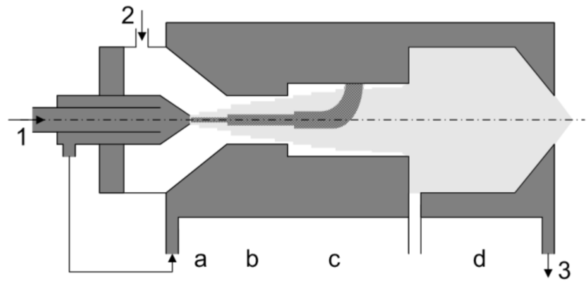



**Fig. 7.1: Constructional layout of a plasmatron with AC power supply**

The cathode emits current carriers in the arc (electrons), a gaseous working medium is usually fed to the cathode area. The cathode material is usually thoriated tungsten or hafnium.

In the plasmatron channel, i.e. in the main working area, the arc discharge burns, causing the heating and transition of the working medium to the plasma state. The plasmatron channel is characterized by high volume density of energy in the order of 10 to 100 $MW \cdot m^{-3}$, therefore it must be intensively cooled down or graduated.

The point of impacting of electrons accelerated by the electric field along the arc, the anode foot, is locally a place with high heat load. To avoid destruction of the anode wall, this anode foot must be continuously moved along the anode surface, at least by tangential flowing of working gas, or by strengthening of the magnetic field. The construction of the area, where the anode foot moves, forms the so-called anode space, from which the plasma stream can flow to free space.

The stilling (stagnation, mixing) chamber is designed for the homogenization of the plasma flow or for an increase in pressure in the arc space. The stilling chamber with output konfuser or with the Laval nozzle uses pressure to increase outlet speed of the plasma flow.

The electric arc, whose shape is stabilized in various ways to burn in the specified area continuously, is used in plasmatrons. This space is usually a cylindrical channel or a nozzle where the arc is blown by working gas or by the vapors of the liquid. Plasma is formed from the stabilizing medium in the arc column continuously and then it flows from the outlet opening in the form of a plasma beam.

The effect of shape stabilization of the arc is manifested primarily by rectifying and limitation of the arc discharge path and by effective cooling of outer layers. In industrially used plasmatrons, macroscopic acceleration of plasma in the axis of the arc occurs. Gas (liquid) between the arc and the wall is heated up only by heat from the surface layers of the arc and its temperature is very low compared to the temperature of the arc. This cooler gas reduces the cross-section of the arc column, therefore, its density as well as its temperature grows at the same value of the flow in the arc. If it is needed to increase the temperature of the arc, it is necessary to intensify cooing of its surface layers or to reduce the cross-section of plasmatron channel.

The arc column at wall stabilization (fig. 7.2a) fills the nozzle up to its wall, which cools down boundary layers of the gas. To ensure axial transport of heat energy by flowing gas, the gas is supplied in small amounts to the electrode chamber.

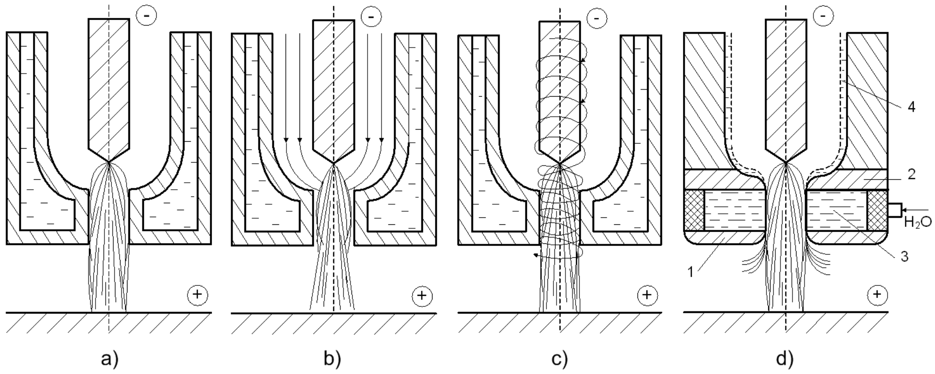



**Fig. 7.2: Ways of shape stabilization of the arc**

Stabilization with intensive axial blowing (fig. 7.2b) differs from the wall stabilization by the fact that the amount of axially flowing gas is so high that there is a relatively cool layer of gas between the hot core of the discharge and the nozzle wall. Working gas must be supplied uniformly around the circumference of the annulus around the cathode; the cathode channel must be short, otherwise there is a risk of a contact between the arc and the wall.

Whirling stabilization of the arc (fig. 7.2c) is more efficient stabilization compared to the previous two types. The tangential gas inlet to the cathode space gives the axial flow of the working medium in the anode channel a rotating component, which proportionally grows along the channel radius, leading to the situation when the arc holds in the axis where it is cooled least. The gas rotation continues also in the nozzle and while the arc is burning, the lighter hot gases get closer to the discharge axis due to centrifugal forces, whereas cooler gas gets to the walls.

With stabilization with the liquid wall (fig. 7.2d) working gas is not supplied to the plasma burner, the arc burns in the channel formed by rotating fluid - most often water supplied at high pressure tangentially at more places along the structural shell of the anodic channel. A water whirlpool develops; it has two diameters, the whirlpool with a small diameter of free opening in the stilling chamber and the whirlpool with a larger diameter in the electrode chamber 4. The nozzles 1 and 2 are protected by a layer of water, which flows into the stilling chamber uniformly to both the sides provided that the diameters of both the nozzles are the same. At the end of the anode channel, most fluid must be exhausted again. Plasma is formed from vapor evaporated by the arc energy from the liquid surface. The advantage is cheap operation.

Another criterion for type classification of the electric arc is a cathode emission mechanism. For cathodes from graphite and refractory metals (tungsten etc.), electrons are emitted by thermal emissions. They are designed in the form of bars and the most suitable material is thoriated tungsten because of its high electron emission and a low grade of erosion in the non-oxidizing atmosphere. Hafnium cathode tablets pressed into a copper body of the cathode are used for the oxidizing environment. They are, however, expensive and the output work for electron emission is considerably higher. Graphite electrodes feature a high erosion grade and they are used as movable electrodes, for example with a water-stabilized arc. Temperature and current density on the surface of cathode crater reaches values of $T >$ 3000 K, or $10^7$ to $10^8$ $A \cdot m^{-2}$, for $W+ThO_2$ at currents of 1 kA to 10 kA and for graphite in the range of 5 kA to 100 kA, respectively.

For cathodes from materials with a lower melting temperature, such as copper, its alloys and steel, electrons are emitted as a result of high intensity of the electric field on the electrode surface. Due to lower temperatures on the emitting surface of the electrode, it reaches values of $0^7$ to $10^8$ $Am^{-2}$at permissible current load of 0.05 kA to 2 kA. Electrodes are designed as cylindrical channels and they are usually equipped with coils for creation of the magnetic field in the arc space, so that rotations of arc feet are ensured.

The selection of electrode material is closely related to the working gas used. From this point of view, argon is the most suitable due to very low erosion of electrodes in the inert atmosphere. To achieve higher efficiency of heat transfer and higher power density, mixtures of argon with gases with higher values of specific enthalpy and thermal conductivity, such as $H_2$, He or $N_2$, are used. Significant electrode erosion in the oxidizing atmosphere, which forms in the arc chamber of the stabilized arc, is the main problem of industrial application of this very promising type of plasmatron. For the time being, the problem is dealt with by a movable, consumable graphite cathode and external copper anode in the form of a water-cooled rotating disc.

With the use of air or another oxidizing atmosphere as working medium with a gas-stabilized arc, reducing of electrode erosion is dealt with the creation of a protective inert atmosphere in the area of the cathode. Argon consumption is minimized to losses incurred by the leak for creation of a slight positive pressure against the main gas circuit. Reduction of cylindrical anode erosion is ensured by the distraction of heat load in the entire circuit of the anode cylinder by rotation of the arc foot by combined action of the tangential supply of the main gas and the magnetic field of the coil installed on the anode.

For the actual operation of the plasmatron, other technological accessories are required: water management (for cooling), gas management (a source of working medium), electric power mains, the actual power source, stabilizing elements (resistance, inductor). In addition, also other auxiliary equipments, such as an ignition circuit to initialize the arc or magnetic rotation of the circuit anode food by means of the coil wound on the anode in the area of the anode foot.

## 7.2. Electron electro-thermal equipment

### 7.2.1. Physical principle of electron heating

It can be said that electron heating is analogous to plasma heating. An electron beam replaces the function of plasma flow generated in the plasmatron; it is a beam of electrons with high kinetic energy. The electron beam is generated in the electron generator (electron-beam gun) in the environment of vacuum, and technological heat is obtained from the kinetic energy of electrons. [10], [11]

All possibilities of electron technologies are based on the features of electrically charged particles. In practice, only electrons are significant because they can be relatively easily released from the shell of the atomic nucleus and then control purposely. They can move in a crystalline grid of some substances, conductors as electric current with magnetic and thermal effects or completely freely in the vacuum where we can control them by the electromagnetic field and use them in various ways.

The physical principle of electron devices consists in the conversion of electric energy to heat by impact of accelerated electrons to the respective material (charge), to which the accelerated electrons give their kinetic energy. Accelerated (initial) electrons impact on the charge and penetrate into it. According to the amount of energy supplied per a time unit onto specific area (power surface density), the electron bundle can have various effects; from simple thermal effects through melting-down up to evaporation of material. Electron penetration depth is several micrometers (event at accelerating voltage 40-100 kV), i.e. only surface heating occurs; further heat is transferred by conduction (in liquid charge also by flowing). The penetration speed of the electron beam under the surface may be many times greater than the speed of conductive heat propagation.

Accelerated electrons are strongly inhibited on their journey because mutual collisions with particles (atoms and molecules) of gas occur. To avoid inhibition of electrons by collisions, the entire process occurs in high vacuum at a pressure of $10^{-2}$-$10^{-4}$ Pa. In this environment, electrons can move without collisions with gas molecules along the path which is several meters long (the mean free path). Accelerated electron energy reaches 10-200 keV.

With penetration of electrons through charge, interaction with the crystalline grid as a whole occur as well as interaction with individual micro-particles (atoms, molecules, electrons) of the crystal. The result is an interaction of fields of all these particles. Their vibration increases. This vibration manifests itself by temperature increase in the respective place outwardly.

In the electric field, electrons gain the kinetic energy

$$W = \frac{m_e v^2}{2} = eU, \qquad (7.1)$$

where $m_e$ is the weight of the electron, $v$ is the velocity of the electron, $e$ is the charge of the electron, and $U$ is the accelerating voltage. The probability of collisions between electrons and gas particles is low in sufficiently high vacuum. The speed of electrons can be expressed by the following equation:

$$v = \sqrt{\frac{2eU}{m_{\mathrm{e}}}} \doteq 5{,}93 \cdot 10^{5} \sqrt{U}. \qquad (7.2)$$

From the above-mentioned equation, one can see linear dependence of electron speed on accelerating voltage *U*; the electron speed for accelerating voltage of 1 V is 593 $km.s^{-1}$. The incident power is directly proportional not only to voltage, but also to the number of electrons n impacting onto the respective material per 1 second.

$$P = n\frac{m_{\mathrm{e}} v^{2}}{2} = neU = IU. \qquad (7.3)$$

The equation for calculation of electron speed (7.2) is valid for accelerating speed up to kV, when the effect of relativity can be neglected because the attained speeds of electrons are relatively small with respect to the speed of light.

Electrons penetrating under the surface of the solid substance give all their kinetic energy on a relatively short path. By calculations as well as by measurement, it can be proved that electrons penetrate in a solid substance to the depth $a$ determined by the following equation:

$$a = 2{,}1 \cdot 10^{-5} \frac{U^{2}}{\rho}, \qquad (7.4)$$

where $\rho$ is density in $g \cdot cm^{-3}$ and *U* is accelerating voltage in kV. The penetration depth of electrons means the path of electrons in material, on which they give their kinetic energy to particles of the solid substance in the affected volume. This is reflected in the first stage by increasing of its temperature, which, according to the circumstances, can lead to changes in the state and other phenomena.

Upon the impact of the electron beam on the surface of a solid substance, a lot of phenomena (Fig. 7.3) happen; not all kinetic energy of primary electrons is transformed into heat. Losses occur. As a result, heating efficiency is lower. As it has been already mentioned, the primary electrons collide on their path not only with atoms and molecules of the crystal, but also with electrons (both free and bound) inside the charge; they have lower energy, but the same weight. Upon collision of the bound electron of the charge with the primary electron, the bound electron receives energy, which it uses for transformation to a higher energy level. It does not remain here for long; it returns quickly to the original level, radiating quanta of energy *W* at frequency

$$f = W/h, \qquad (7.5)$$

where *h* is Planck's constant. The frequency of this radiation is high; it falls within the area of X-ray radiation and it does not participate in heating. Also upon collisions of primary electrons with free electrons of the charge, radiation of x-rays may occur. A certain part of energy supplied by primary electrons is thus consumed for radiated light and heat, or for ionization. Upon some collisions of primary electrons with electrons of the charge, removing of the primary electron or the electron of the charge out of the charge may occur. The so-called secondary emission of electrons occurs and efficiency of heating is reduced. Other losses can occur if the primary electron is removed out of the charge and if it performed only flexible collisions, i.e. if it changed only the direction of movement and it keeps its original energy. Its full reflection occurred and almost all energy of the electron is not used for heating of the charge.

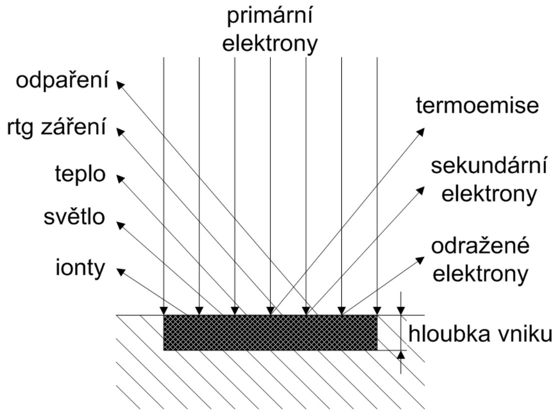



**Fig. 7.3: Surface phenomena**

The proportion of an individual type of losses in the total losses at heating by accelerated electrons in an electron remelting furnace is listed in the following summary:

- losses in the electron-beam gun and during conduction of an electronic beam (approx. 1 %),
- losses in the evacuated area in front of the charge (1-15 %); the losses grow rapidly if the vacuum is low,
- losses caused by reflection of electrons from the charge (approx. 25 %),
- losses by secondary emission of electrons from the charge (approx. 1 %),
- losses by x-ray radiation (1 ‰ – 1 %),
- heat losses by radiation of the heated charge (20 - 30 %),
- energy taken away by water in cooling of a crystallizer (the rest to 100 %).

### 7.2.2. Electron equipment

The basic part of the electric equipment is the electron-beam gun, which is a source of the controlled electron beam, i.e. it is intended for generation, acceleration and focusing of the electron beam. In practice, we require that the electron source provides for a coherent electron beam, which means that the electrons should come from a point source, they should have the same energy, and moreover, their accompanying wave should be in the same phase. [12]-[17]

The electron beam is formed by free electrons and we need a thermal-emission source for its creation. A thermo-emission cathode heated by passing of electric current can be such a source. An amount of emitted and usable electrons, i.e. the emission current, depends on the properties of the material of the cathode, the size of its surface and temperature. The kinetic energy required for emission of electrons from the cathode, i.e. for overcoming of the so-called potential barrier existing at metal surfaces, can be supplied by heating of the cathode to high temperature. The highest usable temperature is specified mainly by the requirements for the duration of the cathode, which is limited by evaporation. A cathode is usually made from tungsten (less often from tantalum). The reason is the requirement for the high temperature of melting of metal used. The cathode is usually directly or indirectly heated; i.e. without any auxiliary cathode or by means of the auxiliary cathode. The service life of the cathode varies and ranges from about 10 to 100 operating hours.

The actual heated cathode does not create the required electron beam. The kinetic energy must be supplied to electrons obtained by emission from the cathode, so that the electron beam has the smallest possible diameter; thus allowing deflection of the beam from its direct path and changing of the place of impact. This can be achieved by a suitable shape of the accelerating electric field, i.e. by the suitable shape of electrodes which it is made of. This requires a lot of systems, for example for acceleration, focusing and deflection of the electron beam. The entire design arrangement is then referred to as the electron-beam gun.

The design of electron-beam guns is affected mainly by the shape and arrangement of the electrode systems; with cathodes, even by the way of heating. According to the arrangement of the electrode system, we distinguish the following electron-beam guns:

- with an axial system,

- with a system of the ring cathode (close or distant),

- with a transverse electron beam.

Electron-beam guns can also be divided according to their functions or a position of the metal subject to remelting with respect to the electric field of the electron-beam gun. The design, when the charge is outside the field of the electron-beam gun (the charge is not an anode), allows a sufficient distance of the charge from electric circuits and eliminate the possibility of secondary discharges in the field. The design of the gun, when the charge is not an anode, is also known as the gun with an independent electron beam. Electron-beam guns, where the conductive charge serves as an anode and must be near the cathode, are guns with a dependent electron beam.

The most versatile gun is the electron-beam gun with an axial system (so-called axial electron-beam gun), which can be designed for powers from several kilowatts to the power of 1.2 MW. In large and powerful devices, only the axial electron-beam guns are used; for low and medium powers, guns with a ring (circular) cathode are used. The life of the cathode of the gun with the ring cathode is much shorter than the life of the axial gun; but they feature very good heat and electrical efficiency, which is a result of the fact that the anode is the actual charge. Analogically according to arc equipment, the electron-beam guns, where the anode is the charge, can be called the electron-beam guns with dependent (direct) action; the electrode-beam guns with independent (indirect) action have a separate cathode and anode.

To secure the required vacuum ($10^{-2}$-$10^{-6}$ Pa), vacuum devices, pumps or vacuum pumps are used. In addition to creating and maintaining of the required vacuum at any time of the technological process, they pump produced gases and vapors in working chambers away.

**Electron-beam gun with a thrust system**

An electron-beam gun with a thrust system use a thermo-emission cathode as a source of electron; the cathode is often heated directly for lower powers (around 25 kW; Pierce gun) and it has a shape of the fiber from tungsten or tantalum wire or tape. For higher powers (up to 150 kW), indirect heating of the cathode is used; i.e. a flat cathode from tungsten or tantalum is heated by electrons from the auxiliary cathode, or a compact cathode heated by radiant heat. The life of this gun is usually up to 300 hours; i.e. about 400 melts of melting processes.

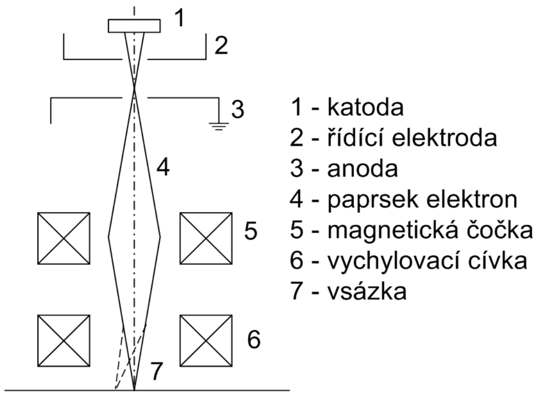
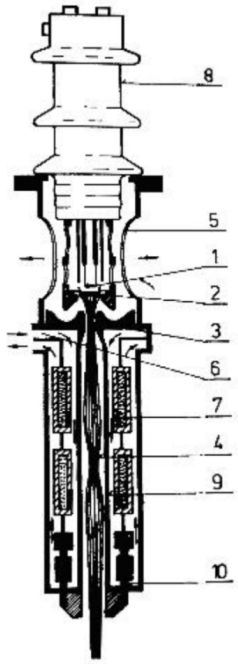



**Fig. 7.4: A simplified diagram of the Pierce electron-beam gun**

In Fig. 7.4, there is a simplified image of the function of the Pierce electron-beam gun The cathode 1 is placed on a HV insulator and the negative pole of accelerating voltage is conducted to it. The anode 3 is on the earth potential; its shape is cylindrical with an opening in the middle. Electrons are accelerated by the electric field (high voltage) between the cathode and the anode. Between the cathode and the anode, there is a control electrode 2, which forms the beam of rays, so that it passes through the opening in the middle of the accelerating anode. The control electrode is also placed on the HV insulator and it features negative bias in relation to the cathode. The change to this bias controls the current of the electron beam 4. The cathode, anode and the control electrode form an electrostatic lens. The coil 5 (magnetic lens) forms the magnetic field, by means of which the beam of electrons is directed to the small tab of the charge 7. The electron beam can be focused by the change to current in the magnetic lens. A deflecting coil 6 deflects in the desired direction. It allows deflection in two axes (e.g. x, a) and it makes precise setting of the electron beam to the weld place or to control of its position possible.

The advantage of the Pierce gun consists in the fact that it is incorporated outside the vacuum area of the furnace; thus being easily replaceable. To increase the total production of the furnace, electric furnaces work with four and more electron-beam guns. Powers of furnaces even over 1 MW can be obtained by parallel cooperation of multiple guns; ingot weights reaches up to 20 tons and the specific consumption of electric energy ranges around 900 $kWh \cdot t^{-1}$. The advantage of furnaces with multiple guns is a possibility of operating upon outage of any of the guns, thus enabling finishing of the melting process without any interruption.

By limiting the impact of the electron beam, the impact on the very small area of 1-0,01 $mm^2$ can be reached; a high amount of heat is released in the area. The thermal effect is determined by the speed of electrons; furthermore it depends on the value of accelerating voltage and on the amount of electrons which bombard the respective area, as it arises from the equation (7.3). A greater amount of electrons can be obtained upon increasing in fiber heating current intensity or upon increasing in the emitting surface.




1 - auxiliary directly heated cathode

2 - main cathode

3 - anode

4 - electron beam

5 - evacuation openings

6 - cooling

7 - magnetic coil

8 - insulator

9 - guide tubes

10 - magnetic deflection

**Fig. 7.5: Pierce electron-beam gun**

An electron-beam gun with a thrust system with indirectly heated cathode (Fig. 7.5) features an auxiliary cathode 1 for heating of the main cathode 2 by impact of electrons. The main cathode is an anode for the auxiliary cathode with a voltage of about +5 kV. The main cathode is heated up and it emits other electrons, whose number depends on the material and temperature of the cathode. The electrode shaping the electric field, so that electrons can fly through the opening in the anode, is connected to the main cathode. Electrons are accelerated by the anode 3. The anode is hollow and its voltage is about + 30 kV. Behind the anode, there is a coil with an iron core and a gap, around which there is a magnetic field focusing electrons in a narrow beam, the so-called magnetic (focusing) coil 7. Focusing coils concentrate a beam of electrons 4 and they prevent its dispersion. The entire system of the gun is in an air-tight housing with a safety enclosure of the vacuum and a place for connection of vacuum pumps. The space under the anode towards the charge is free of the electric field; the electron beam impacts on the charge by inertia with the kinetic energy of up to 100 keV. The area of focusing coils must be cooled by water 6, so that its winding powered by AC current does not overheat and burn. The system contains other coils for focusing and deflection of the beam 10, or another device with coils for magnetic spreading of electrons around the respective area is often fixed to the bottom, output part of the gun. The gun body with the anode is grounded and it is mounted to the working space with the charge without any insulation. High voltage is insulated by insulators 8.

Collisions of electrons with particles of the environment cannot happen, so that electron acceleration is intensive; therefore, the entire process takes place in the vacuum. Some designs feature labyrinth seals separating the space of the highest vacuum around electrodes from the area of lower vacuum, where there are only focusing coils. Since the vacuum in the application chamber (furnace) can be and often is a bit lower, another labyrinth seal is placed in front of the output from the electron-beam gun.

**Electron-beam gun with a system of a ring cathode.**

The electron-beam gun with a system of a ring cathode has the main cathode in the form of tungsten wire (fiber) rolled into a circle (a ring). The main cathode is placed to the center of the so-called Wehnelt cylinder, which features negative bias and thanks to its action a cloud of electrons forms around the emitting cathode. They are gradually exhausted from the Wehnelt cylinder to the anode. There are various structural designs, e.g. with a system of close or distance ring cathode.

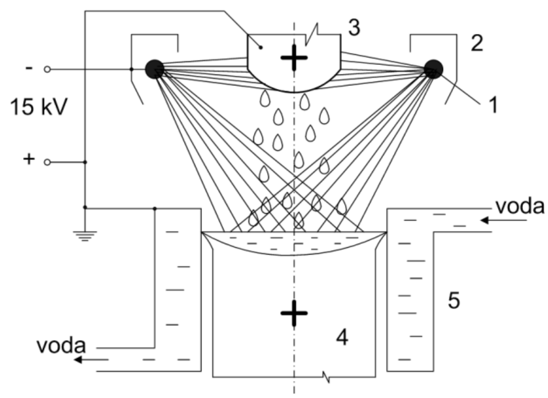



**Fig. 7.6: The electron-beam gun with a system of a close ring cathode.**

The principle of the electron-beam gun with the system of a close ring cathode is shown in Fig. 7.6. The main cathode 1 is directly heated by alternating current. The heating source must be insulated from the ground to the full anode voltage 10-15 kV. The accelerating cathode 2 is conductively connected with the cathode and it helps to direct electrons to the charge. The anode is formed by the charge, i.e. a metal bar 3, which should be remelted, and ingot 4, which is formed in the water-cooled crystallizer 5. The disadvantage of this solution consists in anodic involvement of the melt. Thus, melting of the metal bar as well as the surface of ingot occurs. Thus vapors are formed; they fill the working space. The cathode is directly exposed to vapors and splashing drops of the melt; its life is reduced and the cathode is damaged relatively quickly. In addition, the required high vacuum cannot be maintained between the cathode and the anode.

The electron-beam gun with the system of a distant ring cathode eliminates the deficiency of the close cathode. The cathode recedes itself from the metal subject to melting and it gets to the areas, where there is no risk of metal splashing and away from the reach of its vapors. In addition, there is another anode in the system; it closes individual elements of the cathode, thus securing protection of the hot fiber. It eliminates any electric fields between the gun and the melt. The melting anode is grounded. The directing of the electron beam and adjusting of the impact onto the melting anode and the level of the melt is reached by the magnetic field formed by a pair of coils. The first of them is placed a few centimeters above the melting anode and the other to the vicinity of the ingot. The accelerating voltage and powers of the electron-beam gun with the distant ring cathode are usually higher than with the system with a close cathode.

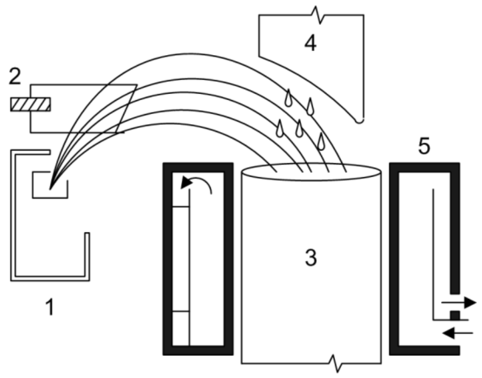



**Fig. 7.7: Electron furnace with a transverse electron beam**

The electron-beam gun with sources, vacuum equipment and accessories forms the electron furnace. There are also other construction designs of melting and remelting furnaces, for example with a transverse electron beam (Fig. 7.7). It is another development type derived from preceding types. The electrode system 1 is formed by the directly heated cathode, the Wehnelt electrode and the anode. The cathode is made from tungsten wire rolled to the U-shape and it is horizontally placed in the Wehnelt ring. The accelerating anode is in the immediate vicinity of the cathode. The generated electron beam is deflected by the transverse magnetic field of the coils 2 to the melting are and it impact onto the surface of the ingot 3 and remelted material 4. The melting area is away from the reach of action of the electric field. Also in this situation the cathode is protected from effects of metal vapors and splashing melt because it is situated outside the axis of the melting area. A remelted ingot solidifies in the crystallizer 5.

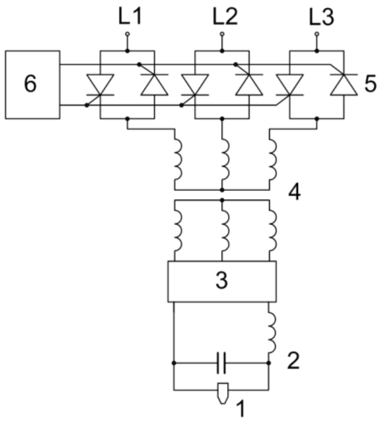



**Fig. 7.8: A power scheme of the electron-beam gun.**

One of the possible ways of supply of electron-beam guns is shown in Fig. 7.8. The electron-beam gun 1 is connected to the secondary winding of the transformer through filters 2 (a choke and a capacitor) and a rectifier 3. On the terminals of the electron-beam gun, rectified and sufficiently smoothed supply voltage is obtained. The secondary voltage of a HV transformer 4 is controlled by thyristors 5, which are controlled by the circuit 6.

## 7.2.3. Application of the electron heating

From the technological point of view, an electron beam can be used for:

- thermal processes - melting of refractory metals and chemically active materials, welding, cutting, soldering, drilling of long holes with small diameters, heat processing of refractory metals, sintering of refractory metals (for powder metallurgy), etc.
- non-thermal processes - an electron beam serves to induce a chemical reaction. It concerns mainly the field of lithographic technologies used in electro-technology for manufacturing of chips. Up to 200,000 structure details can be made on the chip; this cannot be achieved by any other technology.

The electron melting furnaces are used for remelting - refining of metals. An ingot or rod material can be default materials. Electron melting is applied to refractory metals, such as W, Mo, Ta, Ni and also for production of semiconductors, where high purity is required. By controlling the power of the electron-beam gun, the temperature of the bath can be kept at a very high level for a long time because the electron-beam gun features a high density of power input.

Advantages of electron furnaces:

- possibility of melt temperature control within broad ranges,
- production of quality material due to low pressure in the furnace,

- the melting process can be permanently visually monitored.

Disadvantages of electron furnaces:

- increased evaporation of metals with high vapor tension (Mn, Cr, Ti),
- low thermal efficiency (<15 %), high specific consumption of energy,
- more complex structure and operation of the equipment,
- the furnace is the source of X-ray radiation.

Machinability of material by an electron beam is determined by its physical properties and it does not depend on mechanical features. Electron beams can be used for machining mainly of hard machinable materials, such as refractory steels, austenitic steels used for construction of nuclear reactors, zinc alloys with niobium, aluminium and titan alloys, silicon, gem stones, tantalum, tungsten and special alloys used for aviation and astronautics.

An electron beam penetrates into the material up to a certain depth, where the motion of electrons stops. The resulting thermal energy concentrated below the surface causes eruption evaporation of materials. The particles of evaporating material moves at a great speed from the opening. The resulting vapors of the evaporated material are ionized and causes new focusing of the beam in the working area. Material removal happens by repetition of the process.

Out of all above-mentioned technologies, welding by an electron beam is probably used most often. The focused beam heats up the material of joining components to the temperature higher than the melting temperature. The melt is mixed at the interface, usually without any addition of material, and after cooling down, a solid joint occurs. Electron beams feature the ability to penetrate under the surface at high speed. This allows very deep root penetrations which, moreover, excel with minimum width of the affected area. The achieved ratio of the depth to the width of the weld is up to 30:1. Root penetrations achieved by today's top equipment are more than 150 mm deep. The characteristic features of electron welding include low deformation of components after cooling down, minimum excess of weld metal, thus eliminating the possible machining of the weld surface, the strength of the weld close to the strength of the base material, high accuracy and reproducibility, permanent high quality of welds, a possibility of welding of hard weldable metals and welding of combination of various metals which often cannot be welded by any other methods. Electron beams are suitable for welding of hard weldable or completely non-weldable materials. Welds are free of cracks, they feature low porosity and they are metallurgically pure. Thin products with a thickness of 1mm can be welded as well as it is possible to perform welds deep up to 40 mm. Electron welding can be carried out at high vacuum, at partial vacuum or without vacuum.

Nowadays, electron cutting is replaced by laser cutting.

Electron beams are used for drilling of holes of small diameters (from 0.015 mm), at a speed of 4000 holes per second. For drilling of deep holes with the ratio of the length to the diameter of the hole up to 100, the diameter of the beam must be 2-4times smaller than the required diameter of the hole subject to drilling. Tolerance of the drilled-out hole is 5-20% of its diameter.

The electron beam can heat up a small volume of material very quickly and thanks to quick deflection of the beam and very good controllability and reproducibility of the entire process, it brings new possibilities in heat processing of metals. The group of heat processing in the solid phase, i.e. also when no melting of material occurs, include hardening and annealing. For surface hardening, it uses the fact that quick escape of heat from the hot zone to the depth of the material occurs after very short heating of a thin surface layer, thus the surface layer of the material is hardened. Annealing is applied mainly for local surface recrystallization. If material is melted during the process, it is called processing in the liquid phase.

## 7.3. Laser electro-thermal equipment

### 7.3.1. Laser

A laser is one of the biggest inventions of the 20th century. Its name was derived from the initial letters of an English name *Light Amplification by Stimulated Emission of Radiation*, which captures the basic principle of this device. According to the English name we can say that laser is a light amplifier working on the principle of stimulated emission.

A laser is a general name for quantum generators of electromagnetic radiation of specific, precisely defined properties, originally only in the frequency band of light. This name was later adapted even for quantum generators of radiation at other frequency bands, for example for X-ray radiation, $\gamma$-radiation. Laser at frequency bands of optical radiation (in the infrared, visible or ultraviolet spectrum) are suitable for application in thermal technologies; therefore the designation "optical quantum generators" is also used in lasers.

Lasers are generators of strictly monochromatic, very intensive and spatially restricted beam of light rays. Monochromatic radiation in electromagnetic radiation of one, precisely specified frequency. Optical radiation generated by lasers is focused into a very narrow interval of wavelength and it is largely coherent. Coherent light has the same wavelength and the same phase in a certain place and at a certain moment. Common sources radiate incoherent light. Lasers are sources of coherent lights. The actual laser features high emittance and low divergence of a laser beam.

Laser theory is based on the quantum description of both the actual laser system and the surrounding environment. In order to understand the principle of laser design, we have to become familiar with the basic findings of nuclear and quantum physics. [10], [14], [19]

### 7.3.2. The physical nature of laser

Particles (atoms, molecules, ions, electrons) cannot have arbitrary energy, but they can occur only in specific energy conditions, at specific energy levels. According to the quantum theory, a particle can obtain or release energy only in steps, in specific quanta. For example electromagnetic energy of the field can be radiated (emitted, generated) or absorbed only into certain minimum quanta - photons.

In the basic state, a particle has the lowest energy. If we supply sufficient energy to the particle (for example by heating or irradiation), it goes from the lower energy level to the higher level; it is excited. The way electrons are excited to the excited level is called exhausting. A particle remains in the excited state only for a very short period of time and after radiation of one or more photons, it returns to its basic state. At the quantum transition from the state with higher energy $W_2$ to the state with lower energy $W_1$ ($W_2 > W_1$), the energy passes the energy difference $\Delta W = W_2 - W_1$ to the surrounding environment. Otherwise the particle receives the energy $\Delta W$. The quantum transitions of particles can be either radiation or radiationless. The radiation transitions form the essence of functional mechanisms in lasers. Energy quantum (photon) is either radiated, or absorbed at the radiation quantum transition. In accordance with the Planck's law, the energy quantum is determined according to (7.5) by the equation:

$$\Delta W = h\,f, \tag{7.6}$$

where $h$ is the Planck's constant, $f$ is the frequency of radiation absorbed or radiated at the transition from one energy level to the other. The particles which are excited represent the so-called active particles. Particles can swap to the lower energy level by two ways; stimulated or spontaneous emission (Fig. 7.9).

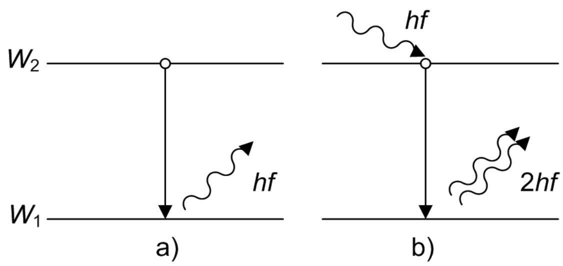



**Fig. 7.9: Spontaneous emission a), stimulated emission b)**

Lasers work on the principle of the stimulated emission (Fig. 7.9b) of electromagnetic radiation of active particles excited by an external power source at a frequency equal to the frequency of a quantum transmission. The direction of propagation, frequency, phase and polarization of the stimulated radiation and stimulating (exciting) radiation are identical. A characteristic feature of the stimulated radiation is a narrow spectral line, time and spatial coherence, high directivity and high radiation intensity.

Contrary, at the spontaneous emission (Fig. 7.9a), the presence of the external power source is not necessary. The excited particles go from the higher energy level to the lower energy level spontaneously. The direction of propagation, phase as well as the polarization of the emitted radiation are random.

Lasers are high-power sources of radiation. The high power is very important to the operation of lasers in order to reach the predominance of the stimulated emission over the spontaneous one. The probability of the stimulated emission is proportional to the number of photons present in the system, i.e. to the radiation intensity. Upon increasing in radiation intensity, the predominance of the stimulated emission over the spontaneous one is achieved.

The fundamental technical elements of lasers include:

- active environment,
- optical resonator,
- exciting power source.

The active environment is a set of particles which are able to generate the stimulated radiation after supplying of energy from an external source. The active energy can be a solid substance (crystal, glass, semiconductor), gas or a gas mixture or the liquid.

In order to strengthen the emission of the stimulated radiation, it is necessary that the active environment abounds in active particles. Since the particles are equally dispersed in the active environment, it is necessary that the path, along which the photons go, is as long as possible. Therefore the active environment is placed into the optical resonator. An example of the optical resonator is Fabry-Perot resonator; it is an optical system formed by two parallel, flat, usually glass plates at a certain distance, in which multiple reflection and multi-beam interference occur. The resonator is usually a system of two mirrors (an open resonator). One of the mirrors features zero transmittance, the whole radiation is reflected back; the other mirror is semi-permeable and it allows a part of radiation to leave the active environment in the form of a laser beam. Radiation occurs only at the resonant frequencies of the resonator used.

The exciting power source or the equipment for pumping of electrons is chosen based on the type of the active environment. If the active environment is a crystal or glass, optical sources, i.e. xenon or krypton discharge lamps, are suitable; their radiation spectrum must be the same as the active environment. For gaseous active environment, an electrical discharge generated either by DC voltage up to 15 kV, or impulse voltage up to 40 kW lasting up to several ms. For the semiconductor active environment, a stream of electrons is used. The sources of excitation energy can also be chemical reactions.

Several conditions must be met, so that the particles in the active environment of the laser maintain their excited state and have amplification properties:

1) the energy states with energy $W_2$ must have more particles than the energy states with energy $W_1$, i.e. $N_2 > N_1$;

2) the particles in the excited state must remain for a longer time (up to $10^{-3}$ s), i.e. up to $10^5$ longer time compared to the particles excited by Joule heat
(in the order of $10^{-8}$s);

3) the particles of the laser active environment must have at least 3 energy levels.

The probability that a photon invokes emission is the same as the probability of its absorption. If the first condition is not met, absorption prevails over emission and the original radiation is not amplified, but weakened. The required distribution of particles is, however, contrary in the heat equilibrium, i.e. the states with higher energy occur less often than the states with lower energy. Therefore, it is sometimes asserted that it is necessary to create an inversion state (inversion) for laser operations. Another formulation of this condition relates to temperature, i.e. negative absolute temperatures are necessary for laser operations.

The excited state, in which the particle can remain longer, is referred to as a metastable state. The transitions between energy levels are governed by selection rules. According to these rules, the radiation quantum transitions from metastable states are prohibited. The probability of quantum transition is low; therefore the particles in the metastable state keep its excitation energy for a relatively long period of time.

In order to create an inverse occupation of electrons in energy levels of atoms and subsequent acquisition of the stimulated emission, the particles of the active environment must have at least three energy levels (Fig. 7.10a). Electron exhaustion occurs from the basic level by energy $W_1$ to the level with energy $W_3$. The life in this band is very short and electrons quickly jump to the level with energy $W_2$, where they remain for a longer period of time. The electrons are in the metastable state. The quick transition of electrons from the band $W_3$ to the metastable level $W_2$ is typical by the fact that it occurs without any quantum of radiation (the radiationless transition). The actual stimulated emission occurs between the levels of $W_2$ and $W_1$. The level $W_2$ must be sufficiently far away from the basic level, so that its filling due to thermal fluctuations of the environment does not happen. A downside of the three-level system is fundamentally low efficiency due to the fact that more than a half of particles must be excited from the basic level to reach the inversion. The operation of a three-level laser, therefore, requires a very intensive exhaustion.


**Fig. 7.10: A system of three and four energy levels**

Upon using the three-level system, you can reach higher efficiency (Fig. 7.10b). Exhaustion takes place between the levels of $W_1 \rightarrow W_4$, the quantum transitions between the levels of $W_4 \rightarrow W_3$ and $W_2 \rightarrow W_1$ are fast, spontaneous and radiationless. The $W_3$ level is metastable. Photon emission (laser radiation) occurs between the levels of $W_3$ and $W_2$. There are considerably fewer electrons on the $W_2$ level than in the basic state, therefore less energy is required to achieve the inversion between the levels of $W_3$ and $W_2$. If we choose sufficiently low temperature, so that electrons are in a steady state gathered only on the $W_1$ level and the level $W_2$ is almost empty, the inversion can be reached in a considerably easier way; this is an inversion against an almost empty level. The $W_2$ level must be sufficiently far away from the basic level, so that its occupation due to thermal fluctuations of the environment does not occur. Such a system is selected that the transition $W_3 \rightarrow W_1$ is prevented.

### 7.3.3. Laser types

Lasers work all on the same principle, i.e. the stimulated emission; however, they can differ in their designs and properties. There are various criteria of division. According to the applied active environment, lasers are divided into:

- solid state (e.g. ruby, fiber, Nd:YAG laser),
- gas (He-Ne, $CO_2$, Ar, excimer laser),
- semiconductor,
- liquid (laser with an organic dye).

A special kind of lasers is a laser with free electrons, where the active environment is accelerated electrons with energy of 10 MeV to 10 GeV. The accelerated electrons move into a periodical magnetic field, which forces the electrons to oscillate in the transverse direction. The electrons in their motion along the curved paths emit electromagnetic waves.

Based on the type of operation, lasers are divided into continuous and impulse. A continuous laser operates in the mode of continuous emitting of laser radiation, and an impulse laser in the mode of impulse radiation. Impulse lasers achieve higher power, i.e. $10^3$ – $10^{16}$ W. Power of continuous lasers is $10^{-3}$ – $10^3$ W. The continuous lasers feature a higher degree of coherence and a narrower spectral line, which is an important property for some special applications (such as holography, interferometry).

According to the number of energy levels, lasers can be divided into three-level (e.g. a ruby laser), four-level (e.g. $Nd^{3+}$) and multiple-level (e.g. gas lasers).

Another criterion of laser division can be, for example, the wavelength, way of particle exhaustion (a discharge lamp, discharge, a stream of electrons, chemical reactions).

For electro-thermal technologies, the solid-state and gas lasers (a $CO_2$ laser) are applied most often.

**Solid state lasers**

The active environment of solid-state lasers are dielectric crystals or glasses containing ions of precious soils ($Nd^{3+}$, $Er^{3+}$, $Ho^{3+}$, $Dy^{2+}$) or transition elements ($Cr^{3+}$, $Ni^{2+}$, $Co^{2+}$, $Ti^{3+}$).

A ruby laser is the first solid-state laser constructed and commissioned by Theodore H. Maiman in 1960. The active environment is a mono-crystal in the shape of a cylinder generating coherent radiation of a wavelength of 694.3 nm (red light). The ruby mono-crystal ($Al_2O_3$ + 0,05 % $Cr_2O_3$) has a diameter of about 1 cm and a length of 5-10 cm. It is an ion laser when the active environment is formed by chrome ions and three energy levels of ruby are used for its operation.

Solid-state lasers are excited optically. The basic issue of exciting of optical generators is the use of the supplied luminous flux. Used sources are not monochromatic, they have a wide spectrum of radiation while only a narrow part of the spectrum is used for excitation of the active environment. Xenon discharge lamps of various shapes (bar, spiral) are used for impulse operation. Spot xenon high-pressure discharge lamps are used for continuous operation.

During action of sufficiently strong optical radiation (approx. 0.5-1 ms), the ruby absorbs yellow-green or blue-purple light through an absorption belt. Thus chromium atoms are pumped from the basic state to the metastable state. The fronts of the ruby roller are covered with material featuring a high coefficient of reflectance and they form the resonator mirrors. One of the mirrors is totally reflective, the other is semi-permeable. If the losses in the resonator are lower that the gain in the active environment, ruby becomes the source of the stimulated radiation. This radiation manifests itself by a short flash coming from the semi-permeable front of the Fabry-Perot resonator which is formed by electroplated faces of the ruby cut. The flash starts with a delay after the start of the pump impulse of the xenon discharge lamp and it usually lasts 1-2 thirds of the time of the discharge lamp pump impulse.

In Fig. 7.11 [20], there is a diagram of one of the first ruby lasers, where the ruby is excited through a spiral xenon discharge lamp. The basis of the laser is a ruby mono-crystal (the active environment) 1, a resonator 2, 3 and an excitation xenon discharge lamp 4. The ruby mono-crystal and the discharge lamp (or several discharge lamps) are placed in the excitation cavity, which most often has the shape of an elliptical cylinder, in one focus of which a discharge lamp is placed, whereas in the other focus, there is the active environment. Its inner part is highly glossy reflecting surface, so that the highest possible amount of light from the discharge lamp can get into the ruby; i.e. the light of the discharge lamp is used for excitation as much as possible.

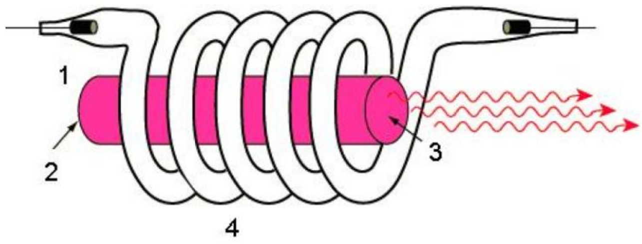



**Fig. 7.11: Ruby laser**

The total efficiency of ruby lasers is less than 1 %. Most of the energy is dissipated in the crystal in the form of heat; the shift of energy levels occurs and generation of laser radiation is interrupted. Therefore, an important part of ruby lasers is cooling.

Nd:YAG laser is currently most widespread solid-state laser, whose active environment is the Nd:YAD crystal. It generates radiation at room temperature in pulse and continuous mode in the infrared spectrum (wavelength 1.064 μm).

**Gas lasers**

The active environment of gas lasers is a gas state. Inversion is achieved between energy levels of some gas components, i.e. atoms (atomic laser), ions (ion laser) or molecules (molecular laser). There are a lot of types of gas lasers; the most important ones include helium-neon laser (He-Ne laser) emitting radiation of a wavelength of 632.8 nm, 1.15 μm or 3.39 μm and a $CO_2$ laser (10,6 μm or 9.6 μm). The first gas laser (He-Ne) was designed in 1961.

Most gas lasers work in a continuous mode. However, some gas lasers working in the impulse mode have been developed. The divergence of the output laser beam is lower than with solid-state lasers; the widths of spectral lines are generally much smaller as well. The disadvantage is relatively low density of gas particles, therefore high-power gas lasers feature either large dimensions, or fast gas flow is provided by an optical resonator. Gas lasers are usually excited not only by an electric discharge, but also by a chemical reaction, photographic dissociation, fast expansion of gas, passing of the fast electron beam or optically.

An He-Ne laser is an atomic gas laser and the active environment is formed by excited active atoms of neon which are excited in a glow electric discharge in the mixture of exciting gas, helium and neon. The electrons of the discharge give a part of their kinetic energy to the inner states of atom during inflexible collisions, causing a transition to higher energy levels. An He-Ne laser can generate radiation either in the visible, or also in the infrared spectra, based on resonator tuning. From a structural point of view, two variants of optical resonator became common; the internal and external ones. The internal optical resonator is placed inside the discharge tube; the disadvantage is, however, gradual eroding of mirror surfaces by discharge products.

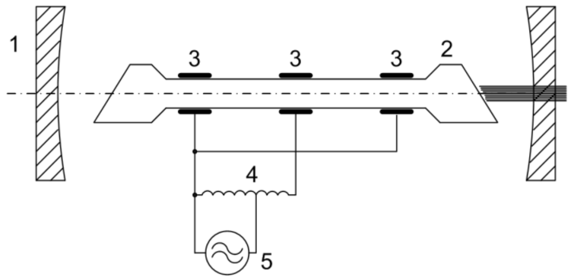



**Fig. 7.12: A diagram of an He-Ne gas laser**

In the external optical resonator (Fig. 7.12), the mirrors 1 are separated from the discharge room. They are placed inside the tube 2. The tube (about 1 m long) is ended by windows tilted under the so-called Brewster angle (the angle, at which light polarization at reflection happens), so the output beam is linearly polarized. Thus it is ensured that the electric component of the field features minimum losses at the interface of glass-air. For each laser transition, it is possible to find other optimum parameters of the discharge tube, i.e. the inner diameter, composition and gas pressure. The tube allowing the work of the laser at an arbitrary transition has a diameter of 5-100 mm, the total gas pressure in the tube is low, approx. 100-200 Pa, the current is 25-50 mA. In the tube, there are 3 electrodes, between which either a high-frequency (power supply through an autotransformer 4, generator 5 features the output of about 100 W at a frequency of 27.12 MHz), or DC (2 kV, 50 mA) pump discharge arises.

A $CO_2$ laser is a molecular gas laser, whose active environment is a mixture of $CO_2$ $N_2$ and He at a pressure of several kilo-pascals generating infrared radiation. This type of laser is usually used in heat applications; it is one of the most widely used lasers at all. It has the largest energy efficiency (about 10-15 %) out of all lasers; thanks to the efficiency, it reaches continuous power of up to 300 kW. The $CO_2$ laser design is made from a discharge tube (the inner diameter of several cm) filled with the mentioned mixture and an open resonator (two mirrors). A HV source is connected to the electrodes; it maintains a longitudinal glow discharge. The output beam comes from the semi-permeable mirror. There are a lot of variations; low-power types ($10^{-3}$ – 10 W) feature small dimensions and long durability. For power of tens to hundreds of watts, the tube is even several meters long. Power of tens of kilowatts is then obtained in some high-volume systems. The discharge is powered either by direct current, or also by alternating current from the power mains 50 Hz.

**Semi-conductor lasers**

The function of semi-conductor lasers is based on the emergence of optical radiation stimulated emission in semi-conductors at the quantum transitions of electrons from conduction to valence energy belt and on the existence of radiative recombination of carriers of charge electrons and holes.

To generate coherent radiation, it must be ensured that inversion is obtained in the active environment of laser, and further, that a positive feedback is created. The inversion is reached by injection of charge carriers by a PV transition (injection laser). Injection semi-conductor lasers enable direct modulation of power of optical radiation by electric current *I*. The positive feedback can be achieved upon modification to a semi-conductor photo-diode to a form of the Fabry-Perot resonator surrounded by two semipermeable mirrors (Fig. 7.13). Semi-conductors with the so-called direct transition, with which radiative recombination without participation of crystalline grid vibrations are suitable for semi-conductor diodes.

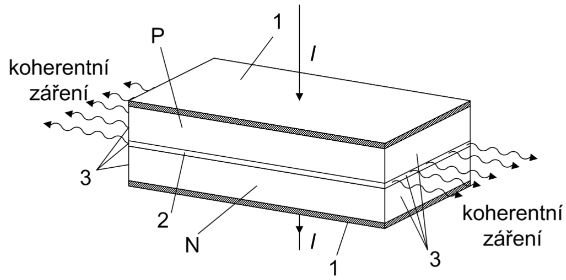



**Fig. 7.13: The basic layout of a semiconductor laser.**

The basis of a semiconductor laser is a light-emitting diode. In the PN transition of the diode during passing of the current in the permeable direction, luminescence - spontaneous emission - occurs. If the transition layer is placed in the resonator, a transition to the stimulated emission, which is characterized by narrowing of the spectral line, occurs. The resonator is created by polishing of front surfaces.

The most commonly used semiconductor laser is a laser with a double heterostructure, whose basis is a ternary semiconductor (formed by three materials) (GaAl)As, working within the band of 0,7 ÷ 0,9 μm and threshold current is in the order of 10 ÷ 100 mA.

Typical power of semiconductor lasers is in the order of milliwatt units at continuous operation at room temperature and it is several times higher at the impulse operation or at lower temperatures; the typical limit modulation frequency is in the order of GHz. Semiconductor lasers GaAs generate radiation in the band of 0.84 μm and quaternary InGaAsP in the band of 1.3 ÷ 1.5 μm.

## 7.3.4. Laser application

The use of lasers is widespread, they are applied in most human activities, not only in industries, but also in medicine, biology, microelectronics, computing technology, measuring technology, in military applications and others. Most lasers we come across in our common lives, are lasers with low power, for example, laser pointers, laser printers, photocopiers, optical mechanics, security devices for laser effects, but lasers are also used in information transfer.

Technological use of lasers is based on application of the ability of lasers to concentrate energy of radiation in the space, time as well as spectrum interval and on the interaction of radiation with a substance. The optical beam coming from the laser can be focused on a small area (diameter 10-100 μm), which results in an increase in the intensity of optical radiation to $10^{12} \div 10^{16}$ $W \cdot cm^{-2}$. The laser beam is able to machine both metal and non-metal materials. Another big advantage of laser application is a possibility of machining without mechanical contact with material, machining of hard-to-reach places and machining of materials, which cannot be machined normally.

In principle, laser heating means absorption of a laser beam in the material to be heated and the subsequent transformation of its energy to thermal energy (technology heat). With metals, the laser beam penetrates to the depth of only $10^{-7}$-$10^{-8}$ m; heat is thus produced in the tiny surface layer, from which it is spread by conduction. Laser radiation penetrates almost wirelessly by air and other gases. Therefore heating can be done in an arbitrary atmosphere and far enough from the laser. Laser heating can be well controlled, thus suitable temperature courses and required depth can be set. The specific beam power and the time of heating (the motion of the beam on the surface) is decisive. The higher thermal conductivity of the material is, the quicker the heating of surface layer must be; thus the premature thermal balancing is prevented. The specific surface power input must be the greater for the respective depth, the faster the heat is dissipated towards the depth.

Upon the interaction of radiation with a solid substance, the substance is firstly heated up, then melted and finally, it is evaporated. These phenomena are the basis for technological application of lasers including laser cutting, drilling, welding, hardening, application of lasers in microelectronics, laser decoration of glass and others.

The most commonly used lasers for laser cutting are continuous $CO_2$ lasers with medium power up to 15 kW. At the point of impact of the laser beam, the material is melted and it is blown away from the cut by a flux of gas. Yet almost no waste occurs and material subject to cutting need not to be clamped; therefore this application is used for high-precision cutting of fragile or easily deformable materials, such as textile or paper. For metal cutting (titanium, low-carbon steel, stainless steel), reactive gases are supplied to the point of cutting (for example oxygen); for non-metallic materials (ceramics, plastic, wood), inert gas is supplied.

Laser drilling is based on removal of material by evaporation. Impulse lasers with impulse length shorter than 2 ms, for example a ruby laser or a Nd:YAG laser, are used. The advantage of laser drilling consists in a possibility of creating of small openings with a diameter of 10-100 $\mu$ even at places, where it is not possible using other methods.

During laser welding the material is melted to the required depth using optical radiation. Compared to laser cutting and drilling, it requires less radiation intensity of the optical beam and longer laser pulse. Compared to other similar technologies (soldering, arc and resistance welding, welding with an electron beam), the laser welding has many advantages: there is no contact with an electrode, localized heating and quick cooling, ability to weld various materials and shapes, ability to weld materials in the respective atmosphere or melted inside the optical transparent material. The laser welding allows creation of a very fine and quality weld. The entire process can be computer controlled. Thank to energy focusing on a tiny surface, even materials with high melting temperature can be welded. Most often, Nd:YAG lasers and continuous $CO_2$ lasers are used.

By hardening, the metal is heated up to the so-called hardening temperature and then it is rapidly cooled down. Thus, it gets better mechanical and physical properties. Laser hardening uses optical radiation of lasers for quick heating. The advantage of laser hardening is the possibility of localized heat processing also in the places where they are impossible with other methods, and practically deformation-free processing. $Co_2$ lasers with powers of several kilowatts are used. The disadvantage of laser hardening is high purchase costs and low energy efficiency. But it has lots of advantages as well as peculiarities, among others mainly a small amount of supplied heat, good controllability and a possibility of local hardening, very fast heating (within several seconds) of the surface layer to the depth of several tenths of millimeters.

Using of lasers in microelectronics (e.g. laser grooving, laser marking, etc.) is based on removal of a thin material layer by evaporation which happens due to radiation by a laser beam. Lasers are used, for example, for repairs to semiconductor memories, during which laser disconnects damaged circuits and connects others instead of them. Lasers also repair damaged matrices (dies) for lithography, removes impurities from material surfaces etc. [10], [18].

Glass decoration by laser is heat processing of a glass surface using laser radiation. At the point of impact, partial evaporation of molten glass and its surface cracking occur. On their edges, light is dispersed, thus luminous appearance is obtained. Lasers emitting infrared light are used for decoration; the IR light is well absorbed by the glass; e.g. $CO_2$ lasers.

In practice, pulse ruby lasers for drilling of hard materials in medicine, for laser location of satellites, etc. are used. A Nd:YAG laser has a wide range of applications. In the industry, it is mainly used for laser machining, cutting, marking and welding. It is also used in medicine, physics and in biology as well as for military applications. Semiconductor lasers are widely applied in optical electronics (optical transmitters) where their small dimensions and the conformity of beam dimensions with dimension of optical elements are used; further in computing technology (optical memories), consumer electronics (CD players), in robotics (sensors), etc.

## 7.4. Infrared electro-thermal equipment

### 7.4.1. Infrared radiation

Infrared radiation (IR radiation) is electromagnetic radiation which is spread directly through a spherical wave. Wavelengths are situated in the optical band, i.e. in the part of electromagnetic spectrum, for which optical laws apply. They are, in particular, Kirchhoff's law, Planck's law, Wien's law and Stefan-Boltzman law.

Infrared radiation is invisible electromagnetic radiation, whose wavelength spectrum is within the range of $\lambda \in <0.74;\ 2000>$ μm, the frequency range is within $f \in <3 \cdot 10^{11};\ 3.8 \cdot 10^{14}>$ Hz. This zone of wavelength is sometimes divided into a close zone (0.74-2.5 μm), medium zone (2.5-50 m) and far (distant) zone (50 μm to 2 mm). Infrared radiation with wavelengths in the close zone is caused by gas atoms or additive ions in the crystal at quantum transitions between excited electron states. Infrared radiation from the medium zone corresponds to quantum transition between vibration states of molecules, whereas infrared radiation from the distant zone corresponds to quantum transitions of molecules. Infrared radiation features the wavelength bigger than visible light (0.38-0;78 μm), but shorter than the micro-wave length (3 mm to 30 cm). Interfaces of each zone are not defined strictly and they may differ in literature.

### 7.4.2. The principle of infrared radiation formation

Infrared radiation features significant heat effects. It is not visible to humans, but we register it as a heat sensation. Surfaces of bodies are heated up by absorption of arbitrary electromagnetic radiation, but, for example, objects at room temperature emit the most radiation in the infrared band 8-12 μm.

The principle of IR formation can be simply described as follows: Let us take a body, which only emits energy - i.e. an absolutely black body. We supply energy (for example, electrical energy) which causes an increase in motion of molecules forming the body. Faster oscillating and vibrating particles have more energy and they may jump to higher energy levels. In this condition, they cannot endure as it is an unstable condition. At mutual collisions of molecules, they pass to lower vibration and rotary energy levels and they emit the energy excess to the surrounding environment as radiation. The energy which the particles lose during the collision equals to energy of the generated quantum of radiation. Much oscillated atoms and molecules emit high-energy quanta, i.e. visible light and UV light. Little oscillated molecules emit low-energy quanta, i.e. infrared radiation.

For the purpose of heating, we will identify as infrared radiation such radiation of solid bodies whose electromagnetic radiation at impact on other body is, more or less, absorbed or changed to thermal energy, which causes heating of such a radiated body. Infrared radiation thus invokes thermal effects upon impact on absorbing material, thus heating the material further. Another heat spreading in the material is a result of heat conduction processes. At temperatures lower than 600 K, the radiation is very weak and heat transfer by flowing begins to apply more. Infrared radiation penetrates only to a slight depth of material due to high frequencies, at which infrared radiation occurs.

The radiant energy depends on the absolute temperature of the radiator, the size of the area and emissivity. The absolute temperature of the radiator surface decisively influences infrared radiation. For radiation of energy particularly in the infrared area, it is necessary to maintain the temperature up to max. 2000 K, when the infrared radiator starts to produce also a part of visible light. It should be noted that radiator power is changed with the 4th power of the absolute temperature. This means that at the change to the absolute temperature by half, the total radiated power drops 16 times. The increase in power at the same temperature can be obtained only upon the increase in radiation surface, what can be complex in many situations. The power proportionally increases with the area. The power of IR radiation can also be increased by surface treatment of the radiator, by means of which we change emissivity.

### 7.4.3. IR radiation sources

IR radiation sources (infrared radiators) are divided according to various criteria, according to the way of IR radiation generation, according to the temperature of the radiator surface or according to the wavelength resulting from surface temperature, thus also the color of the radiator surface:

- according to the generation of IR radiation:

- thermal sources - excited by Joule heat,
- discharge sources - excited by and electric discharge,
- radio-technical,

- according to the temperature (or wavelength) of the radiator surface

- low-temperature sources (long-wave) - $\vartheta_p$ = 400-700 °C, $\lambda_{max}$ = 3-4 μm,
- medium-temperature sources (medium-wave)– $\vartheta_p$ = 700-1200 °C, $\lambda_{max}$ = 2-3 μm,
- high-temperature sources (short-wave) – $\vartheta_p$ = 1200-2500 °C, $\lambda_{max}$ = 1-2 μm,

- according to the radiator colour (resulting from the surface temperature)

- dark radiators,
- light (luminous) radiators.

Out of infra-radiators (according to the way of radiation generation), mainly heat infrared radiators which are excited by Joule heat (passing of current) are applied in practice. If heat energy is obtained from electrical energy, then conversion elements are called IR radiation electric sources (electric infra-radiators). This infra-radiator features two structural elements:

- the active element - e.g. a tungsten spiral emitting radiation,
- the passive element - a reflector rectifying radiation.

The reflectors are mostly thin steel sheets with high reflectance which is obtained by appropriate surface treatment (polishing, galvanic coating, etc.). If the radiator is located in a flask, the reflection layer can be sprayed on the external as well as internal surfaces.

The principle of electric infra-radiators is very simple. Electric current passes through a body with high resistance. The body is heated and it emits infrared rays which spread directly through the space. The heat develops as late as at the impact on the charge and it heats the charge up. On the surface of the charge, the infrared-radiation energy changes to heat again. It is, therefore, a double conversion of energy - heat-radiant and radiant-heat. This transformation is characteristic for infrared heating; it is an indirect way of charge heating. The resulting charge reflects to the required space using reflective surfaces . [21], [22]

Dark electric infra-radiators belong to the category of low-temperature sources; they emit at higher wavelengths. This group also includes, for example, electric radiant panels.

Bright infra-radiators are high-temperature, thus they are sources of short-wave radiation. They are many design types of these infra-radiators. Only some of them are listed:

- An infra-radiator with a metal casing has Ni-Cr wire placed in a fireproof and electric-insulating material that is wrapped by a metal tube. At full voltage, the casing reaches the surface temperature of 650 - 980 °C.
- The infra-radiator with an incandescent reflector lamp is fitted with a tungsten filament. The filament is enclosed in a heat-resistant, transparent, milk or red glass housing, which is partially silvered inside, creating an efficient reflector.
- An infra-radiator with a silicon tube features coiled Ni-Cr wire, which is fixed inside a non-vacuum Si tube, which is closed by porcelain or metal blocks.
- An infrared-radiator with an Si tube lamp consists of a tube of fused silicon, which is filled with inert gas. Inside the tube, there is a coiled tungsten filament separated from the tube walls by tantalum partitions. Filament ends are placed in a sealing material at the end of the housing. The filament works at a temperature around 2230 °C, the surface temperature of the housing is approx. 590 °C.

The typical light infra-radiators include light bulbs with a reduced working temperature of the filament (under 2000 °C). Maximum radiation is about the wavelength of $1.2 \div 1.3$ μm. Thus the proportion of the total share of infrared radiation and simultaneously the durability compared to common light bulb is increased. A common light-bulb is an imperfect light infra-radiator. It emits up to 88 % of energy to the infrared zone and only 12 % of energy to the light zone. For various purposes, various infra-radiators in the shape of light-bulbs have flasks from various types of glass, e.g. quartz, ruby or magnesium.

Radiation intensity depends on the temperature of the radiator and the radiation wavelength. Radiant energy depends on the temperature of the radiator and on the size of the radiator area. If the temperature of a solid body drops, the maximum of the luminosity curve moves depending on the wavelength towards higher wavelengths. A proportion of visible radiation drops and at a temperature of about 700 °C, the radiation is only in the zone of the invisible spectrum. Typical patterns of radiation depending on the wavelength for some types of infrared radiation sources are listed in Fig. 7.14.

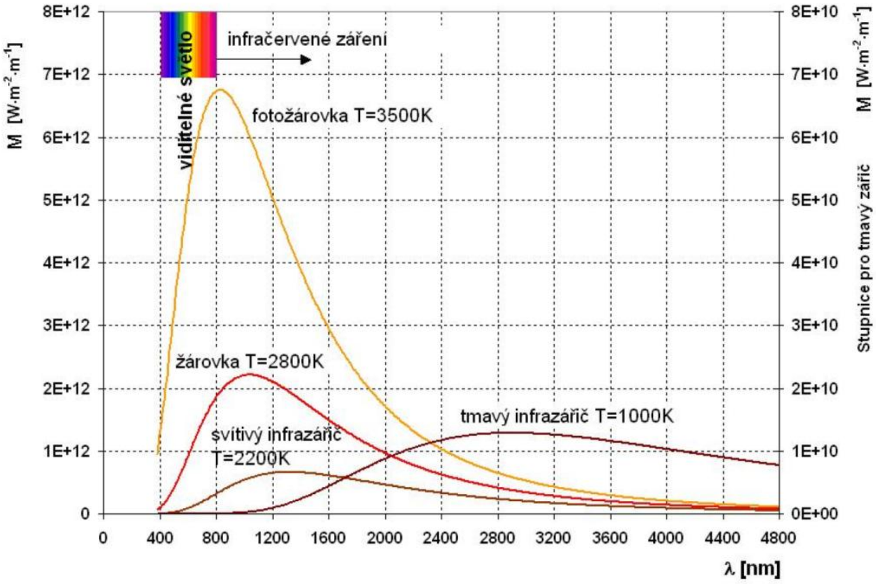



**Fig. 7.14: Typical patterns of radiation for basic types of infrared radiators[11]**

### 7.4.4. Application of the infrared heating

Infrared electro-thermal equipment are made either with a working chamber, or without it, i.e. either as furnaces, or heaters. Furnaces with infrared heating are similar to resistance furnaces in their design; the function of typical heating elements is performed by infra-radiators. Infrared furnaces must have their internal walls made of or modified from materials with high reflectance. They are operated with natural and controlled atmosphere and also in the environment of technical vacuum.

Infrared heating is widely used mainly for its simplicity, reliability and good efficiency. Infrared radiators are used, in particular, for drying of varnishes in the automotive industry, drying in continuous furnaces, in the ceramic industry, the textile industry, etc. In all cases when drying of big surfaces of small thicknesses is performed. In agriculture, it is used in the form of brooders and for drying of grains, etc. Electrical infra-red radiators are among the most effective way of heating as to heat transfer by radiation. A very effective source of heating is obtained immediately after connection of the infra-red radiator to the mains.

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# 8. Electric heating

## 8.1. Issues of thermal comfort of humans in rooms

The main task is to provide favorable heat conditions in closed rooms in a cold winter season when the outdoor temperature is lower than the temperature which is required in the rooms and when other climatic effects (e.g. wind) also cause cooling of rooms. The task is to provide for the so-called thermal comfort.

This means that such thermal conditions must be reached that a human feels good. Thermal comfort of humans is influenced by their health conditions, age, a type of activity which they carry out. The sensation of good thermal comfort is basically determined by the balance of thermal mode of humans, which is necessary for maintaining a constant body temperature of 37 °C.

An important component of thermal mode of humans is sharing of heat from the body surface with the surroundings which is governed by physical laws; therefore they can be expressed mathematically. Within metabolic processes happening in human bodies, a certain amount of heat is released; the amount depends, in particular, on the intensity of physical exertion and on the body weight of the human. This heat must be transferred to the surroundings. The thermal equilibrium, i.e. the condition, in which the surroundings take the amount of heat which is produced by the human from the body, is therefore the first and necessary prerequisite for thermal comfort.

Human bodies are cooled down by conduction, flowing, radiation and also by sweat evaporation and breathing. During little physical exertion, the greater amount of heat is transferred from the body by flowing and radiation - dry cooling of the body. Achieving of thermal equilibrium during dry cooling without excessive sweating is the second prerequisite for the human thermal comfort.

If the ambient temperature rises above the certain threshold or if the production of heat is increased during physical exertion, dry cooling is not sufficient and the excessive heat is transferred away by evaporation - wet cooling of the body.

The condition of the thermal equilibrium can be generally expressed as follows.

$$\Phi_{\mathrm{M}} = \Phi_{\mathrm{V}} + \Phi_{\mathrm{D}} + \Phi_{\mathrm{K}} + \Phi_{\mathrm{S}} \qquad (8.1)$$

where $\Phi_{\mathrm{M}}$ is the heat flux produced by the human body (W), $\Phi_{\mathrm{V}}$ is the heat flux transferred away by evaporation, $\Phi_{\mathrm{D}}$ is a heat flux transferred away by breathing, $\Phi_{\mathrm{K}}$ is a heat flux transferred away by convection (flowing), $\Phi_{\mathrm{S}}$ is a heat flux transferred away by radiation.

The heat flux by convection and radiation firstly passes through the layer of clothing, it is conducted by it, and as late as on the external surface, the heat transfer to the surroundings occurs.

The equation of thermal equilibrium than changes as follows:

$$\Phi_M - \Phi_V - \Phi_D = \alpha \cdot S \cdot (T_h - T_r) = \Phi_K + \Phi_S \quad (8.2)$$

where $\alpha$ is clothing permeability ($W \cdot m^{-2} \cdot K^{-1}$), S is the total surface of the body ($m^2$), $T_h$ is the temperature of the body surface (K), $T_r$ is the temperature of the clothing surface (K).

### 8.1.1. Thermal state of the environment

Several factors decides on the thermal sensations of humans in closed rooms: the degree of physical exertion (internal heat production $\Phi_M$), thermal insulating ability of the clothing (heat permeability α), temperature of the ambient air $\vartheta_v$, the effective temperature of surrounding areas $\vartheta_p$, the moisture of the surrounding air (relative humidity), the speed of air flow.

The factors $\vartheta_v$, $\vartheta_p$, moisture and speed of air flow characterize thermal state of the environment, which manifests itself in the resulting thermal effect of the environment on the human. Mostly we try to express the thermal state of the environment by a single, easily measured value.

**Air temperature in the room**

To assess the thermal state, we need to measure mainly the air temperature $\vartheta_v$ in the area of human living. The air temperature can be considered a satisfactory measure of the thermal state of the environment, where there is an environment with almost calm air and where the temperature of surrounding surfaces is only slightly different from the air temperature. Under these conditions, the air temperature corresponds to the resulting temperature $\vartheta_i$.

The air temperature $\vartheta_v$ is not usually the same in the entire room, therefore local changes and irregularity must be also assessed. Vertical irregularity of air temperature in heated rooms is very important; it arises due to uneven heat supply and uneven cooling of each wall, floor and ceilings in the rooms. Vertical distribution of temperatures in the room with various ways of heating is shown in Fig. 8.1.

In terms of comfortable feeling, the temperature of the lower layer of the air at the place of feet (at the height of 0.1 m above the floor) is decisive. Further, we are interested in air temperature at the head level (at the height of 1.7 m above the floor). Subsequently the difference between these two temperatures,which strongly influences the thermal comfort in the room.

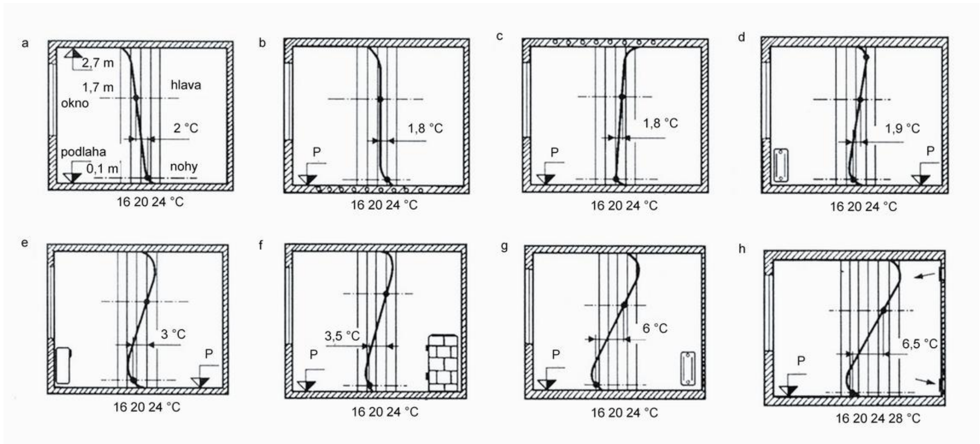


**Fig. 8.1: The vertical distribution of temperatures in the room with various ways of heating**

a - ideal heating, b - floor heating, c - ceiling heating, e - convector heating, f - local heating by tile stove, g - hot-water heating (radiator on the interior wall), h - hot-air heating.

Ideal heating (Fig. 8.1a) is such heating when the temperature at the place of feet is approximately 21 °C and at the height of a standing man the temperature is about 19 °C. In terms of thermal comfort, the difference between the temperatures at the head level and at the foot level should not be more than 2.0 °C for a standing man and 1.5 °C for a sitting man. For each way of heating, the respective temperature difference is always marked in Fig. 8.1. It is evident from the figure that as to the vertical distribution of temperature, the most favorable heating is floor heating (Fig. 8.1b).

**Effective temperature of surrounding surfaces**

To assess the resulting radiant effect of surrounding surfaces by one value, the so-called effective temperature of surrounding surfaces $\vartheta_p$ is often introduced. This temperature is defined as a common temperature of all surrounding surfaces, at which the total heat flux by radiation between the surface of the body and the surrounding surfaces would be the same as in reality. The effective temperature of surrounding surfaces, provided that the temperatures of each surrounding surface are not very different, can be expressed as follows:

$$\vartheta_p = \sum_{i=1}^{n} \varphi_i \cdot \vartheta_i \qquad (8.3)$$

where $\varphi_j$ are ratios of surrounding surface radiation by the human body surface (-)$\vartheta_i$ are temperatures of surrounding surfaces (°C).

The effective temperature thus depends on temperatures of all surrounding areas and on the ratios of radiation related to the surface of the human body. In practice, this requirement is waived and the values are related to the elementary ball, the point, usually placed inside the object.

**The resulting temperature of the room environment**

If we base our thoughts on the relation for thermal equilibrium expressed by means of heat fluxes to the body surface *S* and if we use simplification for a coefficient of heat transfer by convection and radiation $\alpha_k = \alpha_s$ (at the air flow at the speed lower than 0.3 m·s$^{-1}$), we will obtain the following equation for the resulting temperature of the environment required for providing of thermal comfort.

$$\vartheta_{\mathrm{i}} = 0{,}5 \cdot \vartheta_{\mathrm{v}} + 0{,}5 \cdot \vartheta_{\mathrm{p}} \qquad (8.4)$$

As a result, the thermal comfort of humans at the specific internal production of heat and the specific heat permeability of clothing depends only on the air temperature and the effective temperature of surrounding surfaces. The ratio of both the temperatures $\vartheta_v$ and $\vartheta_p$, however, cannot be arbitrary. If it is assumed that in rooms, where the resulting temperature of $_i$ = 18.5 to 21.5 °C is required, the air temperature $\vartheta_v$ should be between 15 and 25 °C, the effective temperature of surrounding surfaces $\vartheta_p$ can change within the limits from 12 to 28 °C. This "thermal comfort zone" is clearly marked by hatching in Fig. 8.2 [1].

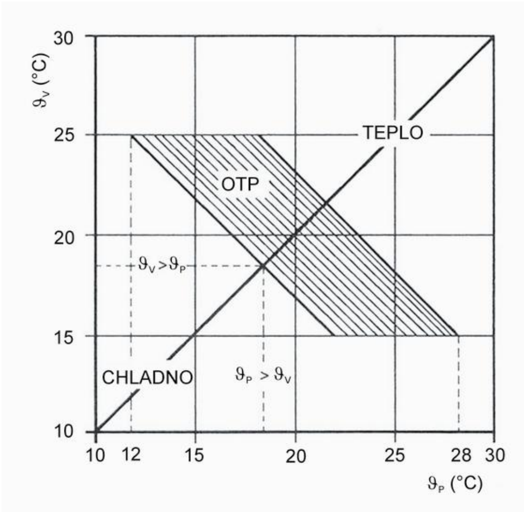


**Fig. 8.2: Thermal Comfort Zone**

## 8.2. Practical Calculation of Heating Equipment

In terms of sizing of the heating system, it is necessary to know the maximum value of heat losses of the building, i.e. the amount of heat which passes from the interior of rooms with temperature $\vartheta_i$ to the colder exterior environment with temperature of $\vartheta_e$. The heating system must be sized for the highest value in the year. When calculating the heat losses, we should base on ČSN 06 0210, Heat Loss Calculation.

The suitability of the building for electric heating shall be assessed on the basis of calculated heat losses and heat consumption per 1 m$^2$ of living space. For calculation of heat losses of building with central heating, the following input documents are required:

- a site plan, from which the position of the building towards the cardinal directions, the height and distance from surrounding buildings, the altitude of the construction site and the prevailing direction and intensity of the wing,
- floor plans of each floor of the building with all major dimensions, including the dimensions of windows and doors, at the scale of at least 1:100,
- sections of the building with specification of all major heights (ceiling height, height of window sills, etc.),
- data on materials and construction of walls, floors, ceiling and roofs for determination or calculation of the heat transfer coefficient,
- data on material and construction of windows and doors required for calculation of heat losses by penetration and infiltration,
- data on the usage of each room for determination of internal temperature $\vartheta_i$,
- description of the intended way of heating of each room.

### 8.2.1. The general procedure of heat loss calculation

The total heat loss of the room $\Phi_c$ according to ČSN 06 0210 equals to the sum of the heat loss by passage through walls $\Phi_p$ and heat loss by airing $\Phi_v$ minus permanent heat gains $\Phi_z$

$$\Phi_c = \Phi_p + \Phi_v - \Phi_z \qquad (8.5)$$

The heat loss by wall passage is determined from heat (basic loss) plus increases according to the following equation:

$$\Phi_p = \Phi_o \cdot (1 + p_1 + p_2 + p_3) \qquad (8.6)$$

where $\Phi_o$ is the basic heat loss by heat passage (W), $p_1$ is an increase for the offset of the effects of cold structures (-), $p_2$ is an increase for speeding of heating and $p_3$ is an increase for the cardinal direction (-).

The basic heat loss $\Phi_o$ equals to the sum of heat fluxes by passage through each walls surrounding the heated room to the exterior environment or neighboring rooms.

$$\Phi_o = \alpha_1 \cdot S_1 \cdot (\vartheta_i - \vartheta_{e1}) + \alpha_2 \cdot S_2 \cdot (\vartheta_i - \vartheta_{e2}) + ... + \alpha_n \cdot S_n \cdot (\vartheta_i - \vartheta_{en})$$
$$= \sum_{j=1}^{n} \alpha_j \cdot S_j \cdot (\vartheta_i - \vartheta_{ej}) \quad (8.7)$$

where $S_j$ is the area of the cooled wall (m$^2$), $\alpha_j$ is the heat transfer coefficient (W·m$^{-2}$·K$^{-1}$), $\vartheta_i$ is a calculating interior temperature (°C), $\vartheta_{ej}$ is a temperature on the external side of the j-th wall (°C).

If any of the walls features a higher temperature than the temperature in the heated room, the heat flux through this wall is negative. In this situation, it is a heat gain $\Phi_z$, which lowers the basic heat loss of $\Phi_o$.

In Fig. 8.1, we list the values of calculating inside temperature $\vartheta_i$ for various types of rooms.

| A kind of heated room | Inside temperature $\vartheta_i$ (°C) of |
|---|---|
| the living room, as the living room, | |
| the bedroom, the study, the children's rooms, | 20 |
| the kitchen, | 20 |
| the bathroom, | 24 |
| the toilet, | 20 |
| the hall, the corridor | 15 |

**Fig. 8.1: Values of calculating interior temperature $\vartheta_i$ for various types of rooms**

An increase to compensate the effect of cold walls $p_1$ allows interior air temperature increasing, so that the desired interior temperature $\vartheta_i$, for which the basic heat loss is calculated, is reached in the heated room at a lower surface temperature of the cooled walls $\vartheta_p$. This increase depends on an average heat transfer coefficient of all walls of the room $\alpha_c$, which can be expressed by the following relation:

$$\alpha_c = \frac{\Phi_o}{\sum S \cdot (\vartheta_i - \vartheta_e)} \quad (8.8)$$

where $\sum S$ is a total area of all structures surrounding the heated room (m$^2$), $\vartheta_e$ is the calculating exterior temperature for the specific area given by the standard (°C).

The increase to compensate the impact of cold structures $p_1$ can be then determined from the relation $p_1 \sim 0{,}15 \cdot \alpha_c$ or determined approximately from Tab. 8.2.

| $\alpha_c$ ($W \cdot m^{-2} \cdot K^{-1}$) | up to 0.1 | 0.1 - 0.9 | 0.9 - 1.5 | 1.5 - 2.0 |
|---|---|---|---|---|
| $p_1$ | 0 | 0.03 - 0.12 | 0.15 - 0.21 | 0.25 - 0.30 |

**Tab. 8.2: The increase to balance the impact of cold structures $p_1$**

The increase to speed the heating $p_2$ in housing developments, hospitals, etc., is taken into account only when continuous operation of heating cannot be provided for even at the lowest outside temperatures. Under normal circumstances, the increase $p_2$ is not considered. For intermittent operation, it is chosen on the basis of the heating time as follows: $p_2$ = 0.1 at the heating time longer than 16 hours a day, $p_2$ = 0.2 at the heating time shorter than 16 hours a day.

The amount of increase for the cardinal direction $p_3$ is influenced by the position of the most cooled building construction; if there are multiple cooled structures, the position of their common corner decides. The value of increase $p_3$ is determined by Tab. 8.3.

| NE direct | J | SW | W | NW | N | NE | E | SE |
|---|---|---|---|---|---|---|---|---|
| $p_3$ | -0.05 | 0 | 0 | 0.05 | 0.1 | 0.05 | 0.05 | 0 |

**Tab. 8.3: The amount of the increase $p_3$ based on the cardinal direction**

The heat loss by ventilation $\Phi_v$ expresses the heat loss caused by natural ventilation by infiltration or at forced vacuum ventilation, and it shall be calculated according to the following relation:

$$\Phi_v = c_v \cdot V_v \cdot (\vartheta_i - \vartheta_e) \quad (8.9)$$

where $c_v$ is a volumetric heat capacity at a temperature of 0 °C, $c_v$ = 1300 $J \cdot m^{-3} \cdot K^{-1}$, $V_v$ is a ventilating air volumetric flow ($m^3 \cdot s^{-1}$).

As you can see, the calculation of heat losses of building according to ČSN 06 0210 is relatively complex. For the forecast of heat losses, while deciding on the way of heating, an approximate determination of heat losses according to Tab. 8.4 is enough. The table indicates heat losses per 1 $m^3$ of heated space. The total heat loss of the building then equals to the sum of heat losses of individual rooms. [1]

| Way of room cooling | Heat losses (W) |
|---|---|
| Middle room (from both the sides of the heated room):<br>a) above the non-heated basement and protected by a | <br>34 – 47 |

| | |
|---|---|
| heated room from above<br>b) above the heated room and protected by a heated room from above<br>c) above the heated room and cooled by the soil from below | 30 - 40<br>37 - 53 |
| A corner room with windows in both the walls:<br>a) above the non-heated basement and protected by a heated room from above<br>b) above the heated room and protected by a heated room from above<br>c) above the heated room and cooled by the soil from below<br>d) above the non-heated basement and cooled by the soil from above | 40 - 58<br>35 - 49<br>44 - 65<br>47 - 73 |
| Bathroom | 40 - 80 |
| Hall | 15 - 30 |
| Staircase | 18 - 35 |
| Average heat loss in 1 $m^3$ of heated space of a detached house | 35 - 60 |

**Tab. 8.4: The approximate determination of heat losses**

### 8.2.2. Heat loss calculation - ČSN EN 12831 standard

The standard stipulates the process of calculation of heat supply required for heating and reaching of the required inside temperature. A new element is the inclusion of heat bridges; on the other hand, no heat gains are considered in the standard. This can be a problem while performing calculations for low-energy up to passive houses.

$$\Phi_{i} = \Phi_{T,i} + \Phi_{V,i} \tag{8.10}$$

where $\Phi_i$ is a heat loss by transfer and ventilation (W), $\Phi_{T,i}$ is proposed heat loss by passage through the structure (W), $\Phi_{V,i}$ is proposed heat loss by ventilation (W).

**The heat loss by transfer and heat bridges**

The proposed heat loss by transfer shall be determined as follows:

$$\Phi_{T,i} = (H_{T,ie} + H_{T,iue} + H_{T,ig} + H_{T,ij}) \cdot (\vartheta_{int,i} - \vartheta_{e}) \tag{8.11}$$

where $H_{T,ie}$ is heat loss by transfer directly outwards ($W \cdot K^{-1}$), $H_{T,iue}$ is heat loss by transfer through non-heated space (($W \cdot K^{-1}$), $H_{T,ig}$ is heat loss by transfer to soil ($W \cdot K^{-1}$), $H_{T,ij}$ is heat loss by transfer through space heated to a significantly different temperature ($W \cdot K^{-1}$), $\vartheta_{int,i}$ is calculating interior temperature (°C), $\vartheta_e$ is calculating outside temperature (°C).

The term “heat bridge” is the main novelty in the standard and it characterizes the heat loss passing through the wall at the point of contact of two different structures.

The heat bridge is characterized by a linear heat transfer coefficient (W·m$^{-2}$·K$^{-1}$) and also by its length (m).

$$H_{\mathrm{T,ie}} = \sum_k S_k \cdot U_k \cdot e_k + \sum_l \Psi_l \cdot l_l \cdot e_l \qquad (8.12)$$

where $S_k$ is the area of construction part in m$^2$, $e_k$, $e_l$ are correction factors exposed to weather effects (-), $U_k$ is a coefficient of heat transfer through construction part in W·m$^{-2}$·K$^{-1}$, $l_l$ is the length of the heat bridge in m, $\psi_l$ is the heat transfer coefficient of the heat bridge in W·m$^{-1}$·K$^{-1}$.

**Heat losses by ventilation**

To determine the heat loss by ventilation, we should base our considerations on the following equation:

$$\Phi_{\mathrm{V,i}} = H_{\mathrm{V,i}} \cdot (\vartheta_{\mathrm{int,i}} - \vartheta_{\mathrm{e}}) \qquad (8.13)$$

where $H_{V,i}$ is a ventilation heat loss coefficient (W·K$^{-1}$).

$$H_{\mathrm{V,i}} = 0{,}34 \cdot V_{\mathrm{i}} \qquad (8.14)$$

where $V_{\mathrm{i}}$ is air exchange in the heated space (m$^3$·h$^{-1}$).

To determine $V_{\mathrm{i}}$, the fact, whether the ventilation is natural or forced, is particularly important. With natural ventilation, sufficient air exchange is determined by infiltration of the cladding of the building and by the hygienic amount of the air which must be exchanged.

$$V_{\mathrm{min,i}} = n_{\mathrm{min}} \cdot V_{\mathrm{i}} \qquad (8.15)$$

where $n_{\mathrm{min}}$ is the minimum intensity of outside air exchange per hour (h$^{-1}$), $V_i$ is the volume of the heated room (m$^3$).

The minimum intensity of air exchange is 0.5 for the basic residential room and 1.5 h$^{-1}$ for a bathroom.

**The proposed thermal power**

$$\Phi_{\mathrm{HL,i}} = \Phi_{\mathrm{T,i}} + \Phi_{\mathrm{V,i}} + \Phi_{\mathrm{RH,i}} \qquad (8.16)$$

where $\Phi_{\mathrm{HL,i}}$ is proposed heat power (W), $\Phi_{\mathrm{T,i}}$ is heat loss by transfer through structure (W), $\Phi_{\mathrm{V,i}}$ is heat loss by ventilation (W), $\Phi_{\mathrm{RH,i}}$ is heat power required at intermittent heating (W).

**Evaluation of comparison of ČSN 06 0210 and ČSN EN 12831 standards**

The main difference in both the standards is the heat loss by transfer through the structure, which differs, in particular, by the presence or absence of heat bridges. Another element, which makes the standards different, is the absence of heat gains in the calculation, either the permanent ones (human presence) or variable ones (solar radiation) in the standard ČSN EN 12831.

Therefore, it is necessary to access to the calculation in a very detailed way to secure the correct design of the heat source to maintain the optimum ration of investment to operating costs, as undersizing leads to investment saving at the expense of higher operational costs and vice versa [14].

### 8.2.3. Heat input calculation

To calculate heat source input, the selected way of heating, the heating mode to the rated or damped temperature, and the way of forced ventilation are decisive. The actual installed power of heaters can be higher compared to the calculated total power input, but no more than:

a) 20 % for input to 50 kW,

b) 10 % for inputs higher than 50 kW.

If the result of calculating power input of the electric heater is at the interval of the first third of the power input difference of the heater type range, the type with lower input shall be chosen. The calculation of the heat input has its specifics for each way of electric heating. Further, we detail the ways of heat input calculations, separately for direct heating, storage and mixed (hybrid) electric heating systems.

**Direct electric heating**

The input of convection or radiant heaters $P_k$ is determined from the relation:

$$P_k = \Phi_c \cdot K \qquad (8.17)$$

where $P_k$ is the input of the convection or radiant heater (W), $\Phi_c$ is the total heat loss of the building (W), K is the coefficient of the heating progress; the value 1.0 should be chosen for continuous operation, 1.1 for heating break not longer than 4 hours, 1.2 for the break exceeding 4 hours, 1.4 for occasional use.

**Storage electric heating**

This type of heating uses withdrawal of electric energy in specified, usually night, hours (charging from 10 p.m. to 6 a.m.) and in selected daily hours (charging for 2 hours and more). The input of the storage heat source can be determined from the total daily need of heat $\Phi_d$, whose amount depends on the total heat losses per hour $\Phi_c$, the required heating time to a full temperature $t_v$ and at the time of reduced heating $t_t$. The start-up time for the desired temperature shall be included in the heating time $t_v$. The dimensioning of the electric power input is the same for continuous as well as distributed charging time $t_n$ = 8 hours.

The operating modes of heating for calculation of heat source input shall be determined from the time of the full heating $t_v$ as $\vartheta_i$ = 20 °C. The storage heating shall be designed for the operating mode defined by the time $t_v$ (h) of the kitchen 10 hrs, the kitchen with the dining room 12 hrs, the living room 14 hrs, the children's room 14 hrs, and other rooms 12 hrs.

The dimensioning of storage heaters shall be performed according to the following relation:

$$P_a = \Phi_d \cdot k_v \tag{8.18}$$

where $P_a$ is the input of the storage heater (stove) (W), $k_v$ is the operation coefficient ($h^{-1}$) see Tab. 8.5.

| Heating break $t_s$ (hrs) | Operating coefficient $k_v$ ($hrs^{-1}$) | | |
|---|---|---|---|
| | Dynamic with a fan III. | Static with a control damper II. | Static without a control damper I. |
| 0 | 0.14 | 0.18 | 0.20 |
| 2 | 0.15 | 0.23 | --- |
| 4 | 0.17 | 0.31 | --- |
| 6 | 0.19 | (0.50) | --- |
| 8 | 0.22 | (1.25) | --- |

**Tab. 8.5: Operating coefficient value**

Daily heat need

$$\Phi_d = \Phi_c \cdot t_v \tag{8.19}$$

Electric central storage heating is designed for the full heating for 12 hours. The remaining daily operation is either reduced, or intermittent. The total daily need for heat $\Phi_d$ for the heat hot-water heating systems shall be determined from the following formula:

$$\Phi_d = \Phi_{dd} + \Phi_{dn} \tag{8.20}$$

$$\Phi_{dd} = \frac{\Phi_c}{\eta} \cdot (t_{vd} + t_{td} \cdot f) \tag{8.21}$$

$$\Phi_{dn} = \frac{\Phi_c}{\eta} \cdot (t_{vn} + t_{tn} \cdot f) \tag{8.22}$$

where $\Phi_{dd}$ is the need for heat during the day (Wh), $\Phi_{dn}$ is the need for heat at night (Wh), $t_{vd}$ is the required heating time to the full temperature by day (hrs), $t_{vn}$ is the required heating time to the full temperature at night (hrs), $t_{td}$ is the required time of reduced heating by day (hrs), $t_{tn}$ is the required time of reduced heating at night (hrs), $f$ is the coefficient of building structure effect, it is considered 0.3 for heavy, 0.4 for medium heavy and 0.5 for light structures, $\eta$ is the efficiency of the heating equipment 0.95.

The required input is thus determined by the following formula:

$$P_a = \frac{\Phi_d}{t_n} \qquad (8.23)$$

**Mixed (hybrid) electric heating**

The mixed heating consists of storage and direct-heating parts. The storage heating withdraws electricity no more than 8 hours a day at night time determined by the electricity supplier. The direct heating works at lower outdoor temperatures at the day out-of-peak time (e.g. from 11 a.m. to 5. p.m.).

The mixed heating allows connection of multiple electric heating equipments to the current distribution network as the simultaneous withdrawing is lower than with purely storage heating. An important factor is also a reduction in dimensions of the equipment, thus saving of acquisition costs.

The design of the electric hybrid heater shall be performed separately for the storage and for the direct-heating part.

$$P_h = 0{,}6 \cdot P_a \qquad (8.24)$$

where $P_h$ is the power input of the hybrid heater (W), $P_a$ is the power input of the storage heater calculated on the basis of the relation for the storage heaters and for the charging time of $T_n$ = 8 hrs (W).

The direct-heating part of the hybrid heater shall be determined according to the relation:

$$P_{ph} = 0{,}4 \cdot P_a \qquad (8.25)$$

The input of the direct-heating part must cover at least 90 % of the heat loss of the room. The central storage heat source for the mixed heating shall be sized to 60 % of the input of the purely storage central heating with an eight-hour charging time. The input of the direct-heating part of the hybrid system must be at least 10 % higher than the heat loss of the room and it equals to about a half of the input of the purely storage source with eight hours of charging [1].

## 8.3. Electric heating systems

The unequal daily withdrawal resulting from the common daily routine of a man led to the effort to use free electric plant capacities at time out of the peak loading of the system. This allowed, at first, introduction of storage appliances for heating or preparation of hot service water which used to be switched on only at night. Further development, however, showed, that the possibilities of the electric system would be exhausted soon, if only storage heat were used; therefore the power supply industry also offers direct-heating and hybrid systems.

## 8.3.1. Storage electric heating

This type of heating uses withdrawal of electric energy in specified, usually night, hours (charging from 10 p.m. to 6 a.m.) and in selected daily hours (charging for 2 hours and more).

Electric energy coverts itself to heat in resistance heating elements or cables which are placed in the storage material. It has the shape of a heater, boiler or it is a concrete part of the constructional structure, usually the floor. Heating requires reliable knowledge of the heating time $t_v$ to the calculating indoor temperature $\vartheta_i$, to which the time of the so-called start-up to the full temperature and the time of reduced heating $t_t$ is included.

There are several possible ways of electric storage heating.

**Storage heaters**

Magnesite or fire clay is usually used as storage material. According to the structural design and the way of heat sharing at room heating (discharging of the heater), we distinguish between three types of storage heaters, as they are shown in Fig. 8.3 [12].

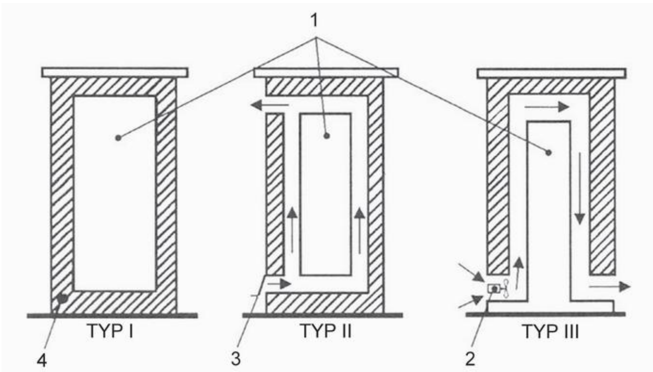


**Fig. 8.3: Three types of storage heaters**

1 - storage substance, 2 - fan, 3 - damper, 4 - thermal insulation.

The central electric storage heating consists of a traditional hot-water heating system - from electric, usually a resistance heat source and from a water accumulator. The heat from the accumulator is transferred to the heated room. Thus the basic principle of electric heating is suppressed; the principle is bringing the energy to the heated room with maximum controllability of its conversion.

Another possible type of storage heating is the so-called large floor storage heating. The heat sources are heating cables placed in the concrete screed floor cover. The surface temperature of the floor should not exceed 25 °C. The condition for the application of the heating is guaranteed long life.

The purely storage system of electric floor heating charged only by night-rate current for 8 hours is suitable mainly for new and refurbished buildings used only in morning hours up to early afternoon hours. It is not very good for rooms heated through the whole day. It is characterized by good bottom heat insulation (a combination of foamed polystyrene and mineral fiber insulation) and mainly by great thickness of the storage plate (90 to 150 mm). The heating plane with heating cables is placed to the lower half of the storage plateFig. 8.4.

This system has the character of a static storage stove; with regard to the thermal inertia of the storage layer and thermal reduction of the walking layer, it is appropriate to add a suitable direct-heating system enabling flexible regulation of the required temperature to it.

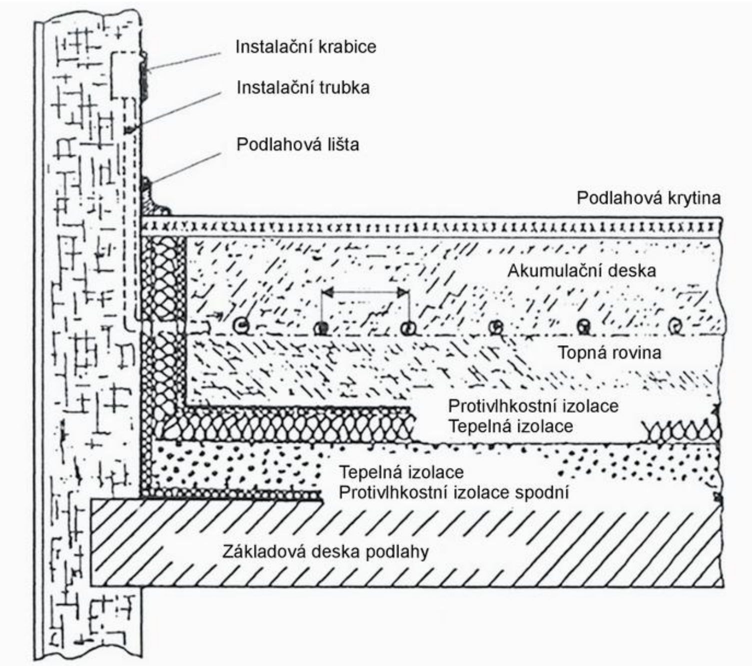


**Fig. 8.4: Storage electric floor heating**

## 8.3.2. Direct electric heating

It is composed of the distribution, heaters with heating elements or electrodes and a control circuit to provide for an optimum cycle of heating. According to the heat source location and the way of heat sharing, the direct-heating systems can be divided as follows:

Local

- convector and hot-air heaters,
- electric floor heating cables,
- radiant heating systems.

Central

- electric hot-water boilers.

**Electric convection heating**

Convectors are electric heaters converting all supplied electric energy to heat. Cold air is supplied to the convector through the bottom part; heated air is released from the upper part; the heated air heats up the entire room by natural circulation Fig. 8.5.

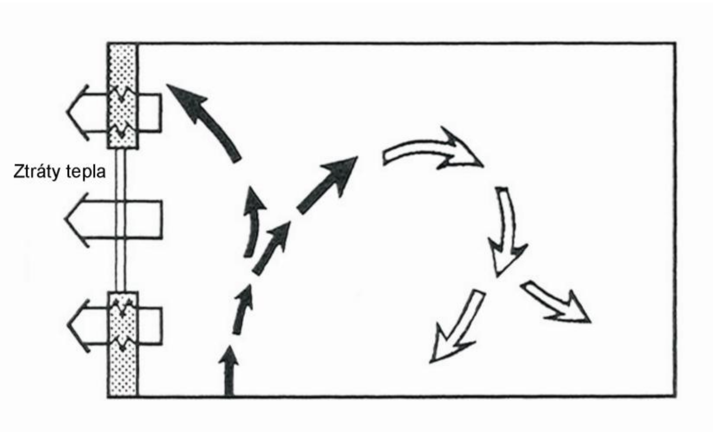


**Fig. 8.5: Air circulation in the room**

Convection heaters with natural convection are mobile, portable or intended for fixing onto the wall. They are either radiators with heated filling, usually oil, or convectors with heating resistance. It is usually a tubular heating body made of stainless steel with pressed-on aluminium grilles, which is adapted for silent operation. The surrounding air is heated up by natural convection around the heater. Modern convectors are fitted with quality control with a possibility of central control of their operation. Convection heaters with forced convection are portable or wall-mounted direct-heating appliances, in which the air is forced around the heating resistors by a fan.

**Floor heating with heating cables**

Large floor systems made by concreting of special electric heating cables into the floor are shown in Fig. 8.6; they are popular mainly for their high efficiency, even distribution of heat around the whole surface, great usage of heated room, relatively simple implementation and creation of heat comfort at lower air temperature than with, for example, convectors.

**Radiant electrical heating**

While with convection heating, the body heats up mainly the air, which passes the heat during flowing on the surface of the heated object, during the radiant heating, heat transfer mainly by radiation occurs Fig. 8.7. Each body radiates electromagnetic energy to its surroundings. Out of the wide range of wavelengths, we are interested only in the ones, which can be absorbed by objects and transformed to heat energy.

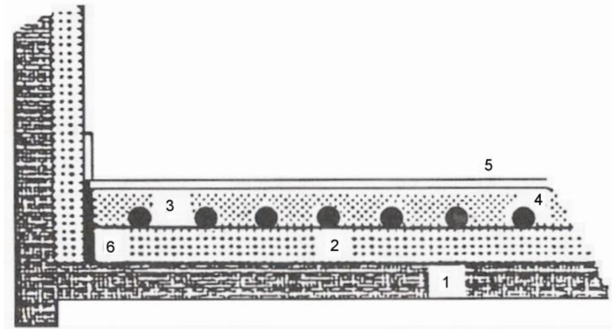


**Fig. 8.6: Floor heating with heating cables**

1- foundation concrete, 2 - polystyrene (approx. 40 mm), 3 - concrete layer (30-50 mm), 4 - heating cables, 5 - tiling, 6 - insulation connection

The radiant heaters can be infrared heaters, whose heating body has the surface temperature higher than 250 °C and radiation is rectified by a reflector in the specified direction.

Low-temperature radiant heating is provided by radiation of the surfaces heated to 25 to 40 °C. Special films or panels are usually fastened onto the ceiling and walls. In such a heated room, the air temperature is lower and the relative humidity is higher than with heating by convection heating bodies. The energy consumption is also lower, mainly due to heating to the lower required temperature in the rooms. Energy savings compared to the convection heating varies approximately between 18 and 24 %.

The application of radiant panels is very broad. They are intended for creation of thermal comfort in workshops with clearance heights up to 3.2 m, for breeding of blooded animals, for tempering of greenhouses, etc. Panels with lower rated power inputs are intended for heating of residential premises.

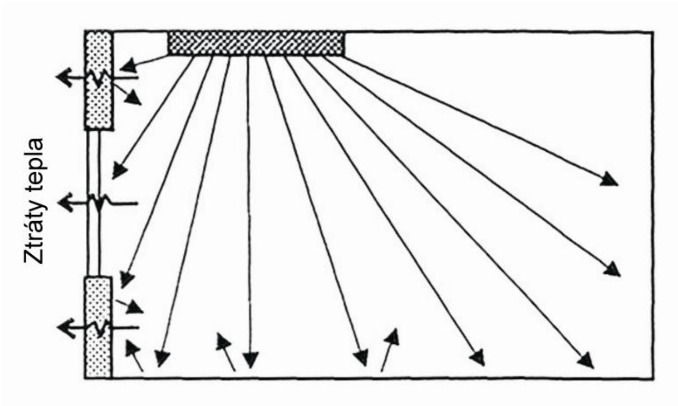


**Fig. 8.7: Radiant electrical heating**

**Heating by means of electric hot-water boilers.**

Electric boilers can be used for heating of new buildings. They are also very suitable as a replacement of solid-fuel boilers in the systems of central hot-water heating of detached houses and terraced houses.

Electric hot-water boilers - the heating medium is water which is heated in a closed container, a boiler, out of which it is distributed through pipes to radiators or similar equipment in individual rooms. Heating of water in the boiler is provided either by heating bodies based on the resistance principle, i.e. a common resistance boiler, or by means of electrodes - an electrode boiler, in which heat develops by passing of electric current through water (electrolyte) between electrodes.

### 8.3.3. Mixed (hybrid) electric heating

Mixed heating consists of storing and direct-heating parts. The storage heating withdraws electricity for max. 8 hrs a day at night time determined by the electricity supplier. The direct-heating part of the heating system works at lower outdoor temperatures at daytime outside the peak. It can be assumed that in the future years, this way of electric heating will be used by many more users than today.

Mixed heating systems can be designed as follows:

- electric hybrid heater,
- combination of central storage heating with direct heaters,
- combination of large floor storage heating with direct heaters.

The mixed heating allows connection of multiple electric heating equipments to the current distribution network as the simultaneous withdrawing is lower than with purely storage heating. An important factor is also a reduction in dimensions of the equipment, thus saving of acquisition costs.

The central storage heat source for mixed heating shall be sized to 60 % of power input of the purely storage central heating with eight-hour charging time. The power input of the direct-heating part must be at least 10 % higher than the heat loss of the room and it equals to about a half of the input of the purely storage source with eight hours of charging [1].

## 8.4. Heat pumps

The operation principle of a heat pump is the same as the reversed operation of a compressor refrigerator. The evaporator takes away heat from the environment with a relatively low temperature and it transfers it to the heating system with a higher temperature using a condenser. It is the closed Carnot cycle, see Fig. 8.8.

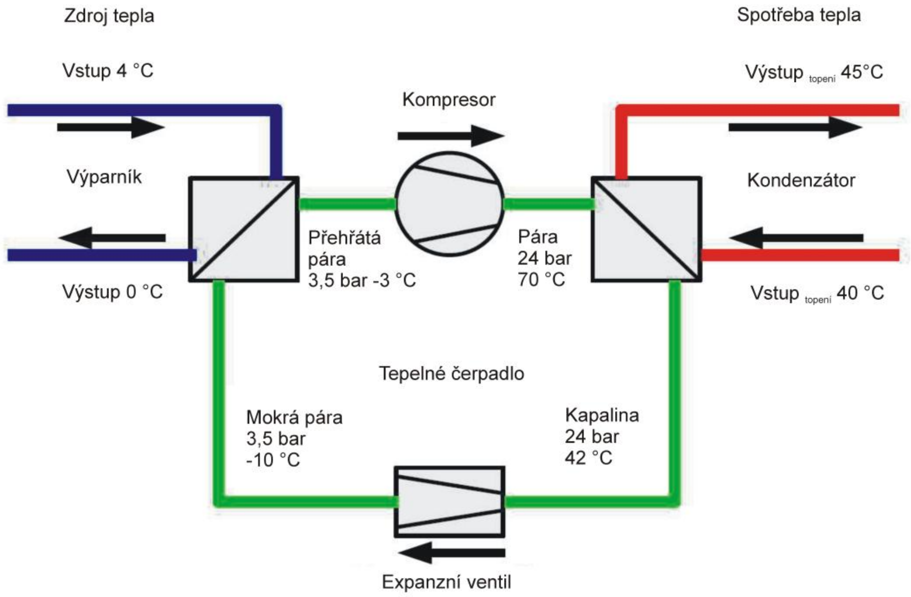


**Fig. 8.8: The operating principle of a heat pump.**

The heat-carrying medium is evaporated at low pressure in the evaporator. The heat required for evaporation is taken away from the surroundings - from stream water or from underground water, from the ground, from the outdoor air. Vapors are suctioned into a compressor and they are compressed. The vapors, having been heated by pressing, are led to the condenser, where the heat is taken from them. The vapors in the condenser condensate and they transfer vaporization heat to the heating system. The condensed medium is led from the condenser through an expansion valve back to the evaporator. As the pressure there is low, the medium begins to evaporate rapidly at a low temperature, while withdrawing the heat. The whole cycle repeats. A part of energy is supplied to this heating cycle through a compressor by compressing. A greater part is, however, withdrawn from the environment with a low temperature (running water, the air, etc.) and transferred to the environment with a higher temperature - the heated room. The compressor is basically used for the transformation of the temperature of the heat-carrying medium during heat transfer.

A power number is specified for a heat pump; it is calculated from the following relation:

$$\varepsilon = \frac{P_p}{P_d} \tag{8.26}$$

where $\varepsilon$ is the power number (-), $P_p$ is the power transferred to the heating system (W), $P_d$ is supplied electric power (W).

The power number is higher than 1; it ranges between 1 and 6. It depends on the actual efficiency of the compressor with its engine, on the heat-carrying medium, and mainly on the difference of temperatures in the evaporator and condenser. It decreases with the increasing difference in these temperatures. In practice, the temperature in the evaporator is not lower than 0 °C. Otherwise the heat transfer through forming frost is worsened. The temperature in the condenser is max. from 60 °C to 75 °C.

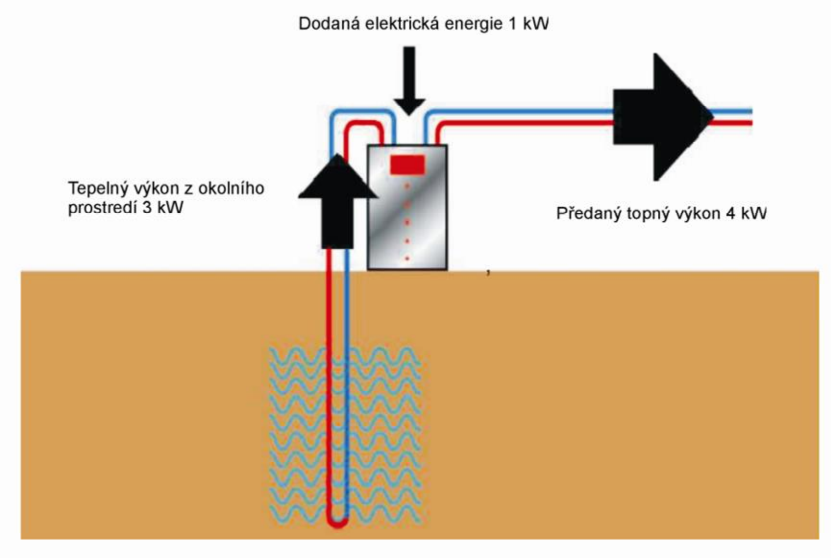


**Fig. 8.9: The principle of the heat pump heating factor.**

The heat pump cools down the surroundings of the evaporator. Therefore the access of new heat must be allowed. It is ideal to place the evaporator in flowing water. As to its dimensions, the evaporator for flowing water is small. The evaporator placed in the ground has almost the same area as the area of the heated rooms. For the evaporator placed in the air, the required area is even larger. The service heat is taken from the condenser usually by water, which is, at a temperature of 45 to 50 °C, distributed to large radiators in the heated rooms.

The heat pumps are proven in large units (in the orders of hundreds of kilowatts and megawatts). The combination of the artificial ice surface - heat pump - swimming pool is particularly advantageous. The evaporator cools down the area of the ice rink and the condenser heats up water in the swimming pool [1].

### 8.4.1. Types of heat pumps

Types of heat pumps are defined by a two-word name mutually separated by a slash, where the first part defines the natural source of heat for the heat pump, whereas the other the carrier medium, which transmits heat to the building.

**Air-water**

Air-water heat pumps do not require any expensive earthworks, therefore the investment in them is less demanding. The disadvantage is relatively strong dependence on the temperature of the ambient air throughout the year. With the increasing temperature of the outside air, the power of the heat pump is higher, and vice versa. As a result, the installation in the so-called bivalent operation, when the required heat energy is secured from an alternate source at critically low temperatures, is necessary. [5], [6]

Waste air - waste air shall mean the air exhausted by the ventilation system of the building. There is a relatively high temperature here; therefore a great amount of energy is accumulated in it. Heat consumption for heating of the ventilation air is one-third of the total heat consumption for heating. The amount of ventilation air is usually limited; therefore the bivalent operation must be introduced here as well.

**Fig. 8.10: Air-water heat pump [7]**

**Air-air**

The air is a medium, which withdraws heat from outside and subsequently the thermal power is transferred to the interior air of the building. Installation of these pumps is rare. They are applied in smaller buildings (such as huts and cottages). They are often used as a recovery equipment. However, their system is more complex.

**Ground-water**

With ground-water heat pumps, thermal energy is obtained from the soil and it is transferred to heating units by means of water circulating in the secondary circuit. These are the most stable heat pumps. They are able to provide for enough energy throughout the year, provided that they are operated correctly. The main disadvantage consists in the earthworks which are required for the installation. They also have an impact on the increase in acquisition costs. According to the way of energy withdrawal from the primary circuit, these heat pumps can be found in the design with a geothermal well or with a flat collector [6].

Geothermal well - these heat pumps use low-potential energy stored in the Earth. The temperature of rocks increases with depth. The depth of the well does not exceed 150 m. If the well is not able to provide for the required energy, multiple wells are designed for one common system. For the reason of sufficiently high temperature, the minimum recommended depth is 50 m. Using modern technologies, this well is not difficult to make. However, it is the most cost-demanding investment in the budget of the entire heat pump. The advantage is that it does not require a large unbuilt surface as it is required with aerial collectors. Another advantage is that its operation does not depend on outside weather conditions; as a result, it can be deployed almost in all areas (the temperature of the well is about 10 °C throughout the year). A well which is deep 12 to 18 m is required for 1 kW of heat pump power. The depth for 10-kw heat pump is approximately 140 m (or 2 x 70 m) [5].

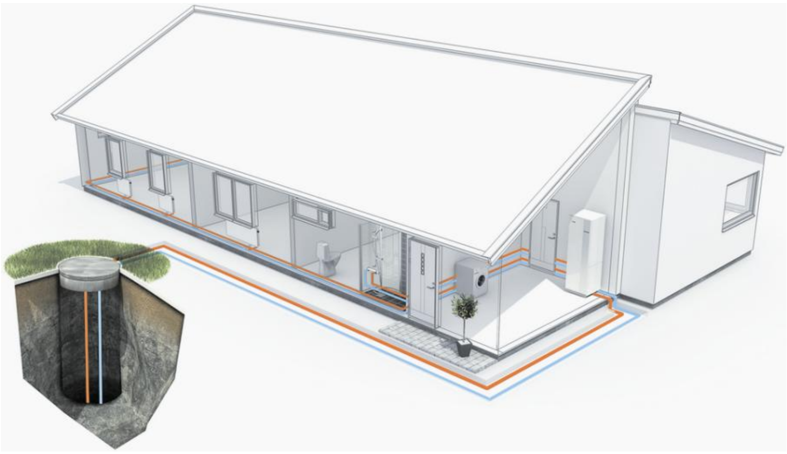

**Fig. 8.11: Heat pump with a geothermal well [7]**

Area collector - accumulated solar energy stored under the earth surface is used here. Excavations for placing of the primary circuit require relatively demanding and long-lasting works. However, if compared with geothermal well, they can be purchased fairly cheaply. Using of the land above the collector is quite limited in some cases. This must be considered. Compared to the geothermal well, lower coefficient of performance is obtained here. Moreover, its value fluctuates throughout the year because the soil temperature varies as well [5], [6].

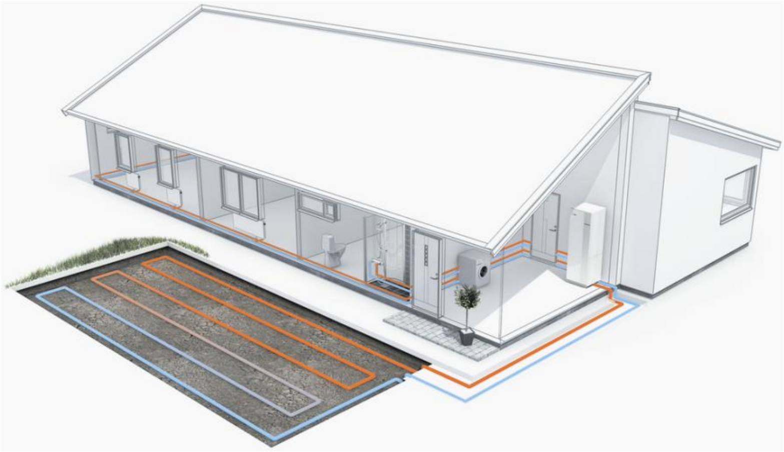

**Fig. 8.12: Heat pump with an area collector [7]**

**Water-water**

For water-water heat pumps, the highest value of the coefficient of performance is obtained. However, the possibility of installation is significantly limited by a lack of locations abundant in the required waters. According to the place of occurence of these waters, these heat pumps are divided into surface ones and underground ones.

Underground - water is the warmest natural source of heat featuring stable temperature around 10 °C throughout the year (sometimes the temperature is even higher). From the technical point of view, two wells are required. These are the so-called heating and soakaway wells. The heating one is a low-potential heat source. Water returns to the soakaway well and to the ground. As for the heating well, it is necessary to verify the yield of the underground water, which should be at least 0.5 0,5 $l \cdot s^{-1}$ for a detached house. On the contrary, the soakaway well must be able to take the same amount of water.

Surface - the surface water shall mean rivers, ponds and other water areas. These heat pumps occur very rarely. Surface waters strongly depends on air temperature fluctuations [5].

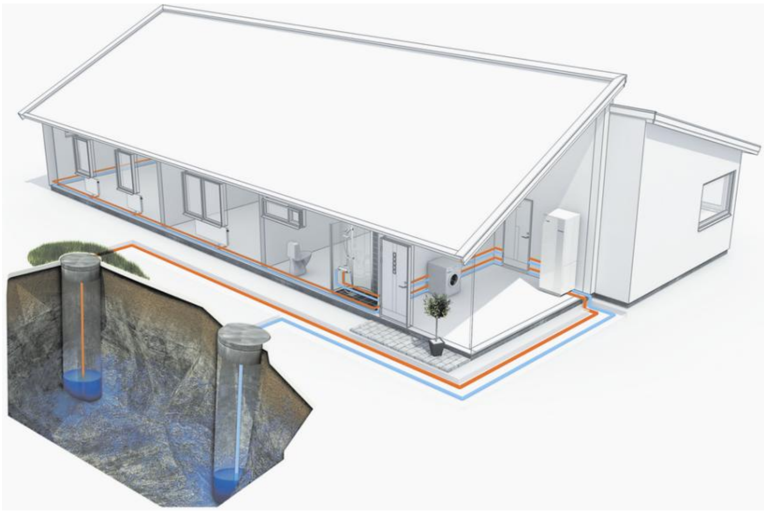

**Fig. 8.13: Heat pump with the occurrence of underground waters [7]**

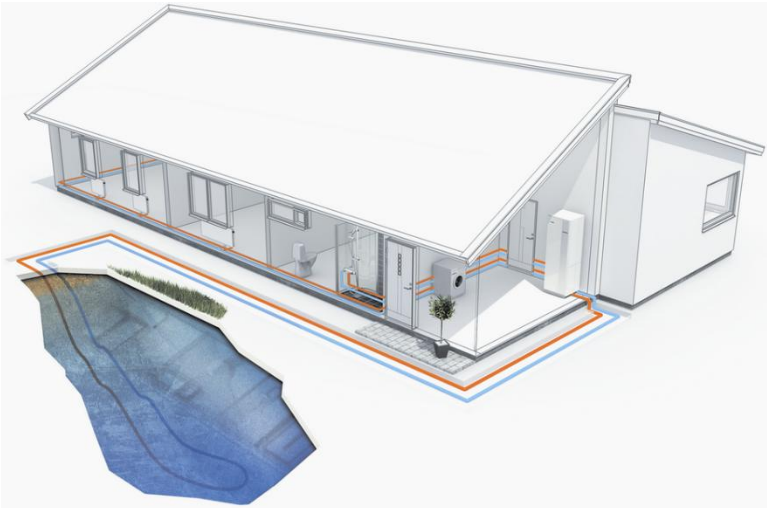

**Fig. 8.14: Heat pump with the occurrence of surface water [7]**

**Energy piles**

They are types of heat pumps pumping low-potential energy from the building construction or foundations of the building. It is used, in particular, for buildings requiring deep foundations. To be specific, these are high-rise (tower) buildings situated on an unsuitable subsoil. This technology can be used only for new buildings. Additional implementation is not possible due to technical reasons. If there is a lack of low-potential energy, the system is combined with the system of deep wells [8], [6].

## 8.4.2. Operation of heat pumps

The total heat losses of the building are only one of the factors determining the suitable selection of a heat pump. It is usually non-economical to size the heat pump to maximum required power. Its proper energy operation influences not only the functions of the entire system, but also the total acquisition and operation costs. According to the percent coverage of heat losses of the building by the power of the heat pump and the type of the alternate source, three operating conditions of heat pumps are distinguished.

Monovalent operation - during this operation, the heat pump covers the required heat of the building even during the most unfavorable periods, i.e. in the spell of low outside temperatures. It is thus 100% coverage of heat losses by the heat pump as the only source of heat. For technical and economic reasons, this mode of operation is used mainly in the ground-water or water-water heat pumps. The operation designed in such a way can be justified only if the building is perfectly insulated with minimum heat losses [5], [6].

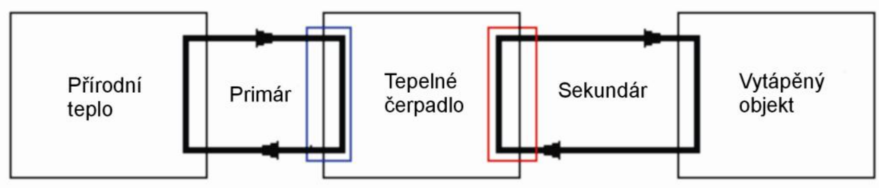


**Fig. 8.15: A drawing of a heat pump in a monovalent operation**

In Fig. 8.15, there is a principle drawing of monovalent operation of a heat pump consisting of three blocks which are thermally associated with the primary and secondary circuits. Natural heat is pumped using the principle of a heat pump to the heated building without any auxiliary heat sources.

Monoenergetic operation - in this operating mode, a small back-up electric source, which is called reheater, is connected to the secondary circuit; the task of the reheater is to cover withdrawal peaks of the required heat power during the coldest days of the year. The monoenergetic operation features the best ration between investment and operating costs from all modes of heat pumps [6].

In Fig. 8.16, a principle diagram of the monoenergetic operation of the heat pump consisting of four blocks which are thermally interconnected by the primary and secondary circuits. Contrary to the monovalent operation, there is an additional fourth block (reheater) on the secondary circuit. Natural heat is pumped using the principle of a heat pump to the heated building, while the reheating is secured by an auxiliary heat source (reheater).

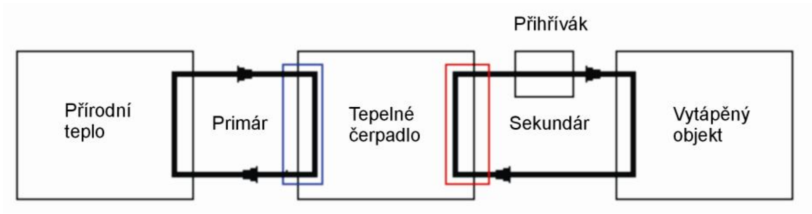


**Fig. 8.16: The drawing of a heat pump in the monoenergetic operation.**

Bivalent operation - this operation is similar to the monoenergetic operation. Heat pumps are also dimensioned with reduced power and power peaks are covered by other heat sources. The difference consists in the fact that the system combines two independent heat sources. Parallel operation of both heat source does not usually occur. When the outdoor temperature falls under the bivalent temperature, the heat pump is put out of order and the total heat load is covered by an equivalent source which does not obtain heat energy from the electric energy. The example may be fossil fuel boiler. The great advantage is the possibility to back-up the heat pump in case of failure or electric outage. [6]

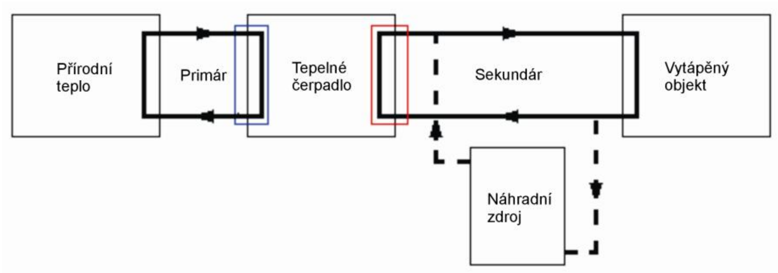


**Fig. 8.17: The drawing of a heat pump in the bivalent operation**

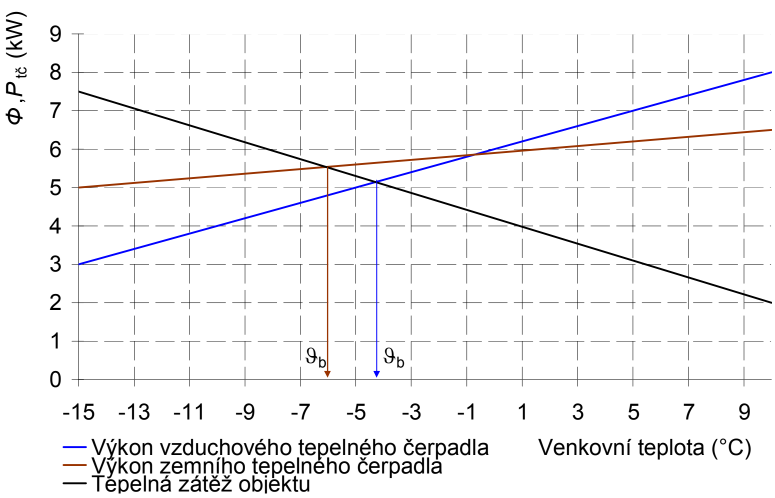


**Fig. 8.18: Determination of the bivalent point**

### 8.4.3. Determination of the bivalent point

The bivalent point indicates the thermal interface, to which it is economical or technically possible to operate a heat pump. At temperatures lower that the bivalent point, the bivalent or monoenergetic modes are applied. This evaluation is graphically shown in **Chyba! Nenalezen zdroj odkazů.** ($\Phi$ is total heat load of the building (kW), $P_{tč}$ is the power of the heat pump (kW)). The vertical line dropped onto the x axis which is determined by the intersection of curves of the total heat load of the building and power of the heat pump indicates the limit temperature of bivalence $\vartheta_b$ [6].

### 8.4.4. Preparation of hot service water

The system of hot service water preparation requires not only the actual container (the reservoir), but also other elements, such as fittings and suitable adjustment of regulation. All of this requires further increases in costs. While designing, these costs must be quantified and compared to other means of heating. In case of electric heating, the special rate cannot be omitted. Some retailers offer heat pumps which integrate the preparation of hot service water. For new buildings, where there is no current system for hot water preparation, the preparation by the heat pump is usually preferable. If the preparation of hot service water by means of a heat pump is not required, it is customary to ensure at least technical possibilities for future connection of hot service water. Growing energy prices (electric energy and gas) or an increase in hot water consumption may lead to this [5].

To specify the required volume of the service water heater, it is necessary to know at least the number of people, for which it is intended. Water consumption per one person and day is within the range of 40 to 60 l. This equals to increase in calculating heat losses by 0.2 kW.

### 8.4.5. Operation with solar connectors

The combination of heat pump operation with solar panels is more and more required. Heat collectors allow to accumulate energy from solar radiation to such an extent that hot service water preparation is possible. The current modern technology allows preparation of hot water throughout the year. In this all-year operation, the collectors are filled an anti-freeze mixture, which means that hot water does not pass directly through the collector, therefore it is appropriate that hot water reservoir includes an integrated exchanger. In a winter season, operation of solar collectors is possible. However, it is not economical to design this system as a separate one. Therefore, alternate sources of heat, which are called add-heating, are normally installed; they cover the lack of energy and allows reaching of the required output temperature of hot water. In this situation, it is a heat pump. In a summer season, a contrary problem may arise: the hot service water tank reaches its maximum temperature (approx. 85 °C) and energy from solar panels is no longer needed. In practice, it is dealt with by installation of a storage tank, which is heated by these collectors. Efficiency of collectors falls with increasing temperature, therefore it is suitable to heat a greater amount of water to lower temperature, but not a lower amount of water to a higher temperature [6].

### 8.4.6. Accumulation of thermal energy

Heat storage in water storage tanks has effective utilization mainly for systems working with low-potential energy, such as heat pumps or solar thermal collectors. Heat pumps use accumulation, in particular for bridging the time when there is not a low tariff of electric energy available. Accumulation is also used for limitation of heat pump switches at lower outdoor temperatures, when the battery is charged and later, heat is withdrawn from it by the heating system. Thus the life of the heat pump is extended. Another system, where heat accumulation must be used, is solar collectors which are used for hot service water preparation and secondary heating. Accumulation is necessary here due to irregularity of solar gains and it allows effective use of these energy yields from solar collectors in cooperation with another source for heating. These ways of accumulation can be called short-term. Heat gains can be stored even in the long term, mainly during summer. They can be subsequently used during the heating season-

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# 9. Air-conditioning, energy savings

## 9.1. Air-conditioning

The origins of air-conditioning based on natural principles of flowing, heat and moisture transfer, can be found in the past. In some areas of India, intensive stable flowing of air was used for modification of temperature and humidity in palace buildings during hot periods. Moistened mats of grass was hanged through the opening on the windward side of building. They served for adiabatic cooling (by evaporation) of supplied air up to temperatures of 20-30 °C. The mats were moistened manually or from perforated gutters supplied with water from reservoirs gravitationally.

The first records of comfortable air-conditioning (with use of ventilation and air cooling) come from Dr. John Gorrie (1802-1855), an American doctor and engineer, who designed and operated equipment for comfortable cooling with ventilation in the first air-conditioned hospital ward in 1844 [3]. Substantial progress in air-conditioning technology was supported by scientific studies in the field of thermodynamics of moist air. In 1911, air-conditioning was recognized as a separate engineering field.

Air-conditioning traditionally deals with thermal and moisture modification of the air by ventilation (exchange of exhausted indoor air for outdoor air, which is fresh) with air flow connected with air filtration. It is a machine modification to air ensuring the required parameters of the environment (temperature, humidity, flow, air purity). Some air-conditioning equipment are not complete. They only provide for cooling and heating or heating and moistening of the air. Air-conditioning relates to ventilation.

Air-conditioning equipment performs the following functions based on its design:

- air exchange in the room by the outdoor room with exhausted of pollutants, i.e. controlled ventilation,
- air filtration or its further special adjustments (ionization, sterilization and others),
- cooling or heating of rooms, which is an air temperature adjustment,
- humidification or dehumidification of the air in the room, i.e. air moisture adjustment.

The condition of the air in rooms can be determined by two factors - either by requirements of people (air-conditioning for comfort) or by technological requirements. These requirements are defined by air parameters (temperature, humidity) either within very narrow limits for all parameters, or broader limits of defining parameters are admitted.

Air temperature, its relative humidity, air flow speed, turbulence intensity and the mean radiant temperature are usually specified as main specifying values of thermal and humidity conditions for air-conditionings for comfort. This type of air-conditioning is always provided by ventilation as well.

Usually air temperature and relative humidity of air are determining variables for air-conditioning for technologies. For most equipment, they are required in a narrow tolerance band.

Air-conditioning can be sorted according to the type of fluid for transmission of heat and cold in the building (air systems, water systems, air/water combined systems, refrigerating systems). Further we can sort according to the number of rooms - zones, in which the air-conditioning system modifies the environment and in which individual changes (thermal and moisture load) occurs (single-zone and multiple-zone systems) [16].

**The main classification of air-conditioning systems:**

a) Air systems
   - single-channel system with a constant air flow
   - single-channel system with a variable air flow,
   - two-channel system with a constant air flow
   - two-channel system with a variable air flow,

b) Water systems
   - a system with fan convectors,
   - a system with cooling/heating surfaces (e.g. ceilings)

c) Combined systems air-water
   - induction system (two-, three-, four-tube - for distribution of water, single-channel for air distribution).

d) Refrigerating systems
   - single-zone system (split) with a constant refrigerant flow,
   - multiple-zone system (multisplit) with a constant refrigerant flow,
   - multiple-zone system (multisplit) with a variable refrigerant flow,

### 9.1.1. Operating modes, functions

Each air-conditioning equipment contains elements allowing creation of cool, which is the main task of each air-conditioning. This equipment for the production of cool can ventilate with this cooler air if there are lower temperatures than the temperature of interiors; thus electric energy as well as extension of life of the device are obtained. For the production of cooling, air-conditionings are used; most often they are compressor cooling circuit working on the principle of a refrigerator, when the temperature is automatically transferred from the substance with a higher temperature to the substance with a lower temperature.

**Cooling principle using a compressor**

This cooling principle, based on the change of the physical state of heat, is usually known as production of cold, and the device, which produces the cold is called the cold source.

The process begins in the compressor placed on the outdoor unit, where cold vapors of refrigerant with low pressure are compressed. Refrigerant with high-pressure and high-temperature comes out of the compressor. It is transferred to a heat exchanger - condenser, which is cooled down by the outdoor air using a fan. Heat from the refrigerant is thus taken away. Condensation occurs at the same time. Behind the condenser, the refrigerant is in a liquid state. The liquid refrigerant is transferred to the indoor unit through the piping. Here, it passes through a throttling capillary tube or an expansion valve, which then decreases the pressure of the refrigerant. The refrigerant temperature falls sharply below the temperature of the refrigerated space. The refrigerant with low temperature and low pressure further passes to the heat exchanger - evaporator. Through evaporator wall, refrigerant takes away heat from the surrounding air, which is forced by a fan through it. On the evaporator walls, the liquid refrigerant evaporates. The refrigerant is leaving the evaporator in a gaseous state at low pressure and low temperature. It is transported by the piping from the indoor unit back to the outdoor unit to the compressor and the entire cycle repeats (Fig. 9.1).

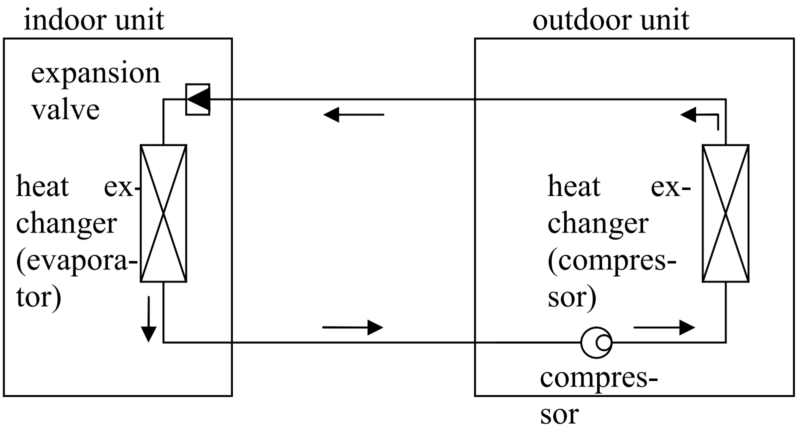


**Fig. 9.1: Cooling principle [2]**

**Heating principle**

In air-conditioning, it is the system of air-air heat pump. The outdoor unit is equipped with a 4-way (reverse) valve, which turns the refrigerant flow direction between indoor and outdoor units. The principle is similar to refrigerating, but the refrigerant passes in the opposite way. Compressed hot refrigerant comes into the exchanger of the outdoor unit and the air in the room is heated. During pressure way of heating, there are savings up to 60 % of energy compared to common electric heating.

Use of air-conditioning in this mode is suitable for additional heating during a transition period in the spring and autumn, as it is the most efficient up to the outdoor air temperature of 0 °C. At lower temperatures, air humidity freezes on the outdoor unit. If its defrosting is ensured functionally, the air-condition can be used in the heating mode up to a temperature of -15 °C.

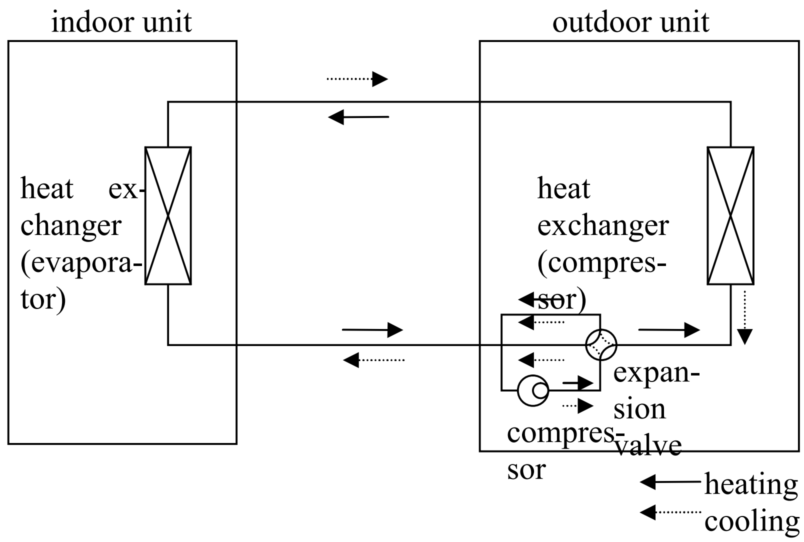


**Fig. 9.2: Heating principle of air-condition [2]**

**Dehumidification function and air filtration**

Air-conditioning is also intended as an air purifier. Dust, odors and allergens are removed from the air. Air filters capture dust, odors and other harmful particles contained in the air. The basic filter is an electrostatic filter, which is positively charged and which captures negatively charged dust. The active carbon filter catches cigarette smoke, odors and pollen. The catalyst filter captures various chemical and harmful particles in the air. In addition, some makes and types of air-conditioning are equipped with state-of-the-art technologies for killing of bacteria, air freshening etc.

In the air-conditioning mode, precipitation of humidity on hot heat exchanger of the indoor unit occurs. It drips into a collecting container. This condensate discharges either by gravity or it must be pumped away using an auxiliary pump.

## 9.1.2. Compressor designs, coolant

- A compressor with constant speed - controlling is to be carried out in two statuses - on/off.
- 2 compressors with a constant speed but with a different power - only one is switched on according to the current requirement for the cooling performance.

- Compressor with variable speed - its speed is controlled by an inverter. The advantage is a more economical operation, lower starting current and a lower noise level.

**Coolant (refrigerant)**

The most important parameter of the coolant is its physical properties. On the one hand, it is the ability to receive and emit heat, further the ability to be compressed, i.e. the pressure. The compression ability need not to be high. In comparison, it is desirable that the ability to receive and emit heat (vaporization heat) is as great as possible. The pressure of the coolant in the air-conditioning circuit is maintained slightly higher than in the surrounding environment, so that no aspiration of the surrounding air into the system, thus adverse changes to the operating process of the air-conditioning, occurs. Other required properties include inflammability and content of substances which are not poisonous. The refrigerant must not cause corrosion.

The oldest coolants are ammonia. Ammonia is flammable and toxic, therefore it cannot be used in air-conditions which are placed in buildings with the presence of humans; therefore it is no longer used. The most common coolants include halogen hydrocarbons. These refrigerants have marking whose number indicates combination of atoms of carbon, hydrogen and fluorine, i.e. R407c, R134A and R410a-Nowadays, the most frequently used refrigerants are R407C and R410a. The refrigerant R410a has up to 50% higher volumetric cooling power than R407c. However, it works at higher pressure levels. Therefore, it requires pipeline with thicker walls.

### 9.1.3. Air systems

These systems are conventional technical solutions. The heat-carrying substance mediating the transfer of heat and cold between sources and the air-conditioned room for covering of the heat load is air conducted through air ducts. For the reason of low heat capacity of air, bigger volumetric flows are required for transfer of thermal energy. It follows from the above-mentioned fact that bigger exchanges of the air in the room are required for ensuring of the indoor environment; it is related to higher speed of air flow through the indoor space, mainly large pipeline conducted between the place of air treatment (machine room) and the air-conditioned room. Air-conditioned systems are applied in many designs, mainly for large rooms of civil and industrial buildings. Air systems can be divided as follows:

**Low-pressure**

- Central - they distribute air to individual air-conditioned rooms with the same level of micro climate. The speed of air flow in the pipeline does not exceed 10 $m \cdot s^{-1}$. Treatment of air forming internal microclimate is performed in the equipment for air treatment usually composed of modular air-conditioning units also allowing re-use of heat. They are suitable for conference and concert halls, theaters, cinemas, department stores, restaurants, diners and for production rooms, buildings intended for animal breeding and laboratories.

- Zone - these systems are suitable for large buildings and premises of industrial buildings, e.g. factory halls, exhibition pavilions, operational complexes, i.e. for premises with various requirements for the level of indoor environment in each zone.
- Unit - units are installed directly in air-conditioned premises; therefore they do not require air-conditioning machine room or the piping network. They feature air treatment performed directly in the air-conditioning room.
- Special - they are intended for creation of interior micro-climate in the premises with precisely specified requirements for the conditions of the indoor environment and low tolerances of their fluctuations, usually temperatures.

**High-pressure**

The air speed flowing through air ducts between the machine room and the air-conditioned room exceeds 12-20 $m \cdot s^{-1}$ in the main sections of the distribution. The higher speed allows decreasing in cross-sections of the air-ducts, thus minimizing the special demands of air-conditioning. On the other hand, this system is noisier and it has higher operating costs. The distribution system consists of high-pressure and low-pressure parts. Air pressure is reduced by units connected to pressure sections of the pipeline before entering the air-conditioned room.

- Single-channel - they used to be use in buildings with larger rooms with the same requirements for indoor micro-climate, e.g. department stores.
- Two-channel - they used to be use in buildings with larger rooms and with various requirements for the condition of the indoor environment, e.g. each floor of the department store.

### 9.1.4. Water systems

The heat-carrying substance is water transported through the piping network from the machine room to end elements, i.e. through fan convectors called fancoils. The mentioned units treat the air in each of the air-conditioned premises of the buildings. Fresh air is conducted to the air-conditioned rooms through air ducts from the central machine room or it is aspired directly by fancoils from the outside. The flow of fresh air is derived from minimum doses of outdoor air for the air-conditioned room.

The advantage of water systems is minimization of air-duct cross-section and a possibility of individual micro-climatic condition control of each room. The systems are applied in many variants, mainly in civil buildings. The current technical solution to water systems is the modification with large heat-exchange elements - the so-called cooling ceiling.

**Cooling ceiling**

This air-conditioning water system of cooling can be characterized as a flat heat exchanger suspended usually under the ceiling of the air-conditioned room. Cold water flows through the exchanged. It takes away the heat, thus covering a part or the whole heat load of the space. If the entire heat load is covered by the cooling ceiling, then only air is supplied to each air-conditioned room; the volumetric flow of the air is based on the hygienically required dose for air exchange for fresh. Cooling ceiling surface temperature is usually about 19 to 20 °C according to the temperature of cooling water and the required microclimatic level. A cooling floor is a modification to the cooling ceiling.

Compared to other systems, the cooling ceiling features a lot of advantages. The crucial advantage consists in ensuring of the state of the environment without air flow inducing draft perceived mainly by users of air systems. In case of cooling ceilings, heat is mainly transferred by radiation between surfaces different temperature and its transmission is not bound to the air-flow.

## 9.1.5. Refrigerating systems

The heat-carrying system is coolant, which transmits heat by means of changes to the physical states forming the so-called thermal cycle. The systems usually work in the cooling mode. Some of them allows heating. In the basic cooling mode of operation, coolant evaporation takes place in the indoor unit. During the mode, air heat of the air-conditioned room is exhausted and in the outdoor condenser unit the condensing heat is transferred to the surrounding air. A typical cooling system is the option known as the split.

## 9.1.6. Combined systems air-water

The air is supplied to the unit through a nozzle and indoor circulation (secondary) air is aspired due to induction. The ratio of the secondary to primary air determines the induction ration, which is usually between 2 and 5. In the induction unit, there is a heat exchanger (cooler and heater), which treats the secondary air as required. After mixing in the induction unit, the air is supplied to the room. The flow of fresh air is derived from minimum doses of the outdoor air of the air-conditioned room.

## 9.1.7. Types of AC units

AC units can be divided into several groups. According to the possibility of their moving, we distinguish mobile and fixed, permanently installed. According to placement on the window AC equipment, we distinguish wall-mounted, ceiling-mounted, window-mounted, channel and separated air-conditions - the so-called Split systems. According to the properties required from the equipment with heating, humidification, cooling, circulation, a water cleaner and others, usually in different combinations.

**Mobile AC equipment**

The advantage of mobile AC units is their mobility. Some AC units consist of two parts (separated), whereas others are compact (monoblock). The separated AC units feature the outdoor part placed at the indoor part as close as possible and it is interconnected with a hose for air transfer. Compact mobile AC units lack the larger part which is placed together with the indoor part in one unit and the exhaust of hot air is ensured by a hose which is led outwards to the exterior. The diameter of the supply hose is usually 0,1 m. To be moved around the interior, the air-condition unit is equipped with castors. The noise level of the AC unit is between 50 and 54 dB (A).

**Window-mounted AC units**

These AC units are manufactured as a single unit. The window-mounted unit is designed to be built into windows or walls in the way that one part of the AC unit is in the interior, whereas the other part is in the exterior. These AC units treat the air only in the room where they have been mounted. Therefore, they are designed for lower power values. For common use, it is approx. 2-6 kW, for larger halls it is up to 10 kW.

**The air-conditioning split system - single-zone**

These are the most common type of air-conditioning equipment. They consist of two parts - the outdoor and the indoor one. They are usually permanently mounted and installed. Their great advantage is a low noise level thanks to the placement of the compressor in the outdoor unit. The cooling power of these devices is usually between 2 and 20 kW (Fig. 9.3).

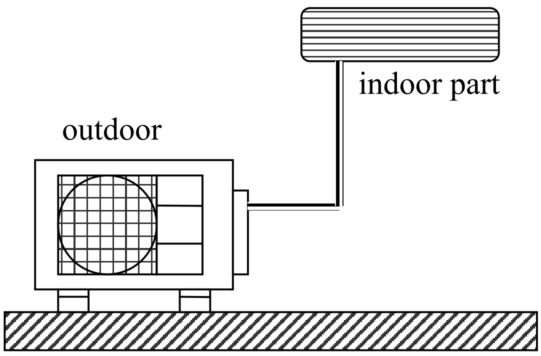


**Fig. 9.3: Two-part AC unit split**

The inner part blows cool or hot air into the room. It is usually connected to 230 V. In terms of location of the indoor part of the air-conditioning unit, we distinguish between:

1) **Wall-mounted units;** the most commonly used. They are mounted on the wall. The flow of cooled air is directed straight ahead from the air-condition unit. According to the number of cooling units of circuits, we distinguish between dual (two units), triple (three units), quadruple and multiple units; generally they are referred to as multisplit. They are usually placed to the

height of 2-3 meters above the ground. The decisive parameter is the ceiling height. Both the unit power and the direction of air flow can be controlled by a remote control or manually.

2) **Ceiling-mounted units** are placed under the ceiling. Usually, they have adjustable blinds, so that they can blow the air in various directions. Also fan speed can be adjusted. They are available in power values of 2-13 kW. One unit with cooling power of 5 kW can cool the room with a volume of approximately 130 $m^3$. If the room has a larger volume, two or more units can be installed.

3) **Console air-conditioners (below window)** are placed below the window or to other places close to the floor. Most console AC units replaced central heating and they are intended mainly for heat generation. For living rooms, they are available with power values of 2-8 kW. Two or more units can be fitted in larger rooms. It is the same with the ceiling-mounted units.

4) **Cartridge units** of air-conditioning consist of a cartridge which is incorporated in the ceiling, so that only the exhaust part can be seen. They usually have a rectangular shape. The number of exhausts is different. Some square air-conditioning units have up to four. Their disadvantage is a higher purchase price.

5) **Channel units** are usually placed under the ceiling. The indoor unit is placed in the false ceiling. The air is usually distributed by pipes around rooms. It is often placed in restaurant facilities. Their power is between 5 and 20 kW for rooms sized up to 600 $m^3$.

The outdoor part of the unit is placed firmly on the external walls or balconies or rooftops. They are placed to the indoor part of the AC units as close as possible, so that the piping, in which the coolant flows are as short as possible. Users usually prefer one outdoor unit for one indoor unit, but there can be two or more. It depends on the importance of the cooling equipment in the interior. The moisture which forms on the walls of pipes with the coolant due to the different temperatures of pipes and the surrounding is discharged by means of another thin pipe, so that the condensed water does not flow on the perimeter wall.

**The multisplit air-conditioning unit - multiple-zone**

Several (usually up to five) indoor units are connected to one outdoor unit. Heat power control is performed according to thermostats in rooms by closing of coolant supply to the indoor units with subsequent disabling of the compressor of the outdoor unit. Some systems have outdoor units with several small independent compressors for each indoor unit or groups of indoor units. Cooling power control is then easier; the controlling is done based on individual needs.

**The AC system multisplit with a variable refrigeration volume (VRV)**

A higher number of indoor units are connected to one outdoor units (sometimes 30 and more). Heat exchangers in indoor units are equipped with throttle valves for the control of coolant flow according to thermostats in each room. The compressor in the indoor unit is operated at variable speed (with a frequency converter) based on the required cooling power. The VRV system is always accompanied by air single-channel system for outdoor air supply according to hygienic requirements.

## 9.2. Possibilities of energy savings - not only for heating

### 9.2.1. Savings on energy in the house

Energy savings are quite often related to house redevelopment, so they can also bring an increase in comfort and the standard of living. The environmental burden is almost always reduced. Since the energy is never free, we expect energy savings as well as financial savings. It is necessary to clarify, what energy savings involve and bring.

Large, if not the largest part of energy, we consume during our lifetime, is related to housing. The energy we consume due to dwelling is used mainly for the following three purposes:

- for heating,
- for water heating,
- for operation of household appliances.

The last mentioned category includes mainly lighting, cooking, washing, cooling, ironing and operation of electronic appliances.

Each house has different energy consumption for heating, which is mainly determined by its structure. It depends on both insulating capabilities of the cladding and on the shape and size of the building. It is clearly seen on panel flats, where most of flats lose heat through the only external wall, whereas their other walls neighbor with other flats or a corridor. By contrast, a detached house loses heat through all walls, the floor as well as the roof. Another example can be a very well-insulated low-energy house, where less than a half of energy (compared to the older house of the same size) escapes through the cladding (Fig. 9.4).

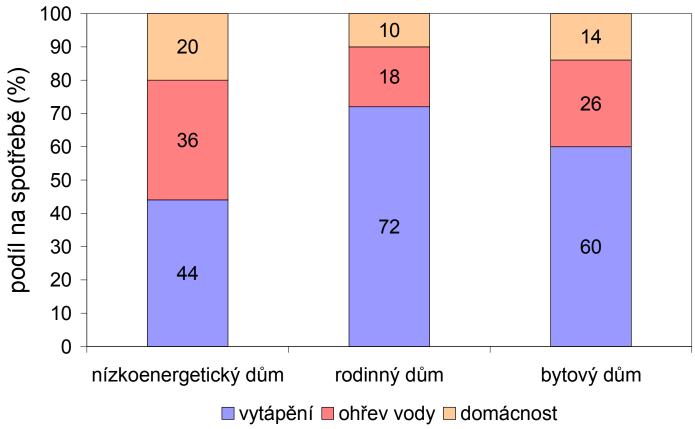


**Fig. 9.4: Approximate consumption structure in various houses [7]**

Energy consumption for operation of household appliances is influenced mainly by their technical development and economical features determined by their structures.

The distribution of energy consumptions according to purposes is important especially because of the fact that each part of consumption can be covered by a different energy, with a different price and with a different environmental impact. For operation of household, we need especially electricity, although we can use, for example, gas or wood for cooking. Water can be heated either by electricity (often the so-called night tariff is used, which is more affordable), or by the same fuel which is used for heating. In practice, it can be a natural-gas (or propane) boiler, which works as a gas-flow heater throughout the year and moreover, it heats during the winter. If we want to heat with coal or wood, we can use the so-called "combined boiler", which is heated by water from the central heating during the heating season and by electricity in the summer (or by the solar system).

### Energy price

It is very complex to determine the actual cost of energy, which would also include the impact on the environment, the health of the population, reduction in fuel reserves for future generation or costs of disposal of spent nuclear fuel. Experts from all over the world deal with the issues. Their results, however, are not always comparable. However, it is evident that when you account for these factors, the price of energy would be much higher. Currently, we can choose from many fuels of different quality and price. There are, of course, some limitations. In some places, there is no natural gas connection, in other places, there is not sufficient electric service lead.

The current prices which are continuously updated can be found in [8]. When compared, for example, the years of 2011 and 2004, we find out, that the prices of coal, electricity and gas increased twice, the price of wood increased three times, the price of propane and light fuel oils remained unchanged.

**How to save on heating bills**

Prior to the decision, where to start saving, it is necessary to consider not only energy flows but also cash flows. In Fig. 9.5, there is an example of a detached house with various combinations of energies for heating, water heating and product ranges of electric energy.

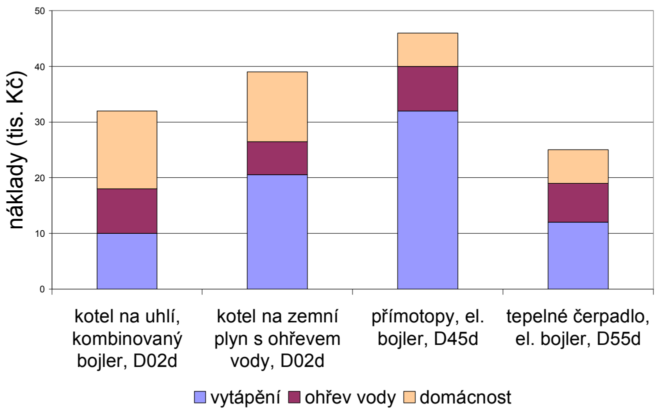


**Fig. 9.5: Comparison of energy costs in a detached house [7]**

Each loss, which is shown in charts in Fig. 9.6, can be reduced somehow. It is obvious that most energy escapes through the cladding. At the same time it is clear that even the best wall insulation cannot reduce the heating bill by a half. Thermal insulation should be performed comprehensively, while taking not only thermal insulation of various structures, but also its price into account.

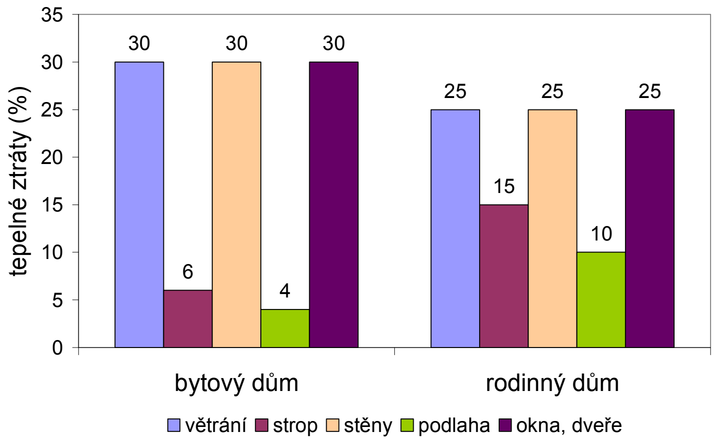


**Fig. 9.6: The share of various structures in heat losses of a house [7]**

It is tempting to search for savings in the reduction in heat losses by ventilation. Window sealing is cheap. However, it is related to major risks. Upon reduction in infiltration under a certain value, the indoor humidity can rise so much that it can lead to damage to the building. Reduction in ventilation can also have effect on residents' health. After each reduction of heat loss, the heating system and powers of heating bodies must be modified. In particular, controlling must be changed. Another option is to use cheaper fuel, for example wood. The costs of boiler change and modification to the heating system are lower than costs of thermal insulation and window replacement. Both options can be combined, so that energy and cost savings match our expectations.

### 9.2.2. Thermal insulation

Additional insulation of the house is usually related to a new facade, roof, attic or other redevelopment of a house. While considering whether thermal insulation pays off or not, the costs of these construction works must be considered.

In addition to a lot of other reasons, thermal insulation is also forced by current regulations. Formerly valid decree no. 291/2001, Coll., determined the maximum specific consumption which could not be exceeded by buildings and redevelopments funded from public funds. These requirements also applied to private houses if they had been subsidized. In addition, the requirements of the decree also applied to completely private buildings and redevelopment of larger buildings with consumption exceeding 700 GJ per year, which usually applies to panel houses. The decree did not apply to detached houses and other private houses with consumption lower than 700 GJ per year.

Since 2007, the decree no. 148/2007 Coll., has been valid, which stipulates:

- the requirement for energy performance of buildings, comparison indicators and the calculation method of determination of the energy performance of buildings;
- the content of energy performance card and the way of its processing, incl. use of already performed energy audits;
- the scope of testing of persons from the details of creation of energy performance card of buildings.

Redevelopment of an older house which will respect the requirements of the standard, should bring the savings of about 40 % by itself. However, it is good to note that the consumption of the house stipulated in the decree is intended mainly for comparison of a specific house with a specific standard; moreover only as far as the cladding is concerned. The actual consumption can be very different from the consumption determined in this way. One of the reasons is that the house can be placed in a warmer climate zone, its heating can be more efficient or it can use heat from waste air.

**Moisture in the house**

Appropriately performed ventilation is the simplest and effective way of removing moisture, which necessarily arises due to stay of people inside. Moisture, similarly to heat, always escapes from the place, where it is more abundant, to the place, where there is less of it - i.e. from the inside out in the winter. Cold winter air contains less moisture than hot air in the flat. Moisture transfer through walls depends on materials used during construction. Some materials, such as glass, metals, most plastics and others do not transmit moisture at all (they feature very high diffusion resistance). Porous materials, such as bricks, wood and concrete, transmit moisture more easily.  These materials can also absorb a certain volume of moisture without any difficulties, and they can release the moisture to the interior later. Thus the climate in the room is balanced, which contributes to better comfort of residents. However, if water steam penetrates to the structure in a larger amount, its condensation can occur. Excessive humidity in the house structure is always a potential source of difficulties. If frozen, it breaks masonry, accelerates corrosion of steel elements, supports rot in wooden structures and molds in interior plasters. Generally, it decreases the durability of the house.

In older brick homes, the moisture is accumulated during the winter and it is evaporated to the interior and the exterior in the summer. The thicker the wall is, the more moisture it is able to accommodate. Thermal insulation thus can be an obstacle for this “breathing”. Therefore, it is necessary that the thermal insulation project always assesses the risk of condensation, the possibility of water evaporation from the structure, and that it proposes such a solution that the moisture is no longer dangerous. The easiest principle is to avoid designing of the structure in the way that the diffusion resistance of materials from inside out drops. This means that moisture hardly gets into the structure from the interior, but if an amount of it penetrates, it can easily escape to the exterior.

**Reduction of losses through walls**

There are two basic types of thermal insulation:

- contact,
- thermal insulation by a ventilated gap.

Each of these ways can be performed as interior and exterior thermal insulation. The structure, however, must always be treated, so that no thermal bridges occur: walls must be insulated not only where there is heated space, but also under the level of floors and above the level of ceilings; the same applies to the thermal insulation of sills, brickwork lining and lintel.

**External thermal insulation**

For most buildings, it is more appropriate to use external (outdoor) thermal insulation. Especially for panel buildings, it is also an effective way to prolong their useful life. Thermal insulation protects from frost in the winter and from sun heat in the summer. Therefore stress of dilations is reduced. Steel connecting elements are more protected from the weather, and therefore from corrosion. Also thermal bridges in joints between panels are dealt with by thermal insulation. If designed properly, they are solutions to window openings as well.

**Advantages:**

- the masonry in “warm” and it is not stressed by temperature fluctuations and by the weather so much,

- accumulation capability of the house is increased,
- elimination of thermal bridges in the structure (such as window lintels, sets, ceilings and others) is easier,
- a risk of moisture condensation in the structure is minimal,
- a building gets a new facade, which leads to savings of maintenance costs,
- during installation, it does not disturb the stay of people inside.

**Disadvantages:**

- need for scaffoldings and the space around the house;
- insulation must be performed on the whole surface of the wall at once;
- the outer contour of the house is extended (may cause interference to a foreign plot);
- higher costs.

**Indoor insulation**

In some cases (for example, if the facade of the house is of historical value), indoor (internal) thermal insulation can be considered. For indoor thermal insulation, insulation with thickness from 15 to 20 cm is commonly used. It is evident that the internal insulation will always be a compromise between the requirement for heat savings and the size of living space. Since we cannot insulate horizontal structures as well, large thermal bridges arise. The original perimeter wall is separated from the warm indoor environment by an insulation layer after thermal insulation, therefore the thermal insulation is much colder. In places, where it is connected to partition walls, ceilings and floor, it cools down these adjacent structures so intensively that mold can occur at the point of contact. A cold zone arises between the original wall and insulation with a very probable occurrence of condensed water inside the structure. The result may be violation of not only the perimeter walls, but also the load-bearing structures, ceilings and floors.

**Advantages:**

- a possibility of insulation of a single room;
- easy access, without any need for scaffolding,
- possible to be installed regardless of the weather,
- easier to be performed within self-help.

**Disadvantages:**

- a risk of moisture condensation in the walls of the house,

- a risk of freezing of the exterior masonry,
- a risk of mold growing, especially in the area of thermal bridges,
- reduction of accumulation capability of the masonry,
- reduction of room space.

**Contact thermal insulation**

It is the most common and well-tested way of thermal insulation. Insulating material is glued to the substrate and anchored by wall plugs (due to gravity and wind, which can tear the material off). Depeter is applied onto the insulating material; the plaster contains a reinforcing mesh. The insulating material that is most commonly used is polystyrene, sometimes rigid boards from mineral fibers (usually in higher floors of buildings due to fire safety). Abroad, also cork boards as natural material are used.

The main advantage is a relatively low price and wide selection of suppliers. Another advantage is that the insulation lacks thermal bridges. Even smaller shaped elements (pilasters, cornices and others) from polystyrene, polyurethane or plaster can be glued to the insulation. Thanks to this feature, the house can have the same appearance as it had before the redevelopment.

The disadvantage is that the system requires solid and well-bearing substrate. It can be hardly used onto old and falling plaster. Contact thermal insulation must never be used for wet masonry! Some technological operations can be performed only during favorable weather, which may extend the construction time.

## 9.2.3. The amount of heat losses

Construction materials feature various abilities to transfer heat. 1cm-thick thermal insulation material (polystyrene) transfers the same amount of heat as the wood with a thickness of 4 cm and perforated bricks with a thickness of 9 etc. (Tab. 9.1)

| **Type of material** | **Thermal conductivity λ ($W \cdot m^{-1} \cdot K^{-1}$)** | **Comparison of material thickness (cm)** |
|---|---|---|
| insulating materials | 0.02 – 0.05 | 1 |
| wood | 0.12 - 0.16 | 4 |
| perforated brick | 0.33 - 0.36 | 9 |
| full brick | 0.78 - 0.82 | 20 |
| reinforced concrete | 1.35 - 1.45 | 36 |

**Tab. 9.1: Thermal conductivity of construction materials [6]**

Thermal insulating properties of the most commonly used materials are shown in Tab. 9.2- Generally, the lighter the material is, the better is its insulating properties. Good insulating materials are those, whose thermal conductivity coefficient $\lambda <$ 0,20 $W \cdot m^{-1} \cdot K^{-1}$.

| Type of material | Thermal conductivity λ ($W \cdot m^{-1} \cdot K^{-1}$) | Appropriateness of application |
|---|---|---|
| foam polyurethane | 0.018 – 0.035 | between the window and the wall |
| foam polystyrene | 0.026 – 0.051 | outdoor walls |
| foam polyethylene | 0.038 – 0.042 | Pipe insulation |
| glass or basalt wool | 0.035 – 0.076 | for higher tempera-tures |
| perlite | 0.058 - 0.062 | to mortar and con-crete |
| expanded clay | 0.090 - 0.110 | to mortar and con-crete |

**Tab. 9.2: Thermal conductivity of heat insulating materials [6]**

## 9.2.4. Economy of thermal insulation

Well-executed thermal insulation should have a lifespan of 40 and more years. For walls, it is necessary to renew paint after several years; it is the same with uninsulated house. Thermal insulation of the roof, ceilings and floors usually lasts up to the time when the building gets a new owner who has different ideas about its appearance and use.

While deciding on the correct thickness of the insulation, we must bear in mind that the costs usually do not rise proportionally with the thickness of the insulating material. For example, for contact thermal insulation, we have to pay for the project, scaffolding, adhesive and plaster materials, surface finishes, regardless of the fact whether we use 5 or 25 cm of the insulating material. Only the costs of the insulation material and anchoring elements rise with the thickness of the thermal insulation. Costs of work can increase slightly. In specific situations, the costs of flashing of sills and other equipment may rise significantly.

The price of the insulating material forms a small part of the total price. On the other hand, the insulating material in the only functional element of the set. This is why it does not pay off to save money on the amount of insulation. Due to the development of energy prices, insulator layers with a thickness of 15, 20 and more cm are used today.

| thermal insulation price ($CZK \cdot m^{-2}$) | 5 cm polystyrene | 10 cm polystyrene | 20 cm polystyrene |
|---|---|---|---|
| preparation of (repairs to) the substrate | 60 | 60 | 60 |
| scaffolding | 350 | 350 | 350 |
| insulator, dowels and strips | 180 | 280 | 510 |

| | | | |
|---|---|---|---|
| adhesive and plaster materials | 80 | 80 | 80 |
| installation | 330 | 330 | 350 |
| total | 1000 | 1100 | 1350 |
| heat transfer coefficients ($W \cdot m^{-2} \cdot K^{-1}$)<br>insulating effect | 0.85<br><br>100 % | 0.43<br><br>200% | 0.21<br><br>400 % |

**Tab. 9.3: Comparison of costs of thermal insulation to the double insulator thickness[7]**

## 9.2.5. Reduction of losses through windows and glazing

Windows are a major source of thermal losses. Heat escapes from both the passage and the radiation through the pane and the frame, and by infiltration together with the air in joints between the casement and the frame. Infiltrations support necessary ventilation. The development of the window design showed great progress, so the new and modern windows are twice as good as the ones we are accustomed to in older buildings.

**Window replacement**

Replacement of old windows for new ones is always so costly that it almost never pays off in terms of energy savings. However, if we decide to replace the windows (for example, due to a bad condition of the original ones), we should not save on glazing. Windows are offered with various types of double-glazing, while the difference between the cheapest and the most expensive type is 10 to 20 % of the window price. By contrast, the difference in the insulation capability is double.

High-quality insulation double-glazing is characterized by the fact that the gap between the panes is filled with argon or a different gas, which is a better insulator than the common air. The so-called vacuum double-glazing, where the air between panes was diluted (it was thus very thick vacuum) are currently surpassed. Another attribute is a microscopic layer of metals (or metal oxides) on the outer side of the inner pane. This layer transmits natural light inside, but not heat outside. We can imagine it as a sieve with mesh, through which short-wave radiation - light - passes easily, but the long-wave radiation - heat - does not pass through it at all (Fig. 9.7).

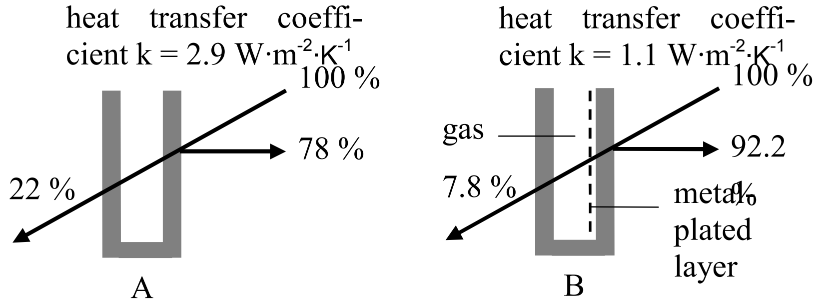


**Fig. 9.7 The difference between a common (A) and a high-quality double-glazing [7]**

Even better insulating parameters can be obtained with the glazing consisting of three panes (triple-glazing). A few years ago, installation of triple-glazing was one of popular saving measures. Over time, however, triple-glazing began to be thrown off because of its high price and the huge weight of windows with higher demands on fixing of window fittings. In addition, double-glazed windows using reflective layers and inert-gas filling, whose parameters were almost identical with triple-glazing and they were significantly cheaper, began to appear. Nowadays, when solutions to reaching of much lower values of the heat transfer coefficient, we can see the renaissance of windows with multiple-glazing. In addition, other solutions, such as replacement of the middle glass with metal-plated transparent foil started to appear. The weight is the same as with the double-glazing and there are no problems with different dilatation of three window panes.

### 9.2.6. Ventilation

Ventilation is a significant source of thermal losses. However, it cannot be easily reduced. A lack of fresh air leads to fatigue and discomfort of people in the building. In the extreme case, it may lead to health difficulties. Ventilation is not only supply of oxygen, but also removal of odors and harmful substances (such as smoke, dust, formaldehydes, or radon, etc.), which are released into the room. Even the actual house benefits from ventilation as it reduces humidity emerging due to stay and activities of people. Sufficient ventilation is also a prevention of mold formation. Reduction of unnecessary ventilation can save approx. 10 to 15% of energy for heating.

**Air exchange intensity.**

It is recommended in standards that the air exchange intensity in the room occupied by people is 0.3 to 0.6 $h^{-1}$, most often value is 0.5 $h^{-1}$. This means that a half of the air volume in the room is exchanged within an hour. It is evident that it is not ideal to base our consideration on the room volume. The amount of air per a person, i.e. 15 to 25 $m^3 \cdot h^{-1}$ seems to be better. If there are no people in the room, the recommended ventilation intensity is at least 0.1 $h^{-1}$ with regard to exhaust of moisture and harmful substances. If radon is present in the house, the ventilation must be very intensive. Should it be the case, machine ventilation is advisable to be used, if possible with heat recovery. Ventilation reduction could have tragic consequences in this case.

### Natural ventilation

Most houses are ventilated through windows. This is actually one of their basic functions. Ventilation should be impact, i.e. with wide-opened windows, at least once in two hours, so that the air can flow quickly. In winter, it is enough to open windows for 3 to 5 minutes. In the spring and the autumn, 10 to 15 minutes is sufficient. Short ventilation is important due to the fact that the walls and the floor are not cooled down unnecessarily. Ventilation through a partly opened window is very inappropriate. A lot of heat escapes without any benefit directly outside and distant parts of rooms are ventilated very little.

### Infiltration

A part of natural ventilation works regardless of a user. Cool air leaks through the frame and the casement of the window inwards. Warm air escapes in calm air through joints in the upper part of windows. In the wing, it escapes through windows on the leeward side. The amount of ventilation air depends on the tightness of windows, the difference of temperatures inside and outside and on the speed of wind. The result is that ventilation by infiltration is never the same as we require. If the room is not occupied, infiltration is mostly unnecessarily high, which increases the consumption of heat. If there are any people in the room, infiltration is usually insufficient, so it is necessary to ventilate through an open window from time to time.

The solution is sealing of the windows or using new sealed windows. Modern windows are up to ten times tighter than ordinary older wooden windows. Additional seals of older windows increases their tightness several times. The consequence, however, is an increase in moisture in the room which is insufficiently ventilated. Then, some molds can occur in corners and at points of heat bridges. In older houses with wet walls, sealing of windows would worsen the problem with moisture significantly. The reasons for moisture must be removed firstly. Savings by window sealing depend on how leaky the original windows were, usually up to 10 %.

### Forced ventilation

Ventilation using fans is nowadays commonly applied only to kitchen hoods and in the bathroom, where there is a need for exhaustion of a greater amount of moisture. Impact operation is assumed; the operation does not have any permanent effects on the environment in the house.

Systematic ventilation, however, is no longer the privilege of industrial and office buildings only. If we ventilate the house with fans, we ventilate only when and only how much we need, which significantly influences energy consumption. Even higher savings, however, can be obtained through heat recovery from the exhaust air. If we equip the house with air-condition, this option offers itself. Another advantage may consist in cooling of the house during the summer. While this does not bring energy savings, it can significantly increase the comfort of the house. Installation of the air-conditioning unit for heat recovery for savings must be carefully considered, as the efficiency of this measure depends on the price of heat we use for heating, on the efficiency of the equipment and on the amount and the price of driving energy consumed by recovery facilities.

**Central ventilation systems**

Ventilation air is supplied into rooms through A/C piping conducted in lower ceilings or in the floor and walls. Air exhaust can be placed either in each room, or centrally, e.g. in the corridor. Doors from rooms, then, must not be air-tight. The core of the system is usually a compact unit with exhaust and supply fan, filters, recovery heat exchanger and air heater (or also with air cooler). The heater can be either electric, or hot-water, which shall be connected to the boiler or another heat source (or via a storage tank). The central ventilation system can be well connected to the house heating. The costs saved on the heating system then balance the installation costs of ventilation. With regard to the extent of construction works, it is suitable for new buildings or for major redevelopments. If we want to have a fireplace or another heating device with a chimney, the design of the ventilation must be slightly overpressure, because vacuum systems exhaust the air through the chimney. This means that more air must be supplied than is exhausted. If the air is only supplied and the exhaust air escapes through windows, it cannot be, of course, used for heat recovery. Generally, it is better to use vacuum systems, so that the moisture flow from the interior to the structures is reduced. Usually it is difficult to provide for different ventilation intensity in individual rooms. It is dealt with in the way that a part of the air in the house circulates, so in terms of ventilation the house is one big room, to which fresh air according to the number of people is supplied. The ventilation system must allow modification of the volume of ventilation air, most often by continuous or stepped change to fan speed. The central ventilation allows very effective using of solar yield from sunny rooms which is distributed throughout the hose, so no overheating of sunny rooms occurs.

Thanks to machine ventilation and heat recovery, up to 80 % of energy for ventilation, i.e. approx. a quarter of the total house consumption, can be saved. A great be benefit is higher comfort of the dwelling and plenty of fresh air.

### 9.2.7. Heat sources

**Wood, biomass**

In the Czech Republic, wood is still one of the cheapest fuels. For larger sources (e.g. block boiler houses of apartment buildings), wood chips, straw or other combustible biomass are used. However, its price is individual.

It applies that the wood should be burnt in special boilers. Wood heating in coal boilers is inefficient. Since wood burns with a long flame (compared to coal, whose flame is short), a great amount of energy is wasted and escapes unused through the chimney.

It is also necessary to stoke with dry wood only. It means with the wood which has been stored in a pile for at least two years. Raw wood significantly reduces boiler durability and more wood must be used. Moreover, its calorific value, compared to dry wood, is half because a great amount of heat is consumed for water evaporation. The need for storing space is one of great disadvantages of wood (together with work required for its preparation). If uninsulated detached house consumes 15-25 stacked cubic meters (scm) of wood, then the demand for the space around the house is really great. Various kinds of woods feature approximately the same calorific value (Fig. 9.8). Hardwood features higher specific weight than softwood. The log of the same size has more energy.

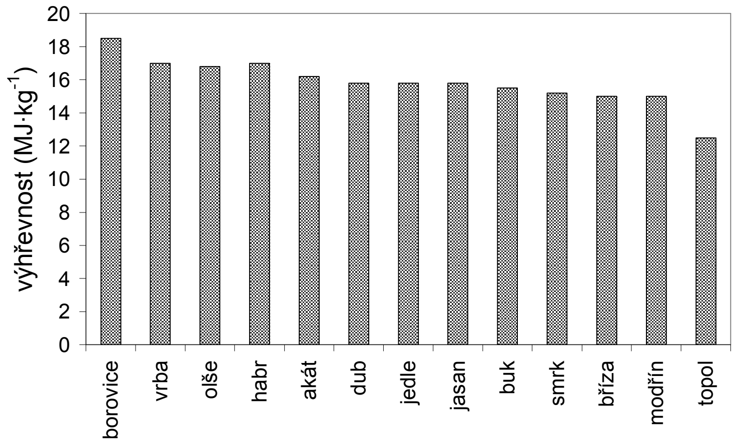


**Fig. 9.8: The calorific value of wood at 20% moisture [7]**

**Natural gas**

It is a very comfortable and relatively environmentally-friendly fuel (emissions of sulfur oxides and dust are practically zero), which can be used with high efficiency. Boilers can be well regulated.

Recently, natural gas has been charged in kWh, not in m$^3$, as it used to be earlier. Gas meters, however, still read in m$^3$, so the gas supplier converts the volume to kWh according to the average of the combustion heat for a billing period. Combustion heat is higher than calorific value (1.11x) - see Fig. 9.9. Both the values stipulate how much energy is contained in the gas

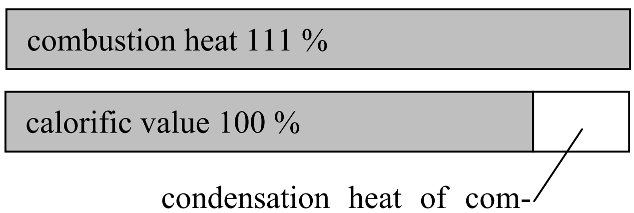


**Fig. 9.9: Maximum efficiency of the boiled depending on cooling of combus-**

**tion products [7]**

If we burn gas, $CO_2$, water steam and a small amount of other products (nitrogen oxides and others) arise. However, if we cool combustion products, the steam condensates and we get heat which was necessary for conversion of water to steam. This water is the reason for the difference between the calorific value (which does not include it) and combustion heat (which includes it). Combustion product condensation is usually undesirable because it causes low-temperature corrosion of steel boilers.

The definition of boiler efficiency is based on the calorific value. It is defined as a ratio between the energy in the fuel (i.e. the calorific value) and the energy we obtain from the boiler (the difference is losses). As a result, modern condensation boilers, which are not endangered by low-temperature corrosion, may feature efficiency over 100 %. Another consequence is that the data on energy supply on the gas bill must be firstly converted to the calorific value if we wish to use it for technical calculations.

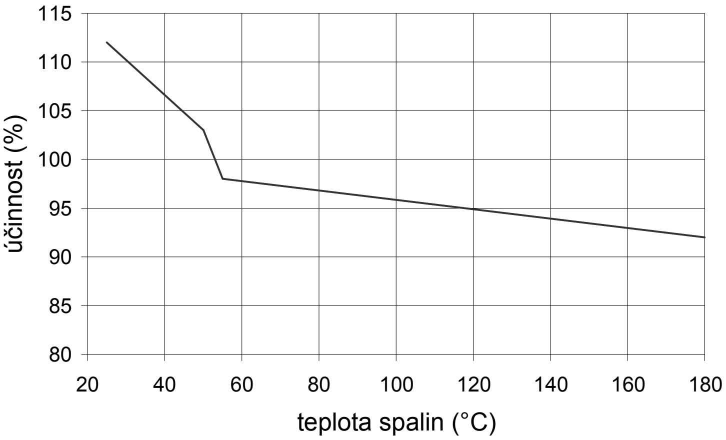


**Fig. 9.10: The maximum efficiency of a boiler depending on combustion product cooling [7]**

**Solar systems**

They are used as auxiliary sources, especially at the beginning and at the end of the heating season. This follows from the fact that there is a lack of sunshine in the winter when heat consumption is the highest. Hot-water collectors can (in addition to hot-water preparation, which is usually the primary function) discharge heat to the floor heating or low-temperature radiators. A storage tank is necessary as the house uses mainly passive yields through windows and glazing at the time of sunshine.

## 9.2.8. Return on investments for thermal insulation

We expect from the investment that it will improve the value of the invested money with a certain profit and a certain risk. We can consider thermal insulation as spending. Both the usable and the market values of the house should be higher after thermal insulation. The decision thus depends only on personal preferences. If the house really needs a new facade, we have to deal with the so-called forced investment. We also consider whether we shall make the common facade with lower costs or whether it is better to make a better, thermal-insulated facade and to pay for heat less than before. Thermal insulation is relatively cheaper as we should exert some costs in any case.

While insulating the building, we can use both measures with fast return on investment, and the non-repayable investments. For basic economic evaluation of ROI, we need to know three parameters:

1) Austerity measure costs - unit prices (e.g. price per $m^2$ of thermal insulation). For most buildings, they are almost the same.

2) The amount of energy savings - the worse the original building is, the more we spend on heating; thus we can obtain savings more easily (thermal insulation on a thin brickwall brings higher savings than on a wall from insulating blocks).

3) Heat price - it depends not only on the price of fuel, but also on the efficiency of the boiler or other equipment. Sometime it is necessary to include other costs in the fuel price - e.g. disposal of ashes for building of a gas connection and the like. In particular, electric heating requires continuous payments regardless the quantity consumed. The less energy we burn, the more expensive is the kilowatt hour. The output of the simple economic assessment is the pure return on investment (ROI). If it is longer than the duration of the measure, the invested resources will never return to us.

In the Czech Republic, ROI in thermal insulation is a lively discussed topic. There are calculations showing that the investment in thermal insulation returns within 7 to 10 years. Of course, depending on the current energy prices. The consumption of a house is assessed according to the consumption of kWh per $m^2$ of living space per year. Older, not insulated houses feature consumption of 200 $kWh \cdot m^{-2}$ per year. The tendencies in future construction clearly head for low-energy heat-insulated buildings with a consumption of only 50 $kWh \cdot m^{-2}$ per year.

Generally speaking, complete thermal insulation, including window exchange, can save between 50 to 75 % of heat for heating in houses built until 1979. The savings built later is usually between 30 to 50 % of heat. The difference is caused by the fact that new standards came into force in this year.

In this chapter, verbatim quotations, charts and figures from references [6, 7] have been used.

## 9.3. Connection of electro-thermal appliances to a low-voltage distribution network

### 9.3.1. Conditions for connection of electro-thermal appliances

Conditions for connection of electro-thermal appliances are determined by the relevant distribution plant. Generally it applies that connection of each electric appliance with the input power of 10 kW and over and connection of electro-thermal appliances if their total input power, incl. appliances which have been already connected, exceeds the value of 5 kW, should be authorized by the respective distribution plant with regard to power throughput of the electric distribution network at the connection point.

Other conditions for connection of electro-thermal appliances are bound with the relevant tariff rates.

### 9.3.2. Protection of electro-thermal appliances

Permanent payments are derived according to the value of the sealed main circuit breaker. Thus the maximum total current of consumer's place is limited.

Another protection of consumer's place circuits must provide for protection of individual power lines and protection of connected appliances. Most electro-thermal appliances feature their own overcurrent protection.

Since electric circuits for electro-thermal appliances are separate for the reason of blocking and controlling, their protection system depends on the input power of electro-thermal appliances and also on the respective conductors (however, they are also dimensioned according to the input power of electro-thermal appliances).

In general, protection against overcurrents and short-circuit currents must be performed.

### 9.3.3. Measuring of electric energy consumption

The measuring device must be fitted on electric gauge boards on in electric gauge switchboards. The location of a measuring device depends on the character of the respective building. The measurement should, if possible, be accessible to employees of the respective distribution plant at any time.

Only electric meters, tariff switch (tariff gauge or ripple control receiver), the breaker in upstream the electric meter, breaking device of the tariff switch circuit, control relays or contactors, zero terminal board or other accessories for measuring can be fitted on electric meter switchboards.

The switchboards must be designed in the way which suits the environment, where they are located. The switchboards are equipped with doors. If opened, no live parts are accessible. Upon closing, they must feature ingress protection of at least IP 30, upon opening the ingress protection must be at least IP 20.

The internal layout of electric meter switchboards must be arranged in the way that the live parts of non-measured distribution are separated from the room for electric meters and tariff switches.

A circuit-breaker must be fitted upstream the electric meter; it must feature the same number of poles as is the number of a phase electric meter. The circuit breaker should be dimensioned, so that it does not limit the input power of installed appliances at the customer's place. The exceptions are the cases, when the limitation is required by the operation of the electrical distribution network or if the current value of the circuit breaker is included in the agreed tariff.

Electricity meter boards or switchboards must be suitable for installation of meters and auxiliary devices with the protection class of I. If the switchboard is fitted with the device of the 2nd insulation class, the protective conductor shall not be connected, but plugged behind the board.

Energy meters for direct measuring are fitted only up to the value of 80 A. For measuring over 80 A, current measuring transformers with conversion of XX/5 A with the accuracy class of 0.5 and nominal load of 15 VA must be used.

Auxiliary relays for controlling of contactors for preparation of hot service water and heating must be in steel-sheet switchboard and installed in the area of main circuit breakers.

## 9.3.4. Blocking of electro-thermal appliances

Blocking of electro-thermal appliances prevent their operation within the specified periods of time. This technical condition allows defining of the respective tariffs for withdrawal of electrical energy. Locking relays or contactors are controlled by a timer or a ripple control receiver receiver.

Blocking against the withdrawal during the high tariff is the prerequisite for storage heating. Blocking must be automatic together with the change to the rate on the energy meter.

Heating during the period of the high tariff, i.e. in the period of energy peaks, is also the condition of blocking for direct heating. Blocking must be automatic. It is the same as with the storage heating.

Direct-heating flow heaters of service water need not to be blocked, but their simultaneous operation with the electric heating must be avoided.

Storage heaters of hot service water with a capacity of over 25 L of water must be blocked against withdrawal during the high tariff. This blocking is technically performed in the same way as with the storage heating.

#### Timer

One of the means of control of electro-thermal appliances is a timer. At the same time, this device switches energy meter rate, and controls and blocks the electro-thermal appliances. The timer consists of a clock machine (which used to be mechanical, but today it is electronic) with a 24-hour mode. The timer controlling energy meters with the evaluation of the maximum features a week mode.

Periods of high and low tariffs are set on the timer. In addition to power supply contacts, the timer features operating contacts of a switching relay. The switching relay switches at the pre-set times, thus it changes the tariff of the electric meter and switching of blocking of electro-thermal appliances.

The disadvantage of the timer consists in the fact that their operation is not synchronized in terms of time. Therefore, accurate switching of high and low tariff zones depends on the accuracy of the clock machine. The timer must be checked and adjusted by workers of distribution plants because these devices are sealed.

Another disadvantage is that setting of periods of high and low tariffs cannot be changed remotely. Any changes can be done only by workers of a distribution place directly at consumer's place. Timer operation cannot thus respond immediately to the current condition in the power supply network.

The timer is used in areas which are not covered by the ripple control signal or where the ripple control signal is not reliable.

**Ripple control receivers**

Ripple control receivers gradually replace timers. They are connected in the same way as the timer on the energy meter switchboard.

The ripple control system is a system of receiver control from one point (ripple control transmitter) without any feedback of command execution. Ripple control transmitters are located in a switching station and its operation is usually controlled from the respective control room. The connection path between the transmitter and receivers is the distribution electric network. The broadcast telegram is encoded at the operating frequency and this signal is superimposed at a common operating frequency of 50 Hz.

The description of the ripple control system, kinds of telegrams and technical implementation of the system is quite an extensive topic. Therefore in this chapter, we mention only the issues directly related to the control of electro-thermal appliance operation.

The ripple control receiver, according to its settings, responds to the respective broadcast telegram of the ripple control and it executes, like the timer, switching of the tariff on the energy meter and controlling (blocking) of electro-thermal appliances.

The advantage of this system consists in the fact that tariff switching can be carried out according to the current condition of the network. The distribution company, however, must adhere to conditions which are binding for the respective rate. There is no problem with time synchronization; neither the daily, nor weekly operation require different types of receiver.

The problem of the ripple control system that due to interference of higher harmonics of current and resonance phenomena prevented extension of this system to industrial areas, where these interferences are significant. By the reduction of the controlling frequency, this system becomes more resistant to interference [9].

# 9.4. References:

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# 10. Numerical methods in electro-thermal technology

## 10.1. Differential operations with vectors

In this chapter, an overview of basic differential operations with vectors, you can come across while modeling electro-thermal processes, is listed. [1]-[10]

The function $u = f(x, y, z)$ defined in the specific area is called the scalar field. The scalar field assigns a numeric value to each point of the respective field (a scalar). The areas $u$ = const. are scalar field levels. An example of the scalar field is a potential electrostatic field. Its levels are called equipotential surfaces.

Partial differential equations are used when we are interested in the influence of several variables on the respective quantity, while separate independent variables usually act independently of each other. A partial differential equation is thus a differential equation of multiple variables, and the following designation is used for the partial derivations of the first order according to individual variables

$$u_x = \frac{\partial u}{\partial x}, u_y = \frac{\partial u}{\partial y}, u_z = \frac{\partial u}{\partial z}, \qquad (10.1)$$

for the partial derivation of the second order, they have the following shape:

$$u_{xx} = \frac{\partial^2 u}{\partial x^2}, u_{yy} = \frac{\partial^2 u}{\partial y^2}, u_{zz} = \frac{\partial^2 u}{\partial z^2}. \qquad (10.2)$$

The integral or solution to the partial differential equation is called the function, which suits the equation identically in a field.

The scalar field gradient $u = f(x, y, z)$ is a vector, whose components are partial derivation of the scalar field according to individual coordinates

$$\mathrm{grad}\, u = \frac{\partial u}{\partial x}\boldsymbol{i} + \frac{\partial u}{\partial y}\boldsymbol{j} + \frac{\partial u}{\partial z}\boldsymbol{k} = \nabla u = \left(\frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}\right)u, \qquad (10.3)$$

where ***i, j, k*** are unit vectors, $\nabla$ is the Hamilton operator nabla (nabla operator) defined by the relation

$$\nabla = \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}. \qquad (10.4)$$

The scalar field gradient is perpendicular to its level at each point. The electrostatic field gradient (at each point) defines the vector characterizing the intensity of the respective field. All these vectors create the vector field. Curves which touch this field at each their point are called lines of forces. The lines of forces are thus always normal to equipotentials. The gradient thus features the direction of the highest increase of this field at each point of the scalar field. The largest drop of the field can be obtained by movement in the opposite direction to the gradient; the zero change in the directions normal to the gradient.

The equation expressing the Fourier's law of heat conduction contains the temperature gradient. It can be said that the temperature gradient determines the increase in temperature in the direction of the normal to the isotherm.

The divergence of the vector field ***a*** is a scalar and it applies that

$$\operatorname{div}\boldsymbol{a} = \nabla\cdot\boldsymbol{a} = \boldsymbol{i}\cdot\frac{\partial\boldsymbol{a}}{\partial x} + \boldsymbol{j}\cdot\frac{\partial\boldsymbol{a}}{\partial y} + \boldsymbol{k}\cdot\frac{\partial\boldsymbol{a}}{\partial z} = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}. \quad (10.5)$$

We can see from the mentioned equation that the divergence of the vector field can be written as a sum of scalar products.

When liquid flows, the divergence of the speed vector expresses volumetric amount of the fluid which drains from the unit volume per a time unit; in other words, the yield of the divergent of the unit volume. The vector field ***a*** is called the “divergent field”, provided that ***a*** ≠ 0 applies to at least one point. The vector field is called divergence-free field, provided that div ***a*** = 0, for example incompressible liquid free of divergences. The flow of this field through a closed surface equals to zero (the same amount leaves the surface and the same amount enters it).

Rotation of the vector ***a*** is a vector

$$\operatorname{rot}\boldsymbol{a} = \nabla\times\boldsymbol{a} = \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right)\boldsymbol{i} + \left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}\right)\boldsymbol{j} + \left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)\boldsymbol{k}. \quad (10.6)$$

Rotation of the vector **a** can be described as a vector product of the nabla operator and the respective vector.

The liquid speed vector rotation expresses the direction of the axis, around which the liquid turns as a whole in the surroundings of the point under consideration. The length of this vector determines double rotation speed (in arc measure) For the irrotational field it holds that ***a*** = 0. Each irrotational field can be simply expressed in continuous areas as a gradient of a scalar field. The rotation can be written as a sum of vector products

$$\operatorname{rot}\boldsymbol{a} = \nabla\times\boldsymbol{a} = \left(\boldsymbol{i}\times\frac{\partial\boldsymbol{a}}{\partial x}\right) + \left(\boldsymbol{j}\times\frac{\partial\boldsymbol{a}}{\partial y}\right) + \left(\boldsymbol{k}\times\frac{\partial\boldsymbol{a}}{\partial z}\right). \quad (10.7)$$

Laplace operator can be obtained by scalar multiplication of the nabla operator by itself.

$$\Delta = \nabla \cdot \nabla = \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \qquad (10.8)$$

It is a linear differential operator of the second order and it consists of partial derivation according to the coordinates. The Laplace operator can be applied to both the scalar and the vector fields. The Laplace operator does not have such a clear significance as the divergence and the rotation, but it is applied in electricity and magnetism, in the science of waves, etc.

All the mentioned differential operators (the gradient, divergence, rotation, Laplace operator, nabla operator) are homogeneous and additive (we obtain the sum of values for the sum of arguments). Further, there is a list of some important relations for calculations with differential operators

$$\begin{aligned}
&\operatorname{div}(\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{b} \operatorname{rot} \boldsymbol{a} - \boldsymbol{a} \operatorname{rot} \boldsymbol{b}, \\
&\operatorname{div}(\operatorname{grad} u) = \nabla \cdot (\nabla u) = \Delta u, \\
&\operatorname{rot}(\operatorname{rot} \mathbf{a}) = \operatorname{grad}(\operatorname{div} \boldsymbol{a}) - \Delta \boldsymbol{a}, \\
&\operatorname{rot}(\operatorname{grad} u) = \mathbf{0}, \\
&\operatorname{div}(\operatorname{rot} \boldsymbol{a}) = 0.
\end{aligned} \qquad (10.9)$$

The last two equations express a significant finding that the potential field (grad *u*) is always irrotational (its rotation identically equals to zero) and the field rot ***a*** is necessarily divergence-free (its divergence is identically zero).

In the electric thermal technology, tasks for cylindrical bodies are often dealt with; therefore basic differential operation will be stated in cylindrical coordinates of $r$, $\varphi$, $z$. The result of vector divergence and the Laplace operator of the scalar function is the scalar.

$$\begin{aligned}
&\operatorname{div} \boldsymbol{a} = \frac{1}{r}\frac{\partial (r a_r)}{\partial r} + \frac{1}{r}\frac{\partial a_\varphi}{\partial \varphi} + \frac{\partial a_z}{\partial z}, \\
&\Delta u = \nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial z^2},
\end{aligned} \qquad (10.10)$$

the vector coordinates of grad *u* in the directions of $r$, $\varphi$, $z$ are as follows:

$$\frac{\partial u}{\partial r}, \quad \frac{1}{r}\frac{\partial u}{\partial \varphi}, \quad \frac{\partial u}{\partial z}, \qquad (10.11)$$

the coordinates of the rot ***a*** vector in the directions , $\varphi$, $z$

$$\frac{1}{r}\frac{\partial a_z}{\partial \varphi}-\frac{\partial a_\varphi}{\partial z},\quad \frac{\partial a_r}{\partial z}-\frac{\partial a_z}{\partial r},\quad \frac{1}{r}\frac{\partial (r a_r)}{\partial r}-\frac{1}{r}\frac{\partial a_r}{\partial \varphi}. \tag{10.12}$$

Note: The following number (scalar) is called as the scalar product of two vectors $\boldsymbol{a}(a_x, a_y, a_z)$, $\boldsymbol{b}(b_x, b_y, b_z)$ (designation $\boldsymbol{a}\cdot\boldsymbol{b}$ or $\boldsymbol{ab}$)

$$\boldsymbol{a}\cdot\boldsymbol{b} = a_x b_x + a_y b_y + a_z b_z. \tag{10.13}$$

If ***a, b*** are non-zero vectors and their angle is $\varphi$, then the following applies to their scalar product:

$$\boldsymbol{a}\cdot\boldsymbol{b} = |\boldsymbol{a}|\cdot|\boldsymbol{b}|\cos\varphi. \tag{10.14}$$

The non-zero vectors ***a***, ***b*** are normal to each other just if the following applies:

$$\boldsymbol{a}\cdot\boldsymbol{b} = 0. \tag{10.15}$$

The vector product (design. $\boldsymbol{a}$ x $\boldsymbol{b}$) of two vectors $\boldsymbol{a}$ $(a_x, a_y, a_z)$, $\boldsymbol{b}$ $(b_x, b_y, b_z)$ is the vector and using the basic vectors **i**, **j**, **k,** the vector product can be written in the following form:

$$\boldsymbol{w} = \boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}. \tag{10.16}$$

The vector product of two linear-dependent vectors is a zero vector. The vector product of two linear-independent vectors ***a***, ***b***, which form the angle $\varphi$, has the following properties:

- it is perpendicular to both the respective vectors ***a***, ***b***, i.e. $\mathbf{w}\cdot\boldsymbol{a} = 0$, $\mathbf{w}\cdot\boldsymbol{b} = 0$,
- its length numerically equals to the area of a parallelogram defined by the vectors ***a, b***

$$|\boldsymbol{a}\times\boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}|\sin\varphi. \tag{10.17}$$

## 10.2. Curvilinear (contour) integral

The curvilinear (contour) integral of the scalar or the vector fields along the specific curve. Consider a spatial oriented curve *c* which is smooth in parts, which is closed. The curvilinear integral along this closed curve is called the vector field circulation ***a*** along the closed curve *c* and it has the following form:

$$\oint_c \boldsymbol{a} \cdot \mathrm{d}\boldsymbol{s}, \tag{10.18}$$

where d***s*** = ***t***d*s*, *s* is a parameter of length and *t* is a unit tangent vector at a point of curve under consideration. The integral along the closed curve in the potential field equals to zero.

$$\oint_c \boldsymbol{a} \cdot \mathrm{d}\boldsymbol{s} = \oint_c \mathrm{grad}\, u \cdot \mathrm{d}\boldsymbol{s} = 0. \tag{10.19}$$

The Stokes law expresses the relation between the curve integral of the vector field over the closed curve *c* and the surface integral, where the surface *A* is the surface enclosed by the curve *c*. The following applies

$$\oint_c \boldsymbol{a} \cdot \mathrm{d}\boldsymbol{s} = \iint_A \mathrm{rot}\, \boldsymbol{a} \cdot \mathrm{d}\boldsymbol{A}, \tag{10.20}$$

where d***A*** = ***n***d*A*, ***n*** is the external vector of the normal. Physical interpretation of the Stokes law: The flow of the vector rot ***a*** through the surface *A* equals to the circulation of the vector ***a*** along the curve *c*.

The Gauss-Ostrogradskiy theorem relates the surface and the volume integral.

$$\iiint_V \mathrm{div}\, \boldsymbol{a} \cdot \mathrm{d}V = \iint_A \boldsymbol{a} \cdot \mathrm{d}\boldsymbol{A}. \tag{10.21}$$

The integral on its right side is the surface integral along the closed surface, whose inside is the volume *V.* Physical meaning: The flow of the vector ***a*** through a closed surface equals to the volume integral from the divergence of the vector ***a.***

## 10.3. Bessel Function

Cylindrical functions are used for modeling of some physical fields, e.g. for the analysis of temperature and electromagnetic fields in the cylindrical environment. Using these functions, distribution of temperature in cylindrical charge, the electrical field in the cylinder during dielectric heating are examined, surface phenomena in an electrically conductive cylinder by action of the electromagnetic field are analyzed, etc. There are several types of cylindrical functions. They include the Bessel functions which are classified according to the type and the order.

Let us consider the following differential equation

$$x^2 y'' + xy' + \left(x^2 - n^2\right) y = 0, \tag{10.22}$$

then the solution to the equation is the Bessel function, where the *n* number is designed as an order of the function. The function $y = J_n(x)$ is the Bessel function of the first kind of the *n* order and it is defined by the following relation:

$$J_n(x) = \left(\frac{x}{2}\right)^n \sum \frac{(-1)^k}{k!\Gamma(n+k+1)} \left(\frac{x}{2}\right)^{2k}, \qquad (10.23)$$

where $k$ = 0, 1 , 2, .. For the integer number $n$, the following applies

$$J_{-n}(x) = (-1)^n J_n(x). \qquad (10.24)$$

In the relation (1.20), there is the $\Gamma$Euler's totient function of the second order or the gamma function, for which it applies

$$\Gamma(x) = \int_0^\infty \mathrm{e}^{-t} t^{x-1} \mathrm{d}t \qquad (10.25)$$

the following applies$\Gamma$ ($n$+1) = $n$!, i.e. for $n$ = 0 it holds that $\Gamma$(0+1) = 0! = 1, for $n$ = 1 it holds that $\Gamma$(1+1) = 1! = 1 etc.

Bessel functions with real and complex arguments are used in application of electric heatings. At first, the Bessel function of the zeroth and the first orders for the real argument are introduced.

- The Bessel function of the first kind, zeroth order ($n$ = 0) with a real argument

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{(2\cdot 4)^2} - \frac{x^6}{(2\cdot 4\cdot 6)^2} + \frac{x^8}{(2\cdot 4\cdot 6\cdot 8)^2} - \cdots, \qquad (10.26)$$

- The Bessel function of the first kind, the first order with the real argument

$$J_1(x) = \frac{x}{2} - \frac{x^3}{2^2\cdot 4} + \frac{x^5}{(2\cdot 4)^2\cdot 6} - \frac{x^7}{(2\cdot 4\cdot 6)^2\cdot 8} + \cdots \qquad (10.27)$$

It also applies that the derivation of the function $J_0(x)$ ia a negative value of the function $J_1(x)$

$$\frac{\mathrm{d}[J_0(x)]}{\mathrm{d}x} = -J_1(x). \qquad (10.28)$$

The Bessel functions with a complex argument $(x\sqrt{-j})$ are expressed by means of two functions ber $x$ and bei $x$, where the function ber $x$ includes the sum of all real elements and the function bei $x$ the negative sum of imaginary elements. Then we can write:

- The Bessel function of the 1st kind, 0th order with a complex argument

$(x\sqrt{-j})$

$$J_0(x\sqrt{-j}) = 1 - \frac{(x\sqrt{-j})^2}{2^2} - \frac{(x\sqrt{-j})^4}{(2\cdot 4)^2} + \frac{(x\sqrt{-j})^6}{(2\cdot 4\cdot 6)^2} - \cdots$$

$$J_0(x\sqrt{-j}) = 1 + j\frac{x^2}{2^2} - \frac{x^4}{(2\cdot 4)^2} - j\frac{x^6}{(2\cdot 4\cdot 6)^2} + \cdots \qquad (10.29)$$

$$J_0(x\sqrt{-j}) = \mathrm{ber}\,x - j\mathrm{bei}\,x,$$

- The Bessel function of the 1st kind, 1st order with a complex argument $(x\sqrt{-j})$

$$J_1(x\sqrt{-j}) = \frac{\mathrm{d}\left[J_0(x\sqrt{-j})\right]}{\mathrm{d}(x\sqrt{-j})} = -\sqrt{j}\left(\mathrm{ber}'\,x - j\mathrm{bei}'\,x\right). \qquad (10.30)$$

The Bessel functions of the 2nd kind (Neumann) are distinguished by the designation $N_n(x)$ and they are the function of the Bessel function of the 1st kind. In the following relations, $\gamma$ is the Euler constant, $\gamma$ = 0,57721566:

- The Bessel function of the second kind, the zeroth order with the real argument

$$N_0(x) = \frac{2}{\pi}\left(\gamma + \ln\frac{x}{2}\right)J_0(x) - \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^m}{(m!)^2}\left(\frac{x}{2}\right)^m\sum_{m=1}^{\infty}\frac{1}{k}, \qquad (10.31)$$

- The Bessel function of the second kind, the first order with the real argument

$$N_1(x) = -\frac{\mathrm{d}\left[N_0(x)\right]}{\mathrm{d}x}, \qquad (10.32)$$

- The Bessel function of the second kind, the zeroth order with the complex argument

$$N_0(x\sqrt{-j}) = \mathrm{ner}\,x - j\mathrm{nei}\,x, \qquad (10.33)$$

- The Bessel function of the second kind, the first order with the complex argument

$$N_1(x\sqrt{-j}) = \frac{\mathrm{d}\left[N_0(x\sqrt{-j})\right]}{\mathrm{d}(x\sqrt{-j})} = -\sqrt{j}\left(\mathrm{ner}'\,x - j\mathrm{nei}'\,x\right). \tag{10.34}$$

The Bessel function of the second kind with a complex argument are expressed analogically as with the first kind and the functions including the sum of all real elements and the negative sum of imaginary elements, the ones labeled ner *x* and nei *y*, have been introduced. The entries of ber‘ *x*, bei‘ *y*, ner‘ *x* and nei‘ *x* express derivations of these functions.

## 10.4. Maxwell's equations

The Maxwell's equations are the basic equations for the electromagnetic field which explain all categories of electromagnetic phenomena. They are the theoretical basis for the explanation of the function of electromagnetic equipment. The equations describing electric, magnetic and electromagnetic fields can be derived from them. The significance of these equations also consists in the fact that they remain without any changes in all surveillance systems.

Maxwell's equations determine the wave character of the electromagnetic field. The essence of wave phenomena is the final speed of propagation of these phenomena. If the speed was infinitely large, the phenomenon would be in the whole space immediately, the propagation would not be gradual and no waving would exist.

Basic quantities defined in these fields are used to describe the electromagnetic field; electric and magnetic intensity and induction (*E, H, D, B*). The intensity vectors of electric and magnetic fields ***E, H*** are perpendicular to each other.

Maxwell's equations are expressed either in the integral form or in the differential form. The reality directly describes the integral shape because it relates to the final lengths, surfaces and volumes, and they generally apply.

$$\begin{aligned}
&\oint_c \boldsymbol{H} \cdot \mathrm{d}\mathbf{l} = J + \frac{\mathrm{d}\Psi}{\mathrm{d}t},\\
&\oint_c \boldsymbol{E} \cdot \mathrm{d}\mathbf{l} = -\frac{\partial \Phi}{\partial t},\\
&\oint_A \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{A} = Q,\\
&\oint_A \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{A} = 0.
\end{aligned} \tag{10.35}$$

In the first two equations, the curve integral represents the integral of the vector circulation (***H*** or ***E***) along the closed oriented curve c. In the remaining two equations, the surface integral refers to the vector discharge from the closed area *A*. The first equation expresses the relation between the circulation ***H*** and the time change of the electric induction flow *Ψ* and the total current *J* passing through the area bounded by the curve *c*. In the second equation, the relationship between the circulation ***E*** and the time change of the induction magnetic flux *Φ* passing through the respective area. Gauss's law for the electric field is the third equation and it expresses the relationship between the electrical induction ***D*** and the total electric charge *Q* inside this area. The last equation is the Gauss's law for the magnetic field and it stipulates that the total magnetic flux through any closed surface equals to zero.

The first Maxwell's equation of the electromagnetic field in the integral form expresses generally applicable relation between the magnetomotive force in the closed curve and between the conductive current and the time change of the electric induction flow. The current and the flow pass through the inside of the respective closed curve. It follows from the stated equation that the magnetomotive force may arise even without the conductive force provided that the electric induction flux through the inside of the curve changes. Similarly we can say that a time variable magnetic flux induces the electromotoric power in the closed curve, whose inside it passes through (the second Maxwell's equation).

The differential form of Maxwell's equations converts integral quantities to specific quantities (density), thus concentrating the validity of mathematical relations to the element, i.e. to the specific point. These Maxwell's equations are the basic partial differential equations of the electromagnetic field, which describe its macroscopic laws. They are defined by the following relations:

$$\begin{aligned} &\operatorname{rot} \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}, \\ &\operatorname{rot} \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \\ &\operatorname{div} \boldsymbol{D} = \rho_0, \\ &\operatorname{div} \boldsymbol{B} = 0. \end{aligned} \tag{10.36}$$

Such modified equations apply only at such points, where the quantities are continuous and continuously differentiable functions of the position. The first two equations contain surface densities, the other two space densities. It follows from the mentioned equations, why the field curls (rotation) or force line divergences occur. The first equation expresses the fact that the curl magnetic field is induced not only by conductive currents (***J***), but also by displacement currents ($\partial \boldsymbol{D}/\partial t$). The second equation is actually the law of the electromagnetic induction, i.e. the time change of the magnetic field is always related to the existence of the curl electric field. The third equation confirms the existence of a free electric charge with a density of $\rho_0$ ($\rho_0$ is the volume density of the charge) and it is generally applicable. The fourth equation says that the free magnetic charge does not exist and that magnetic force lines are always closed. If there is not a free electric charge in the electromagnetic field, then the following applies:

$$\operatorname{div} \boldsymbol{D} = 0. \tag{10.37}$$

If we complete the four main Maxwell's equations by four complementary equations expressing the effect of the environment of the phenomena ongoing in the electromagnetic field

$$\boldsymbol{J} = \gamma \boldsymbol{E}, \quad \boldsymbol{D} = \varepsilon \boldsymbol{E}, \quad \boldsymbol{B} = \mu \boldsymbol{H}, \quad \boldsymbol{F} = Q(\boldsymbol{E} + \boldsymbol{c} \times \boldsymbol{B}), \tag{10.38}$$

we will get the full system of Maxwell's equations, by means of which the electromagnetic field is clearly determined in the respective field for all times $t > 0$. In the relations, properties of the environment, $\gamma$ conductivity ($\mathrm{S \cdot m^{-1}}$), $\varepsilon$ permittivity ($\mathrm{F \cdot m^{-1}}$) and $\mu$ permeability ($\mathrm{H \cdot m^{-1}}$) are included. The last equation is marked as the Lorentz force and it determines the force acting on the point charge $Q$ (charged particle) in the electromagnetic field and it equals to the vector sum of the electrostatic and electrodynamic forces.

An important part of Maxwell's equations in the differential form is limit conditions which are prerequisites for the existence of the clear solution to the electromagnetic field in the respective field. The limit conditions shall mean the boundary conditions, condition on the interface of two environments and initial conditions.

## 10.5. The energy of the electromagnetic field

If we wanted to analyze the electromagnetic field energy transfer, for example, from the inductor to the charge during induction heating, we would have to create the energy balance of these processes.

The electromagnetic field energy $W$ is composed of two components, the electric component of the energy $W_e$ and the magnetic component of the energy $W_m$. The total energy of the electromagnetic field in the volume $V$ is then determined by the contribution of these two components.

$$W = W_\mathrm{e} + W_\mathrm{m} = \frac{1}{2}\int_V \boldsymbol{E} \cdot \boldsymbol{D}\,\mathrm{d}V + \frac{1}{2}\int_V \boldsymbol{H} \cdot \boldsymbol{B}\,\mathrm{d}V \tag{10.39}$$

The electromagnetic field energy drops in time, then the energy drop in the volume $V$ together with time $t$ results in the power.

$$P = -\frac{\partial W}{\partial t} = -\varepsilon\int_V \boldsymbol{E} \cdot \frac{\partial \boldsymbol{D}}{\partial t}\,\mathrm{d}V - \mu\int_V \boldsymbol{H} \cdot \frac{\partial \boldsymbol{B}}{\partial t}\,\mathrm{d}V. \tag{10.40}$$

The equation can be modified using the first and the second Maxwell's equation (1.33) and we will obtain

$$P = \int_V \gamma\, \boldsymbol{E}^2 \mathrm{d}V + \int_V \left(\boldsymbol{H}\,\mathrm{rot}\,\boldsymbol{E} - \boldsymbol{E}\,\mathrm{rot}\,\boldsymbol{H}\right)\mathrm{d}V. \tag{10.41}$$

The expression in the second integral on the right side of this equation can be modified based on the vector analysis (1.9) and we will get

$$P = \int_V \gamma \boldsymbol{E}^2 \mathrm{d}V + \int_V \operatorname{div}(\boldsymbol{E} \times \boldsymbol{H}) \mathrm{d}V. \tag{10.42}$$

The respective equation is a general equation of the energy balance and it expresses the loss of field energy in a time unit and the causes for this loss. The first relation on the right side of the equation expresses the heat that is developed in the volume in a time unit, i.e. power losses (the expression $\gamma \boldsymbol{E}^2$ represents the elementary form of the Joule Law). The other relation expresses the radiation power of the electric field.

If the electromagnetic field is generated by the external source with the power of $P_z$, then the following applies:

$$P_z = \frac{\partial W}{\partial t} + \int_V \gamma \boldsymbol{E}^2 \mathrm{d}V + \int_V \operatorname{div}(\boldsymbol{E} \times \boldsymbol{H}) \mathrm{d}V. \tag{10.43}$$

On the basis of the mentioned relation we can say that the power of the external source will be consumed for an increase in the electromagnetic field energy in time unit, for covering of Joule losses in the specific volume of the field and on the radiation of the energy from the unit volume of the field.

If the environment is non-conductive, $\gamma = 0$ (for example, dielectric of a capacitor), the component of Joule losses will be zero. The following applies to the non-conductive environment with an external source

$$P_z = \frac{\partial W}{\partial t} + \int_V \operatorname{div}(\boldsymbol{E} \times \boldsymbol{H}) \mathrm{d}V. \tag{10.44}$$

## 10.6. Poynting radiation vector

The Poynting radiation vector (sometime abbreviated to the Poynting vector) is a very convenient tool for calculations of the electromagnetic field, for example for induction heating, supplementary losses in the winding of electrical machines, etc.

The Poynting radiation vector $\boldsymbol{S}$ (W·m$^{-2}$) is a vector product of intensity vectors of the electric and magnetic fields.

$$\boldsymbol{S} = \boldsymbol{E} \times \boldsymbol{H}. \tag{10.45}$$

The intensity vectors of the electric field ***E*** and the magnetic field ***H*** are perpendicular to each other as well as to the direction of electromagnetic wave propagation. The vectors ***E***, ***H*** and ***S*** form a dextrorotatory system and the Poynting vector is normal to the plane, on which ***E, H*** lie (Fig. 10.1). The energy flow with the density of ***E*** x ***H*** through the closed area *A* to or from the volume *V* surrounded by this area equals to the speed of an increase or decrease in the field energy. The Poynting vector represents the flow of radiant energy through a unit of the area per time unit.

According to (10.14), its size equals to the area of a rectangle with sides *E* and *H.*

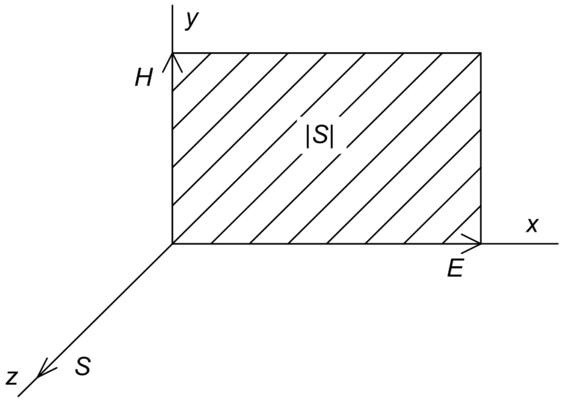


**Fig. 10.1 The direction of the Poynting vector in the electromagnetic field and its size.**

If we use the Poynting vector in the equation of the energy balance (10.39) and if we modify the equation on the basis of Gauss-Ostrogradsky theorem (10.18) which converts the volume integral to the surface integral, we will obtain the following:

$$P=\int_V \gamma \boldsymbol{E}^2 \mathrm{d}V+\int_V \operatorname{div}\boldsymbol{S}\,\mathrm{d}V=\int_V \gamma \boldsymbol{E}^2 \mathrm{d}V+\oint_A \boldsymbol{S}\cdot \mathrm{d}\boldsymbol{A}. \qquad (10.46)$$

Based on this equation, we can say that the divergence of the Poynting vector, i.e. the volume density of the energy flow, equals to the amount of energy which comes out from the unit volume of the electromagnetic field per a time unit.

$$\operatorname{div}\mathbf{S}+q_{\mathrm{J}}+\frac{\partial w}{\partial t}=0. \qquad (10.47)$$

The amount of heat, to which the electromagnetic energy is transformed per a time unit in the unit volume $q_{\mathrm{J}}$ determines the Joule heat. The time derivation of the volume density of energy indicates the increase in energy of the unit volume of the electromagnetic field per a time unit. The mentioned mathematical formula actually expresses the law of energy conservation.

The total power coming from the volume *V* is determined by the equation

$$\int_V \operatorname{div}(\boldsymbol{E}\times\boldsymbol{H})\mathrm{d}V=\int_V \operatorname{div}\boldsymbol{S}\,\mathrm{d}V=\oint_A \boldsymbol{S}\cdot \mathrm{d}\boldsymbol{A}. \qquad (10.48)$$

If there is a electromagnetic energy source (for example, a radiant body) at any point, the divergence of the Poyinting vector is positive at this place and the flow of this vector through an arbitrary closed area *A* (surrounding the source) is also positive.

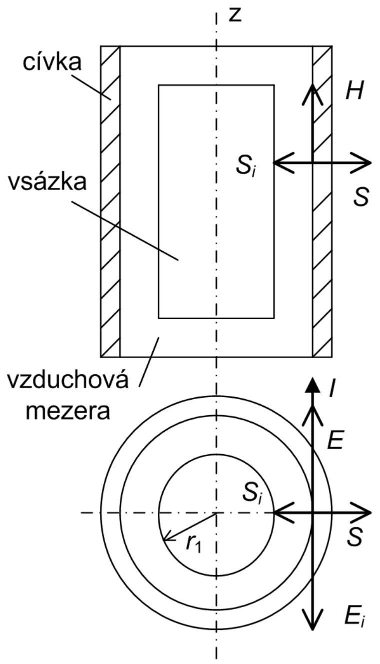


**Fig. 10.2 The Poynting vector in a coaxial system of a cylinder coil and charge .**

The significance of the Poynting vector will be demonstrated on the coaxial system of the cylindrical coil and conductive charge during induction heating. If the respective system is connected to an AC source, the current ***I*** starts to flow through the coil winding; the electromagnetic field intensity ***E*** is the same as the current direction. Due to the passing current, the magnetic field of the intensity ***H*** is induced in the core of the coil, which generates the induced eddy current in the winding. The induced eddy currents feature the opposite direction as the inducting current of the coil and they close along the perimeter of the charge in the planes normal to the common axes of the coil and the charge. The induced voltage per a height value of the coil $\boldsymbol{E}_i$ thus features the opposite direction than the intensity ***E.*** Using Flemming left hand rule, we can specify two phasors ***S*** and $\boldsymbol{S}_i$, which determine two different energy flows of the electromagnetic field. The energy flow density ***S*** is determined by the relation (10.42), its direction is outwards the coil and it has a physical meaning of electrical losses. The other energy flow density $\boldsymbol{S}_i$ is a vector product of the electric field intensity $\boldsymbol{E}_i$ as a result of the generated eddy currents and the magnetic field intensity.

$$\boldsymbol{S}_\mathrm{i} = \boldsymbol{S}_\mathrm{i} \times \boldsymbol{H}. \qquad (10.49)$$

The energy flow density $\boldsymbol{S}_i$ is directed to the coil axis and it represents the energy radiated from the unit volume per a time unit to the coil core and incident on the charge surface. The obtained energy is absorbed by the charge surface; it is consumed for generation of the eddy currents, the energy is stifled and heat develops according to the Joule law in the charge.

The induced eddy currents in the coaxial system of the coil and charge close along the perimeter of the charge in the planes perpendicular to the common axis of the coil and the charge. The current density decreases towards the axis.

The Poynting vector is expressed as a complex number for the harmonically variable field; it is determined by the vector product of phasors ***E*** and ***H****, while ***H**** is the complex compound phasor. Thus we obtain the real and the imaginary part of the Poynting vector.

$$\overline{\boldsymbol{S}} = \boldsymbol{E} \times \boldsymbol{H}^{*} = \mathrm{Re}(\boldsymbol{S}) + j\,\mathrm{Im}(\boldsymbol{S}). \qquad (10.50)$$

where Re(***S***) represents the active component of the field energy flow density per time unit, i.e. the active power per area unit, Im(***S***) is the reactive component of this density, i.e. the reactive (magnetizing) power per area unit. The real component has its physical meaning like in a time-stable field; the imaginary component expresses the part of field energy which changes periodically from the electric energy to the magnetic energy and vice versa.

## 10.7. The wave equations of the electromagnetic field

The energy transfer of the electromagnetic field and its conversion to Joule heat is related to the issues of the electromagnetic waves. When the current passes through a planar radiator, planar electromagnetic waves arise in its surroundings; the cylindrical coil radiates cylindrical electromagnetic waves into its core. Based on the configuration of the field in the charge, the electromagnetic waving can be longitudinal or transverse; according to the environment, through which the wave spreads- conductive or non-conductive.

The wave equations of the harmonic electromagnetic field can be obtained upon transformation of Maxwell's equations to the equations which will be expressed by only one variable of the component of the electromagnetic field intensity, i.e. either by the electric ***E*,** or the magnetic ***H*** one. The system of equations of electromagnetic wave spreading applies to the conductive as well as non-conductive electrical environment and to the arbitrary time course of the electromagnetic field variables. They have the following form:

$$\begin{aligned} \nabla^2 \boldsymbol{H} - \gamma\,\mu \frac{\partial \boldsymbol{H}}{\partial t} - \varepsilon\,\mu \frac{\partial^2 \boldsymbol{H}}{\partial t^2} &= 0, \\ \nabla^2 \boldsymbol{E} - \gamma\,\mu \frac{\partial \boldsymbol{E}}{\partial t} - \varepsilon\,\mu \frac{\partial^2 \boldsymbol{E}}{\partial t^2} &= 0. \end{aligned} \qquad (10.51)$$

The wave equations for time harmonic course of both the components then express propagation of the magnetic and the electric components of the same harmonic wave in the electrically arbitrary environment and they have the following form:

$$\nabla^2 \overline{\boldsymbol{H}} + \left(\omega^2 \varepsilon\, \mu - j\omega\gamma\, \mu\right)\overline{\boldsymbol{H}} = \nabla^2 \overline{\boldsymbol{H}} + \boldsymbol{k}^2 \overline{\boldsymbol{H}} = 0,$$
$$\nabla^2 \overline{\boldsymbol{E}} + \left(\omega^2 \varepsilon\, \mu - j\omega\gamma\, \mu\right)\overline{\boldsymbol{E}} = \nabla^2 \overline{\boldsymbol{E}} + \boldsymbol{k}^2 \overline{\boldsymbol{E}} = 0. \tag{10.52}$$

In the above-mentioned equations, $\overline{\boldsymbol{E}}$ and $\overline{\boldsymbol{H}}$ are rotating phasors in the complex plane, to which the following applies:

$$\overline{\boldsymbol{E}} = \boldsymbol{E}_m e^{j\omega t}, \quad \frac{\partial \boldsymbol{E}}{\partial \boldsymbol{t}} = j\omega\overline{\boldsymbol{E}}, \quad \frac{\partial^2 \boldsymbol{E}}{\partial \boldsymbol{t}^2} =, -\omega^2 \overline{\boldsymbol{E}},$$
$$\overline{\boldsymbol{H}} = \boldsymbol{H}_m e^{j\omega t}, \quad \frac{\partial \boldsymbol{H}}{\partial t} = j\omega\overline{\boldsymbol{H}}, \quad \frac{\partial^2 \boldsymbol{H}}{\partial \boldsymbol{t}^2} = -\omega^2 \overline{\boldsymbol{H}}. \tag{10.53}$$

Electric properties of the environment and angular speed are summarized in the wave propagation constant ***k***, the so-called wave number. Generally, the wave number is a complex number, which means that it has the real and the imaginary components.

$$\boldsymbol{k}^2 = \omega^2 \varepsilon\, \mu - j\omega\gamma\, \mu = -j\omega\, \mu\left(\gamma + j\omega\,\varepsilon\right),$$
$$\boldsymbol{k} = \sqrt{-\, j\omega\, \mu\left(\gamma + j\omega\,\varepsilon\right)} = \alpha - j\beta. \tag{10.54}$$

The real component of the wave number $\alpha$ is called the phase constant and the imaginary component $\beta$ is the constant of damping (the damping factor).

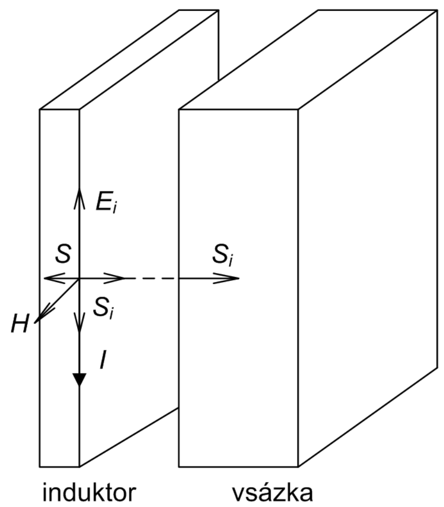


**Fig. 10.3 Electromagnetic waving between two flat plates**

### 10.7.1. Plane electromagnetic waving

If the shape of the inductor and the charge are fat (e.g. for mutually parallel plates Fig. 10.3), the resulting waving features the character of the plane electromagnetic waving. The generation of the electromagnetic field in the plate inductor and the transfer of its energy into the charge is similar to the cylinder-shaped inductor (Fig. 10.2). Upon connection to an AC power source, the current *I* flows through the plate inductor; the current is invoked by the electric component of the *E* field. This current generates the magnetic component of the field with the intensity *H* which induces the electric field with the intensity $E_{i,}$ featuring the opposite direction compared to the intensity *E*. The plate generates the wave and on the basis of knowledge of the Poynting vector, it arises that it radiates the energy flow of the field in two directions. Inwards the plate inductor with the density *S* and it has the character of electric losses and outwards the plate (towards the charge) with the density of $S_i$. This density of energy flow comes out of the plat in the perpendicular direction and it is radiated through the non-conductive environment to the surface of the conductive charge.

The important criteria for the assessment of the electromagnetic waving is whether it is a one-sided or double-sided waving and the thickness of the respective charge with regard to the relative penetration depth of the electromagnetic waving to the electrically-conductive charge *a*.

As an example, the plane one-sided waving is given. The magnetic and the electric waves are determined by the superimposing of two waves, the direct one ($E_p$, $H_p$) and the back one. For the plane one-sided waving in the direction of the axis *x*, we can get the general solution to wave equations (10.49) for the magnetic and the electric waves in the following form:

$$\begin{aligned} \overline{\boldsymbol{E}} &= \boldsymbol{Z}\left(\boldsymbol{A}e^{-jkx} - \boldsymbol{B}e^{jkx}\right) = \overline{\boldsymbol{E}}_{\mathrm{p}} + \overline{\boldsymbol{E}}_{\mathrm{z}}, \\ \overline{\boldsymbol{H}} &= \boldsymbol{A}e^{-jkx} + \boldsymbol{B}e^{jkx} = \overline{\boldsymbol{H}}_{\mathrm{p}} + \overline{\boldsymbol{H}}_{\mathrm{z}}, \end{aligned} \tag{10.55}$$

where the complex integration constants ***A, B*** are determined from the conditions of the clarity of the task and ***Z*** is characteristic impedance of the charge.

$$\boldsymbol{Z} = \sqrt{\frac{j\omega\mu}{\gamma + j\omega\varepsilon}}. \tag{10.56}$$

If the environment is electrically non-conductive ($\gamma = 0$), the characteristic impedance is defined by the real number for the electrically-conductive environment (provided that the following applies: $\gamma > \omega\varepsilon$) by the complex number.

$$Z = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}}, \quad \boldsymbol{Z} = \sqrt{\frac{j\omega\mu}{\gamma}}. \tag{10.57}$$

For one-sided waving of the conductive charge with a sufficient thickness *d* ($d \rightarrow$

∞) for the direct components $\overline{\boldsymbol{E}}_{\mathrm{p}}$ and $\overline{\boldsymbol{H}}_{\mathrm{p}}$ dampen earlier than they fall onto the back wall of the plate charge. Therefore, no back components occur and they are thus zero. The solution is thus easier for both the components of the wave; the following applies to the density phasor of the inductive current

$$\overline{\boldsymbol{E}} = \overline{\boldsymbol{E}}_{\mathrm{p}} = \frac{1+j}{a\gamma}\overline{\boldsymbol{H}}_0 \mathrm{e}^{-x/a}\mathrm{e}^{-jx/a}, \quad \overline{\boldsymbol{H}} = \overline{\boldsymbol{H}}_{\mathrm{p}} = \overline{\boldsymbol{H}}_0 \mathrm{e}^{-x/a}\mathrm{e}^{-jx/a},$$
$$\overline{\boldsymbol{J}} = \gamma\,\overline{\boldsymbol{E}} = \frac{1+j}{a}\overline{\boldsymbol{H}}_0 \mathrm{e}^{-x/a}\mathrm{e}^{-jx/a}, \tag{10.58}$$

where $\boldsymbol{H}_0$ is a value on the surface of the charge, *x* is the distance from the surface of the charge. The following applies to the variable expression of these quantities:

$$\frac{\overline{\boldsymbol{E}}}{\overline{\boldsymbol{E}}_0} = \frac{\overline{\boldsymbol{H}}}{\overline{\boldsymbol{H}}_0} = \frac{\overline{\boldsymbol{J}}}{\overline{\boldsymbol{J}}_0} = \mathrm{e}^{-x/a}\,\mathrm{e}^{-jx/a}, \tag{10.59}$$

where the first exponential function expresses the proportional decrease in the amplitudes of individual phasors with the distance of x (for x = a the decrease is to 36.8 % of the value on the charge surface). The second exponential function expresses the spatial rotation of individual phasors in the distance of x against the phasors on the surface of the charge. For the distance of x = λ = 2πa, individual phasors rotate by 360°, i.e. they are just in the phase. The courses of individual variables are listed in Fig. 10.4 and they are completed with the dependence expressing the proportional amount of the induced heat in the charge of the unit volume per a time unit.

The charge thickness is usually expressed against the penetration depth *a*. For $x=2\pi a$ the components of the electromagnetic wave (as well as the current density) reach less than 0.2% of the value on the charge surface. The charge with a thickness of $d \geq 2\pi a$ has a sufficient thickness (we do not have to take the back component of the wave into account); otherwise (for $d < 2\pi a$), the charge is thin and the propagating wave also contains back components; as a result, the relations (10.55) and (10.56) does not apply to them.

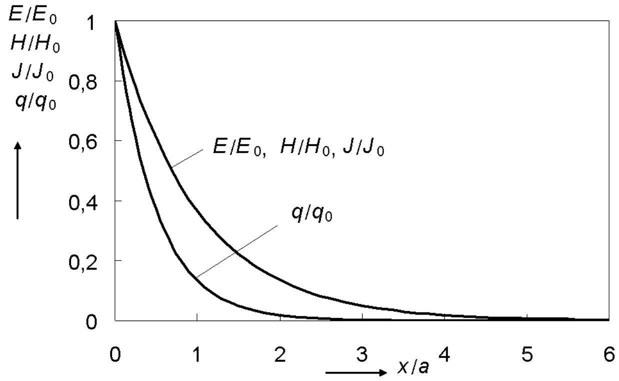


**Fig. 10.4 Amplitude courses *E, H, J* in the charge with a great thickness**

## 10.7.2. Cylindrical electromagnetic waves

The cylindrical charge is usually applied to induction heating The transfer of energy in the system of inductor-charge is performed by means of electromagnetic waves.

The transfer of energy of the electromagnetic field in the coaxial system of the cylinder and the charge and the process of heat generation in the charge is explained in the preceding chapter (Fig. 10.2). The induced eddy currents in this coaxial system close along the perimeter of the charge in the planes perpendicular to the common axis of the coil and the charge. The current density decreases towards the axis. Due to the damping of the waves in the conductive charge towards the axis, reduction of the magnetic component of the field intensity form the maximum value on the charge surface to the minimum value in the charge surface occurs. In this layout, there are the axial (longitudinal) magnetic field and the radial electric magnetic field.

Equation in cylindrical coordinates are expressed for cylindrical electromagnetic waves and for one-dimensional task, transformation is used

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} \qquad (10.60)$$

and the wave equations of the cylindrical electromagnetic wave for a general environment have the following form:

$$\frac{\partial^2 \overline{\boldsymbol{H}}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{\boldsymbol{H}}}{\partial r} + \boldsymbol{k}^2 \overline{\boldsymbol{H}} = 0, \quad \frac{\partial^2 \overline{\boldsymbol{E}}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{\boldsymbol{E}}}{\partial r} + \boldsymbol{k}^2 \overline{\boldsymbol{E}} = 0. \qquad (10.61)$$

The wave propagation constant ***k*** is dependent on material properties according to (1.51) and it is defined by the components *α* a *β*. The transformation of a plane wave to the cylindrical one does not have any impact on the material properties of the environment, where it spreads. The transformation of the Laplace operator to cylindrical coordinates can be applied only to such a vector quantity, which features the direction of the *z* coordinate (Fig. 10.2) in the transformed system. This is fulfilled by the magnetic wave phasor $\overline{\boldsymbol{H}}$; therefore, the equation containing the phasor of the electromagnetic wave $\overline{\boldsymbol{E}}$ is applicable to another configuration, where this phasor features the direction *z*.

Regardless of the configuration of the electromagnetic field, the equations (10.58) are the Bassel differential equations (see chapt. 10.3) and upon their solution, we will obtain the linear combinations of cylindrical functions of the complex arguments $\left(x\sqrt{-j}\right)$.

The general solution to the electromagnetic field propagating in the conductive environment with the longitudinal magnetic component (Fig. 10.2) is

$$\overline{\boldsymbol{H}} = \boldsymbol{A}\, J_0\left(x\sqrt{-j}\right) + \boldsymbol{B}\, N_0\left(x\sqrt{-j}\right),$$
$$\overline{\boldsymbol{E}} = \sqrt{-j}\,\frac{\sqrt{2}}{a\,\gamma}\left[\boldsymbol{A}\, J_1\left(x\sqrt{-j}\right) + \boldsymbol{B}\, N_1\left(x\sqrt{-j}\right)\right] \qquad (10.62)$$

The electric and the magnetic waves have both the direct and the back components. The direct components include the Bessel function of the first kind and the back components include the Bessel function of the second kind; in both the cases of the zeroth and the first order, respectively.

## 10.8. The fundamentals of mathematical-physical modeling of thermal plasma

The models of thermal plasma flow (for example the interaction of an electric arc with the flow of cooling medium) are based mainly on solutions to the equations of continuum mechanics, i.e. the continuum equations, equations of moments and equations of energy.

Basic laws below are considerably general and it is only assumed that the liquid is homogeneous (continuum, for which the local properties, i.e. density $\rho$, pressure *p,* temperature $\vartheta$ and velocity *c* have physical sense) and without any extreme gradients of status variables.

The equation of continuity - the law of mass conservation (kg·m$^{-3}$·s$^{-1}$) is determined by the following relation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\rho\, \boldsymbol{c}\right) = 0, \qquad (10.63)$$

where the first component on the left side of the equation expresses the time change of the mass in the volume unit, whereas the other component is the divergence of the mass flow. It follows from the equation that the time change in the volume unit equals to the divergence of the mass flow (the sum of all mass flows inwards and outwards from the unit volume).

**The equation of moment** - the law of momentum conservation is defined as follows:

$$\rho\frac{\partial \mathbf{c}}{\partial t}+\rho(\boldsymbol{c}\cdot\nabla)\boldsymbol{c}=-\nabla p+\nabla\cdot\vec{\boldsymbol{\tau}}+\boldsymbol{J}\times\boldsymbol{B}, \qquad (10.64)$$

where the first component on the left side of the equation is the inertia force, whereas the other component is the convection force. On the right side of the equation, firstly there is the pressure (compression) force (pressure gradient), the second component is the friction (viscous) force ($\vec{\boldsymbol{\tau}}$ is the tensor of Reynold's stress) and the third component is the electrodynamic power (the vector product of the current density ***J*** and the magnetic induction ***B***). The pressure gradient and the electromagnetic force represent the driving forces in the equation of momentum conservation; the inertia force, the convection force and the friction force act opposite. The equation of moments is a vector equation which is to be broken down into three scalar equations separately for each component - in Cartesian (orthogonal) coordinates, the x, y and z components, or in cylindrical coordinates radial, azimuthal and z components.

**The equation of energy (the equation of energy conservation)** ($W\cdot m^{-3}$) is based on the first law of thermodynamics and it expresses the fact that the increase in the internal energy *h* is at the expense of heat supply from the surrounding, the increments of the heat by compression and the internal friction and energy supplied by action of external forces. The following relation applies to the energy equation

$$\rho\frac{\partial(h+\frac{c^2}{2})}{\partial t}=-\nabla\cdot\rho\,\mathbf{c}\,(h+\frac{c^2}{2})-\nabla\cdot\boldsymbol{J}+\nabla\cdot(\vec{\boldsymbol{\tau}}\cdot\boldsymbol{c})+\boldsymbol{JE}, \qquad (10.65)$$

where there is the time change of enthalpy on the left side, and there are individual components on the right side, namely (from the left) the convection element (the enthalpy flow), losses by heat conduction, friction (viscous) losses and Joule losses (***E*** is the vector of the electric field intensity). Joule losses represent the electric power supplied to the arc plasma due to ohmic losses by the passage of electric current. Other elements include discharge of the energy from the arc, or losses. The vector ***J*** includes losses due to heat conduction $\boldsymbol{J}_q$ and due to radiation $\boldsymbol{J}_r$, i.e. $\boldsymbol{J} = \boldsymbol{J}_q + \boldsymbol{J}_r$. The convection element includes the energy taken by the plasma flow (the enthalpy flow). In the elements with the enthalpy, also the kinetic energy of the flowing plasma per a weight unit ($c^2/2$), which forms a negligible part of the energy flow at common speeds of flow, is included.

The basic relations from thermodynamics (stated in the previous chapter) and generalized Ohm's law are added to these equations.

$$\boldsymbol{J} = \gamma \left[ \boldsymbol{E} + (\boldsymbol{c} \times \boldsymbol{B}) - \beta_{\mathrm{H}} (\boldsymbol{J} \times \boldsymbol{B}) \right], \tag{10.66}$$

where $\gamma$ is conductivity and $\beta_H$ is Hall's constant. The expressions in the square bracket are intensities, from the left - of the electric field from the applied voltage, from the induced voltage due to movement in the magnetic field and the intensity of the electric field from the Hall voltage.

The following simplification is carried out when the mentioned system of relations for the description of phenomena in flowing plasma, e.g. for the analysis of the cooled arc which burns in the cylindrical channel of the plasmatron. The element of electromagnetic forces in the equation of moments (10.61), the losses of friction forces and the kinetic force of the plasma in the equation of energy (10.62) and, with regard to the great intensity of the electric field of the applied voltage on the arc, the two remaining elements on the right side of the Ohm's law (10.63) are neglected. The cylindrical symmetry is usually assumed. The radial components of gradients (axial velocity and temperature) are decisive for Raynold's (viscous) stress $\tau$ as well as for heat conduction $J_q$ (heat flux).

$$\tau = \eta \frac{\partial c_z}{\partial r}, \quad J_q = \Phi = -\lambda \frac{\partial T}{\partial r}. \tag{10.67}$$

The radiation element of the equation of energy equals to the radiated energy from the unit of volume (radiation coefficient $\varepsilon_E$)

$$\nabla \cdot \boldsymbol{J}_r = \varepsilon_E. \tag{10.68}$$

Using the mentioned simplifications, the equation of mass preservation in cylindrical coordinates can be rewritten as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho c_z) + \frac{1}{r}\frac{\partial}{\partial r}(r \rho c_r) = 0, \tag{10.69}$$

the equation of momentum conservation for the z-component can be rewritten into the following form:

$$\rho \frac{\partial c_z}{\partial t} + \rho c_z \frac{\partial c_z}{\partial z} + \rho c_r \frac{\partial c_z}{\partial r} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left( \eta r \frac{\partial c_z}{\partial r} \right). \tag{10.70}$$

In most tasks subject to solving, we come across rotationally symmetrical cases, when the azimuthal component of the momentum speed is zero and the radial component does not play a significant role. The equation of energy conservation can be expressed as follows:

$$\rho \frac{\partial h}{\partial t} + \rho c_z \frac{\partial h}{\partial z} + \rho c_r \frac{\partial h}{\partial r} = \gamma E^2 - 4\pi \varepsilon_E + \frac{1}{r} \frac{\partial}{\partial r}\left( \lambda r \frac{\partial T}{\partial r} \right). \quad (10.71)$$

Ohm's law with consideration given to the mentioned neglecting can be expressed by the relation:

$$i = E \int_0^r \gamma \, 2\pi r \mathrm{d}r, \quad (10.72)$$

where $i$ is the current passing through the arc, and $E$ is the intensity of the electric field in the arc.

It follows from the above-mentioned system of equation, that the properties of substances for the analyzed gaseous systems must be known.

The analysis of the phenomena in the channel is to be carried out on the basis of measured integral values (e.g. voltage and current of the arc, radial losses, flow of the cooling medium) and knowledge of thermodynamic and transport properties of the used medium.

The simplified model of the axially cooled electric arc is actually the arc burning in the cylindrical channel whose walls are cooled down with water. The electric arc also features a cylindrically-symmetrical shape.

For the need of the simplified model, the entire volume of the anodic channel of the plasmatron is divided into two main areas: into the zone of the arc and the zone of cold gas. The division is based on the differences of the calculations inside the arc and outside it. The processes occurring inside the arc are more comprehensive than in the zone of the cold gas; the zone of the electric arc is electrically conductive, whereas the cold zone is formed by neutral gas. The dividing line (the border of the arc) between these areas is the surface of the electric arc and it is determined by the temperature, at which the working gas begins to be electrically conductive.

An important part of the modeling of phenomena is specification of temperature distribution in the axial and the radial directions of the electric arc.

## 10.9. References:


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INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

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