B. COMPUTATIONAL PART

B.1. PHYSICAL BASIS OF HEAT PROPAGATION

B.1.1. Quantities, symbols, units

Temperature, temperature difference

ϑ .... temperature °C .... degree Celsius
Θ .... thermodynamic temperatureK .... kelvin
$\Delta\vartheta = \vartheta_2 - \vartheta_1$ .... temperature difference °C, K
$\Delta\Theta = \Theta_2 - \Theta_1$ .... temperature difference °C, K

Both temperature and temperature difference are scalar quantities. The temperature field is a scalar field. Relationships between temperatures :

$$°C + 273.15 = K$$

Heat

Q .... heat J .... joule

Heat is a form of energy. Relationships between units of :

unit	**J**	**Wh**	**cal**	**kpm**	**erg**
J	1	$2.778 \cdot 10^{-4}$	0.239	0.102	10^{7}
Wh	3600	1	860	367.1	$3.6 \cdot 10^{10}$
cal	4.186	$1.163 \cdot 10^{-3}$	1	0.427	$4.186 \cdot 10^{7}$
kpm	9.807	$2.724 \cdot 10^{-3}$	2.343	1	$9.807 \cdot 10^{7}$
erg	10^{-7}	$2.778 \cdot 10^{-11}$	$2.389 \cdot 10^{-8}$	$1.020 \cdot 10^{-8}$	1

Heat capacity (stored heat)

$$Q = m\, c \cdot \cdot \Delta\vartheta \qquad (J ; kg , J \cdot kg^{-1} \cdot K^{-1} , K)$$

m .... weight of the body
c .... specific heat capacity (specific heat)
$\Delta\vartheta$ .... temperature difference

Specific heat capacity

c .... specific heat capacity ($J \cdot kg^{-1} \cdot K^{-1}$)

Relationships between units :

unit	**$J\cdot kg^{-1}\cdot K^{-1}$**	**$kJ\cdot kg^{-1}\cdot K^{-1}$**	**$cal\cdot kg^{-1}\cdot K^{-1}$**	**$kcal\cdot kg^{-1}\cdot K^{-1}$**
$J\cdot kg^{-1}\cdot K^{-1}$	1	10^{-3}	0.2389	$0.2389\cdot 10^{-3}$
$kJ\cdot kg^{-1}\cdot K^{-1}$	10^{3}	1	238.9	0.2389
$cal\cdot kg^{-1}\cdot K^{-1}$	4.186	$4.186\cdot 10^{-3}$	1	10^{-3}
$kcal\cdot kg^{-1}\cdot K^{-1}$	4186	4.186	10^{3}	1

Heat output

Heat output is heat per unit time. It is a scalar.

P .... heat output W .... watt

Heat flux density

Heat flux density is the heat output per unit area. It is a vector - it has a direction given by the normal to the area element dA under consideration.

q .... heat flux density ($W\ m\cdot^{-2}$)

$$q = dP / dA$$

Example 1 :

How many kcal/hr is 10 W ?

Solution :

10 (W) = 10 (J/s) = $10\cdot 3600 / 4186$ (kcal/hr) = 8.6 (kcal/hr)

Example 2 :

How many cal is the value of 5 Wh ?

Solution :

5 (Wh) = 5/3600 (W/s) = $5/3600\cdot$ cal/4.186 = 4300 (cal)

Example 3 :

What will be the specific resistance of aluminium in$\Omega\cdot$ m if it is equal to 0.03 in$\Omega\cdot mm^2 /m$?

($3\cdot 10^{-8}\Omega\cdot m$)

Example 4 :

What will be the current density in A/m^2 , if it is equal to 5 in A/mm^2 ?

($5\cdot 10^6\ A/m^2$)

Example 5 :

How many kpm is 3 cal ?

(1.278 kpm)

B.1.2. Relationship between thermal and mechanical energy

For practical purposes, it is useful to realise the relatively significant mechanical work involved in heat energy of the order of one kilocalorie. This will be documented by the following examples :

Example 1 :

How much cement could be loaded onto a 2m high truck using the energy required to heat 1 litre of water by 20°C ? The loading efficiency is $=\eta$ **a**, 100 %

b, 50 %

Solution :

Thermal energy required :

$Q = m\,c \cdot \cdot \Delta\vartheta = 1 \cdot 4.186 \cdot 10\ ^{3} \cdot 20 = 8.372 \cdot 10^{4}$ J

Energy required for loading :

$W = m\,g \cdot \cdot\ h / \eta$

g .... gravitational acceleration

h .... loading height

η .... loading efficiency

From the equation Q = W determine the mass of the load :

a, $m = Q \cdot \eta\ /\ (\ g \cdot h\ ) = 8.372 \cdot 10^{4} \cdot 1\ /\ (\ 2 \cdot 9.806\ ) = 4.267 \cdot 10^{3}$ kg

b, $m = 8.372 \cdot 10^{4} \cdot 0.5\ /\ (\ 2 \cdot 9.806\ ) = 2.134 \cdot 10^{3}$ kg

The results show that the energy required to brew a few cups of tea would be enough to load a few tens of cents of cement onto a car or wagon.

Example 2 :

How many times more energy intensive is a litre of hot tap water than a litre of cold water ? Both waters are drawn from the same source at a temperature of $\vartheta_1 = 10$ °C up to a height of h = 100 m. The cold water is taken directly at the point of consumption, the hot water is heated at the point of consumption to $\vartheta_2 = 70$ °C.

Solution :

We consider the efficiency of pumping by pump with electric motor in relation to the primary energy $\eta_{č} = 0.15$ (η power plant = 0.3 ;η motor with pump = 0.5). We consider heating by coal with efficiency $\eta_o = 0.5$.

Energy required for cold water (based on 1 litre):

$W_s = m\,g \cdot \cdot\ h / \eta_{č} = 1 \cdot 9.806 \cdot 100 / 0.15 = 6538$ J

Energy required for hot water (based on 1 litre):

$W_t = m\,g \cdot \cdot\ h / \eta_{č} + m\,c\ (\cdot \cdot \vartheta_2 - \vartheta_1\ )\ /\ \eta_o$

$W_t = 1 \cdot 9.806 \cdot 100 / 0.15 + 1 \cdot 4.186 \cdot 10^{3} \cdot (70 - 10) / 0.5 = 6\,538 + 502\,320 = 508\,858$ J

$n = W_t / W_s = 508\ 858 / 6\ 538 = 77.8$

Hot water is almost 78 times more energy intensive than cold water.

Example 3 :

What wattage would a direct-fired electric instantaneous heater have to have to make hot water flow out of a 10 mm diameter tap ϑ_2 = 60 °C at v = 2 m/s ? The water is heated from a temperature of ϑ_1 = 10 °C. The heating efficiency is 97%. How many fluorescent lamps of 40 W could shine at this wattage ?

(33.5 kW , 838 fluorescent lamps)

Example 4 :

How many times more energy does it take to heat 10 litres of water by 10°C than to raise that 10 litres of water to a height of 10m ? Consider both the heating efficiency and the lifting efficiency to be 100% .

(427 times more)

Example 5 :

By how many degrees Celsius will the water in a 200 meter high waterfall heat up if all of its positional energy is converted to heat ? From what height would the water have to fall 0 °C warm to boil ?

(0.47 °C, 42 692 m)

Example 6 :

Fill a bath with 100 litres of 37°C warm water heated from 10°C. How high would we have to raise this water to make the position energy of the water equal to the energy required to heat it ? The heating efficiency η_o is equal to the lifting efficiency η_z .

(11 527 m)

Example 7 :

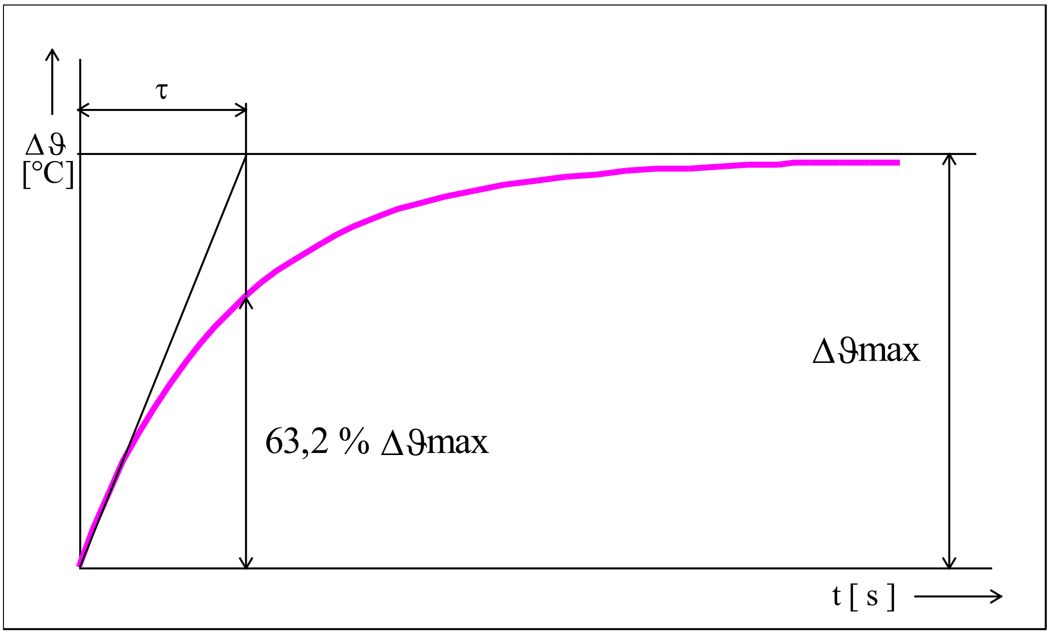
By how many °C will 1kWh of energy heat 20 litres of water at a heating efficiency of 90% ? How many people weighing 80 kg will 1 kWh of energy transport from the ground floor to the fifth floor (23 m) in a lift with an efficiency of 60% ?

(38.7 °C , 120 people)

B.1.3. Warming and cooling processes

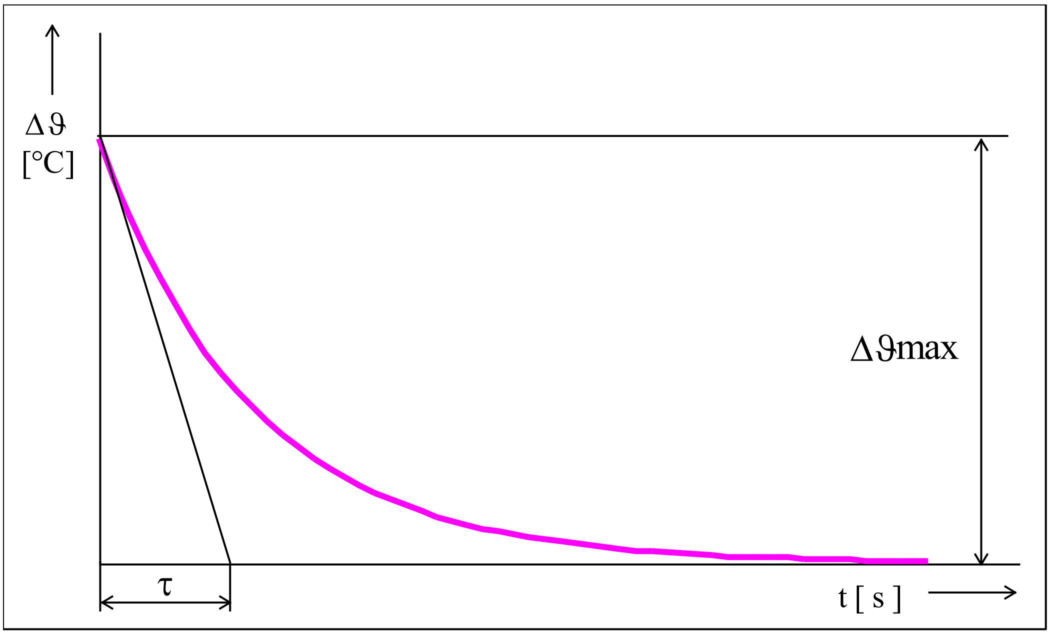
The dependence of temperature on heating time is expressed by the warming curve :

$$\Delta\vartheta = \Delta\vartheta_{max} \cdot (1 - e^{-\frac{t}{\tau}})$$



The dependence of temperature on cooling time is expressed by the cooling curve :

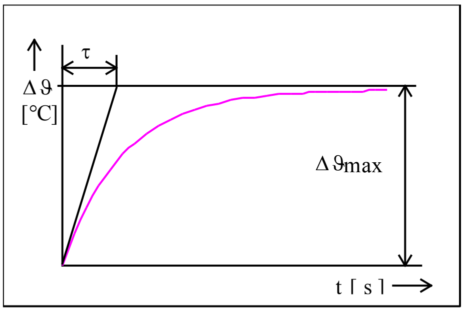
$$\Delta\vartheta = \Delta\vartheta_{max} \cdot e^{-\frac{t}{\tau}}$$



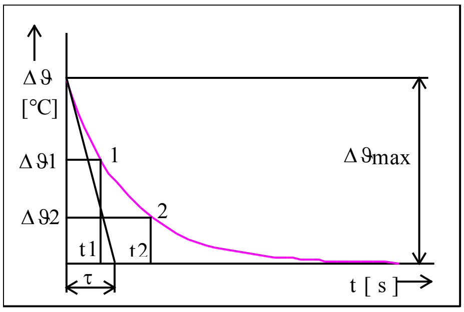
Example 1 :
How long does it take to heat water from 20 °C to 100 °C if it cools from 40 °C to 30 °C in 10 minutes ? The cooling process takes place between 100 °C and 20 °C, the time constant of warming is equal to the time constant of cooling. Consider the completed process in terms of three time constants.

Solution :

Warming curve :



Cooling curve :



We know two points on the cooling curve that must satisfy its

equation: $\Delta\vartheta = \Delta\vartheta_{max} \cdot e^{-\frac{t}{\tau}}$

Point 1 : $$\Delta\vartheta_1 = \Delta\vartheta_{max} \cdot e^{-\frac{t_1}{\tau}} \qquad (\mathbf{1})$$

Point 2 : $$\Delta\vartheta_2 = \Delta\vartheta_{max} \cdot e^{-\frac{t_2}{\tau}} \qquad (\mathbf{2})$$

Dividing equation (**1**) by equation (**2**) gives an equation of one unknown :

$$\frac{\Delta\vartheta_1}{\Delta\vartheta_2} = \frac{\Delta\vartheta_{max} \cdot e^{\frac{t_1}{\tau}}}{\Delta\vartheta_{max} \cdot e^{\frac{t_2}{\tau}}} = e^{\frac{t_2 - t_1}{\tau}}$$

We logarithm this equation and calculate the unknown :

$$\ln\frac{\Delta\vartheta_1}{\Delta\vartheta_2}=\frac{t_2-t_1}{\tau}$$

Where $\Delta\vartheta_1 = \vartheta_1 - \vartheta_0 = 40 - 20 = 20$ °C

$\Delta\vartheta_2 = \vartheta_2 - \vartheta_0 = 30 - 20 = 10$ °C

$t_2 - t_1 = 10$ min = 600 sec

$$\tau=\frac{t_2-t_1}{\ln\frac{\Delta\vartheta_1}{\Delta\vartheta_2}}=865.6 \quad \text{sec}$$

$3 \cdot \tau = 3 \cdot 865.6 = 2596.9$ sec

B.1.4. Heat transfer by conduction

The heat is spread in three ways, either separately or, more purely, in various combinations :

1. by conduction (conduction)
2. by flow (convection)
3. radiation (radiation)

For heat transfer by conduction, we define the thermal conductivity coefficientλ as a material constant characterizing the ability of a given substance to transfer heat by conduction (this ability is directly proportional to the magnitude of this coefficient).The unit of the thermal conductivity coefficient is W m $K\cdot^{-1}\cdot^{-1}$ and its values for different materials are given in the table :

For heat conduction, the relation :

$$P=\int_S \bar{q}\cdot\bar{d}\bar{S}=\int_S -\lambda\cdot \text{grad}\ \Theta\cdot\bar{d}\bar{S}$$

which for a homogeneous temperature relation goes to the form :

$$P=\lambda\cdot\frac{S}{l}\cdot\Delta\vartheta$$

The following examples show how to solve some specific cases of heat conduction.

Example 1 - Plane wall :

Determine the heat output through a wall with thickness l = 50 mm and area S = 1 m^2 . The temperature on the outer surface of the wall is ϑ_1 = 100 °C , on the inner surface ϑ_2 = 90 °C. The wall is :

a, steel , λ = 40 W . m^{-1} . K^{-1}
b, concrete , λ = 1.1 W . m^{-1} . K^{-1}
c, diatomaceous , λ = 0.11 W . m^{-1} . K^{-1}

Solution :

$$P = \lambda \cdot \frac{S}{l} \cdot \Delta\vartheta \qquad (W ; W.m^{-1} .K^{-1} , m^2 , m , K)$$

a, $P = 40 . \frac{1}{0,05} . (100 - 90) = 8\,000$ W

b, $P = 1.1 . \frac{1}{0,05} . (100 - 90) = 220$ W

c, $P = 0.11. \frac{1}{0,05} . (100 - 90) = 22$ W

Example 2 - Composite plane wall :

Determine the heat flux through the boiler wall. The wall is covered with a layer of soot with a thickness of l_1 =1 mm, λ_1 =0.08 $W.m^{-1} .K^{-1}$ and on the water side there is a boiler stone with a thickness of l_3 =2 mm, λ_3 =0.8 $W.m^{-1} .K^{-1}$. The boiler wall has a thickness of l_2 =12 mm, λ_2 =50 $W.m^{-1} .K^{-1}$. The wall temperature on the water side is ϑ_4 =206°C , on the heating side ϑ_1 =685°C. Determine the heat flux density q , the temperatures at the interface of the layers , the mean temperatures of the layers. The boiler wall has an area S=10 m^2 .

Solution :

Heat flux density :

$$\overline{q} = \frac{\vartheta_1 - \vartheta_4}{\frac{l_1}{\lambda_1} + \frac{l_2}{\lambda_2} + \frac{l_3}{\lambda_3}} = \frac{685 - 206}{\frac{0,001}{0,08} + \frac{0,012}{50} + \frac{0,002}{0,8}} = 31\,430 \text{ W. } m^{-2}$$

Interface temperatures :

soot - boiler

$$\vartheta_2 = \vartheta_1 - q \cdot \frac{l_1}{\lambda_1} = 685 - 31\,430 \cdot \frac{0{,}001}{0{,}08} = 292.12\ °C$$

water stone - boiler

$$\vartheta_3 = \vartheta_4 + q \cdot \frac{l_3}{\lambda_3} = 206 + 31\,430 \cdot \frac{0{,}002}{0{,}8} = 284.58\ °C$$

Mean layer temperatures :

soot

$$\vartheta_S = \frac{\vartheta_1 + \vartheta_2}{2} = \frac{685 + 292{,}12}{2} = 488{,}56\ °C$$

boiler wall

$$\vartheta_{SK} = \frac{\vartheta_2 + \vartheta_3}{2} = \frac{292{,}12 + 284{,}58}{2} = 288{,}35\ °C$$

boiler stone

$$\vartheta_{KK} = \frac{\vartheta_3 + \vartheta_4}{2} = \frac{284{,}58 + 206}{2} = 245{,}29\ °C$$

Heat flux :

$$P = q \cdot S = 31\,430 \cdot 10 = 3{,}143 \cdot 10^5\ W$$

Example 3 - Composite plane wall ,λ temperature dependent :

Determine the heat loss through the double-layer wall of the heating furnace. A base fireclay layer with a thickness of l_S = 230 mm ,λ_{S0} = 0.971 W.m^{-1} .K ,$^{-1}\xi_S$ = 0.00058 is insulated with a porous fireclay with a thickness of l_{iz} = 115 mm ,λ_{izo} = 0.23 W.m^{-1} .K ,$^{-1}\xi_{iz}$ = 0.0002 . On the inside of the masonry the temperature isϑ_1 = 930 °C , on the outside of the insulation the temperature isϑ_3 = 70 °C. This is$\lambda = \lambda_o + \xi \cdot \vartheta_{stř}$, where$\vartheta_{stř}$ is the mean layer temperature.

Solution :

1, Estimate the temperature at the layer interface - e.g.ϑ_{20} = 500 °C
2, Calculate the mean temperature of the layers :

Fireclay : $$\vartheta_{sI} = \frac{\vartheta_1 + \vartheta_{20}}{2} = \frac{930 + 500}{2} = 715\ °C$$

insulation : $$\vartheta_{izI} = \frac{\vartheta_{20} + \vartheta_3}{2} = \frac{500 + 70}{2} = 285\ °C$$

3, Calculate the thermal conductivity at a given mean layer temperature :

Fireclay : $\lambda_{sI} = \lambda_{S0} + \xi_S \cdot \vartheta_{sI} = 0.971 + 0.00058 \cdot 715 = 1.386$ W.m^{-1} .K^{-1}

insulation : $\lambda_{izI} = \lambda_{iz0} + \xi_{iz} \cdot \vartheta_{izI} = 0.23 + 0.0002 \cdot 285 = 0.287$ W.m^{-1} .K^{-1}

4, Calculate the heat flux density :

$$\overline{q} = \frac{\Delta \vartheta}{\sum_{i=1}^{2} \frac{l_i}{\lambda_i}} = \frac{930 - 70}{\frac{0,23}{1,386} + \frac{0,115}{0,287}} = 1517,7 \quad \text{W m}^{-2}$$

5, Calculate the temperature at the interface :

$$\vartheta_{2I} = \vartheta_1 - q \,.\, \frac{l_S}{\lambda_S} = 930 - 1517.7 \,.\, \frac{0,23}{1,386} = 678\ °C$$

Since the calculated temperature at the interfaceϑ_{2I} = 678 °C differs significantly from the estimated temperatureϑ_{20} = 500 °C, we repeat procedure **1**,÷ **5**, with an input temperature at the interface of the layersϑ_{2I} =678 °C. Enter the individual values in the table.

Greatness	ϑ_2	ϑ_S	ϑ_{iz}	λ_S	λ_{iz}	**q**	ϑ_2
Step	°C	°C	°C	W.m^{-1} .K^{-1}	W.m^{-1} .K^{-1}	W.m^{-2}	°C
I	500	715	285	1,386	0,287	1517,7	678
II	678	804	374	1,437	0,305	1601,2	673
III	673	801,5	371,5	1,436	0,304	1597,2	674

Example 4 - Cylindrical wall

Determine the heat flux density q (W m^{-1}) through the wall of a refractory steel pipe with dimensions d_1 = 32 mm , d_2 = 42 mm. The thermal conductivity coefficient of the material of which the pipe is madeλ = 14 W.m^{-1} .K^{-1} . The temperature of the outer wall of the pipeϑ_1 = 580 °C, the temperature of the inner wall of the pipeϑ_2 = 450 °C.

Solution :

For a composite cylindrical wall, the heat transfer through the conduction is given by :

$$q = \frac{2 \cdot \pi \cdot \Delta \vartheta}{\sum_{i=1}^{n} \frac{1}{\lambda_i} \cdot \ln \frac{d_{i+1}}{d_i}} \qquad (\text{W.m}^{-1} ; \text{K} , \text{W.m}^{-1} .\text{K}^{-1} , \text{m})$$

For a single-layer wall and the values of our assignment :

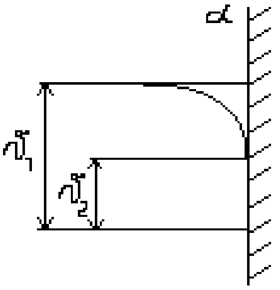
$$q = \frac{2 \cdot \pi \cdot (580 - 450)}{\frac{1}{14} \cdot \ln \frac{42}{32}} = 42052 \text{ W.m}^{-1}$$

B.1.5. Heat transfer by flow

Let us introduce the heat transfer coefficientα with unit W m K$\cdot^{-2.-1}$, which determines how much heat flux (power) flows through a unit area at a temperature difference of 1 °C. Heat transfer in this way is applied when heat is transferred from a solid surface to the surrounding environment or vice versa (usually in combination with radiation).

Heat propagation by flow is one of the most difficult computational problems in thermal engineering. It is dealt with in many scientific literatures. In important cases, it is best to determine the heat transfer coefficientα ourselves by measuring it on a model as appropriate to our case as possible using the given relations in whichα occurs.

Newton's law applies to the transfer of heat through the flow:



$$P = \alpha \cdot S \cdot \Delta\vartheta \quad (W ; W m K\cdot^{-2.-1} , m^2 , K)$$

Example 1 - Heat propagation by net flow :

Determine the heat loss through a vertical wall of area S = 1 m^2 . Wall temperatureϑ_1 = 60 °C, ambient temperatureϑ_2 = 10 °C.

and, by natural convection $\alpha = 4 \cdot (\Delta\vartheta)^{0,13}$, $v_0 = 0$ m s$\cdot^{-1}$
b, by blowing $\alpha = 5,8 + 3,95 \cdot v_0$, $v_0 = 5$ m s$\cdot^{-1}$

in$_0$ is the flow velocity of the medium at the wall

Solution :

a, $P = \alpha \cdot S \cdot \Delta\vartheta = 4 \cdot (\Delta\vartheta)^{0,13} \cdot S \cdot \Delta\vartheta =$
$= 4 \cdot (60 - 10)^{0,13} \cdot 1 \cdot (60 - 10) = 332,6$ W

b, $P = \alpha \cdot S \cdot \Delta\vartheta = (5.8 + 3.95 \cdot v_0) \cdot S \cdot \Delta\vartheta =$

$$= (5,8 + 3,95 \cdot 5) \cdot 1 \cdot (60 - 10) = 1277,5 \text{ W}$$

Example 2 :

Determine graphically the temperature in the wall of the room. The indoor temperature isϑ_1 = 20 °C, the outdoor temperatureϑ_5 = -20 °C. The inner wall is brick with a thickness of$_1$ = 0.36 m, thermal conductivity coefficientλ_1 = 0.464 W m K$\cdot^{-1.-1}$, and there is a layer of concrete with a thickness of$_2$ = 0.13 m, thermal conductivity coefficientλ_2 = 1.102 W m K$\cdot^{-1.-1}$. The heat transfer coefficient of the inner surface isα_2 = 17.4 W m K$\cdot^{-2.-1}$, the heat transfer coefficient of the outer surface isα_1 = 5.8 W m K$\cdot^{-2.-1}$.

Solution :

1, Draw to scale a section through the composite wall through which the heat flux passes.

2, Mark the indoor and outdoor temperature on the vertical axis.

3, At the internal temperature level, choose the P pole to the right of the wall.

4, Calculate the unit thermal resistances corresponding to the given heat propagation method and the given parameters.

5, On the vertical semi-line at any point between the pole P and the composite wall, we will apply the unit thermal resistances from the indoor temperature level towards the outdoor temperature level in a gradual scale:

- flow on the inside of the composite wall
- conducting layer of bricks
- conduction through the concrete layer
- flow on the outside of the composite wall

6, Connect the pole P to the ends of the thermal resistors thus plotted.

7, At the point where the junction of the pole with the end point of the last thermal resistance intersects the outdoor temperature level, construct a semi-vertical line in the vertical direction.

8, The intersections of the pole junctions with the endpoints of the unit thermal resistances with the semi-line so constructed give us the temperatures at the interface of the individual layers :

- on the inside of the folded wall
- at the interface between two layers of a composite wall
- on the outside of the folded wall

9, Plot these temperatures in the appropriate locations on the composite wall.

10, By combining these temperatures we get the desired graphical representation of the temperature progression.

Calculation of unit thermal resistances :

- flow at the inner surface of the composite wall

$$R_{q1} = \frac{1}{\alpha_2} = \frac{1}{17{,}4} = 0{,}0575 \ \mathrm{W}^{-1} \cdot \mathrm{K}$$

- heat conduction through the brick layer

$$R_{q2} = \frac{s_1}{\lambda_1} = \frac{0{,}36}{0{,}464} = 0{,}776 \ \mathrm{W}^{-1} \cdot \mathrm{K}$$

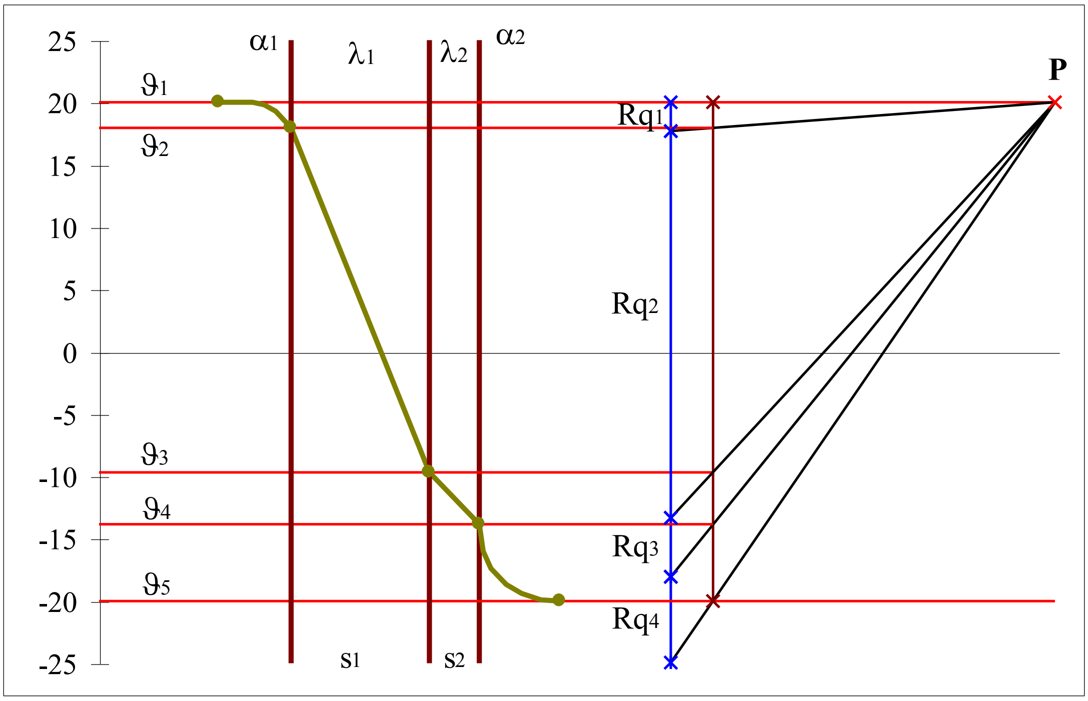
- heat conduction through the concrete wall

$$R_{q3} = \frac{s_2}{\lambda_2} = \frac{0{,}13}{1{,}102} = 0{,}118 \ \mathrm{W}^{-1} \cdot \mathrm{K}$$

- flow at the outer surface of the composite wall

$$R_{q4} = \frac{1}{\alpha_1} = \frac{1}{5{,}8} = 0{,}172 \ \mathrm{W}^{-1} \cdot \mathrm{K}$$

Graphic design :



B.1.6. Radiant heat transfer

Any body whose temperature is above 0 K radiates thermal energy through its surface. It is an electromagnetic wave that follows the laws of geometrical optics.

The laws governing the propagation of heat by radiation :

a, Stefan-Boltzmann law :

$P_{č} = \sigma_{č} \cdot \Theta^4$ ($W \cdot m^{-2}$; $W \cdot m^{-2} (K/100)^{-4}$, K)

Stefan-Boltzmann constant $\sigma_{č} = 5.6697\ W \cdot m^{-2}\ (K/100)^{-4}$

b, Planck's law :

$$M_{\lambda č} = f(\Theta, \lambda) = \frac{c_1}{\lambda^5 \cdot \left(e^{\frac{c_2}{\lambda \cdot \Theta}} - 1 \right)} \qquad (W \cdot m^{-4} ; m , K)$$

$c_1 = 3.73 \cdot 10^{-16}\ W \cdot m^2$

$c_2 = 1,438 \cdot 10^{-2}\ m\ K \cdot$

c, The law of Wien :

$$\lambda_m = (\frac{2892}{\Theta} \quad \mu\, m ; K)$$

d, Heat output transferred between two parallel, equal sized surfaces. Each with a surface A, one with temperature Θ_1 and emissivity ε_1 and the other with temperature Θ_2 and emissivity ε_2 :

$$P = \frac{A \cdot \sigma_{Ź}}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \cdot \left[\left(\frac{\Theta_1}{100} \right)^4 - \left(\frac{\Theta_2}{100} \right)^4 \right] \qquad (W)$$

e, Two surfaces, of which A_2 completely surrounds the smaller A_1 :

$$P = \frac{A_1 \cdot \sigma_{Ź}}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2} \cdot (\frac{1}{\varepsilon_2} - 1)} \cdot \left[\left(\frac{\Theta_1}{100} \right)^4 - \left(\frac{\Theta_2}{100} \right)^4 \right] \qquad (W)$$

Example 1 :

Determine $P_{č}$, λ_m , $M_{\lambda m č}$ of an absolutely black body with area $S = 300\ cm^2$ and temperature $\vartheta = 1200$ °C

Solution :

Heat flux (power) :

$$P_{č} = \sigma_{č} \cdot \Theta^4 \cdot S = 5.6697 \cdot \left(\frac{1200 + 273.15}{100}\right)^4 \cdot 300.10^{-4} = 8000 \quad W$$

The wavelength at which the maximum spectral density of the radiation intensity is :

$$\lambda_m = 2892 / \Theta = 2892 / (1200 + 273.15) = 1.96 \text{ m}\mu$$

Spectral intensity density of radiation at wavelength 1.96μ m :

$$M \; W \; m_{\lambda m č} = \frac{c_1}{\lambda_m^5 \cdot \left(e^{\frac{c_2}{\lambda_m \cdot \Theta}} - 1 \right)} = \frac{3.73 \cdot 10^{-16}}{\left(1.96 \cdot 10^{-6}\right)^5 \cdot \left(e^{\frac{1.438 \cdot 0.01}{1.96 \cdot 10^{-6} \cdot 1473.15}} - 1 \right)} = 8.9 \cdot 10^{10} \,.^{-3}$$

Example 2 :

Determine the heat output radiating from a body of area A_1 = 1 cm^2 , temperatureϑ_1 = 1000 °C, emissivityε_1 = 0.9 to a body of area A_2 = 10 cm^2 , temperatureϑ_2 = 0 °C, emissivityε_2 = 0.9. The second body completely surrounds the first in space.

Solution :

$$P = \frac{A_1 \cdot \sigma_{\acute{Z}}}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2} \cdot (\frac{1}{\varepsilon_2} - 1)} \cdot \left[\left(\frac{\Theta_1}{100}\right)^4 - \left(\frac{\Theta_2}{100}\right)^4 \right]$$

$$P = \frac{1 \cdot 10^{-4} \cdot 5.67}{\frac{1}{0.9} + \frac{1}{10} \cdot \left(\frac{1}{0.9} - 1\right)} \cdot \left[\left(\frac{1273}{100}\right)^4 - \left(\frac{273}{100}\right)^4 \right] = 13.25 \qquad W$$

B.2. RESISTIVE ELECTROTHERMAL DEVICES

B.2.1. Heating element design

For the design of heating cells of circular or rectangular cross-section of the resistive conductor, we use the following relations :

a, circular cross section of the resistive conductor :

$$d = \sqrt[3]{\frac{4 \cdot \rho \cdot P^{2}}{10 \cdot \pi^{2} \cdot p \cdot U^{2}}}$$ (mm ;Ω· mm m$^{2\cdot-1}$,W,W· cm^{-2} ,V)

Where ρ .... specific resistance of the conductor material
P .... power of one phase of resistance furnace
p .... voltage on the heating element

b, rectangular cross section :

$$b = \sqrt[3]{\frac{P^{2} \cdot \rho}{20 \cdot \beta \cdot (\beta + 1) \cdot U^{2} \cdot p}}$$ (mm ; W,Ω· mm m$^{2\cdot-1}$, H,W· cm^{-2} ,)

$a = b \cdot \beta$

Where β .... the aspect ratio of the rectangle

The length of the heating conductor is designed either from the relation :

$$l = \frac{U^{2} \cdot S}{P \cdot \rho}$$ (m ; V, mm^{2} , W , Ω· mm ^{2}· m^{-1})

whereS .... heating wire cross section

or from a relationship

$$l = \frac{P}{O \cdot p}$$ (cm ; W , cm , W· cm^{-2})

whereO .... heating wire circumference in cm

Example 1 :

Determine the dimensions (a , b , l) of the heating strip for heating the resistance furnace if the input power of the furnace is P = 75 kW. The heating elements are connected in a triangle, the furnace operates in a 3x380/220 V voltage system. The specific surface load of the resistive conductor is p = 1.2 W· cm^{-2} , the specific resistance of the resistive conductor material isρ = 1.2 Ω· mm ^{2}· m^{-1} . The aspect ratio of the rectangular cross sectionβ = 5 .

Solution :

$$b = \sqrt[3]{\frac{P^2 \cdot \rho}{20 \cdot \beta \cdot (\beta + 1) \cdot U^2 \cdot p}}$$

$$b = \sqrt[3]{\frac{1.2 \cdot \left(\frac{75000}{3}\right)^2}{20 \cdot 5 \cdot (5+1) \cdot 380^2 \cdot 1.2}} = 1.93 \quad \text{mm}$$

a = β· b = 5· 1.93 = 9.66 mm

$$l = \frac{U^2 \cdot S}{P \cdot \rho} = \frac{a \cdot b \cdot U^2}{P \cdot \rho} = \frac{1.93 \cdot 9.66 \cdot 380^2}{25000 \cdot 1.2} = 89.9 \quad \text{m}$$

Example 2 :

The power of the annealing furnace is P = 60 kW, the annealing temperature is 850 °C. The clamp voltage is 3x380/220 V, the heating elements are connected in a triangle. The specific surface load of the heating conductor is p = 1.2 W· cm^{-2} , the specific resistance of the resistive conductor material at 850 °Cρ = 1.2 Ω· mm ^{2}· m^{-1} . Determine the length and diameter of the heating wire of circular cross section for one phase of the furnace.

Solution :

$$d = \sqrt[3]{\frac{4 \cdot \rho \cdot P^2}{10 \cdot \pi^2 \cdot p \cdot U^2}}$$

$$d = \sqrt[3]{\frac{4 \cdot 1.2 \cdot \left(\frac{60000}{3}\right)^2}{10 \cdot 3.14^2 \cdot 1.2 \cdot 380^2}} = 4.82 \quad \text{mm}$$

$$l = \frac{U^2 \cdot S}{P \cdot \rho} = \frac{\pi \cdot d^2 \cdot U^2}{4 \cdot \rho \cdot P} = \frac{3.14 \cdot 4.82^2 \cdot 380^2}{4 \cdot 1.2 \cdot \left(\frac{60000}{3}\right)^2} = 111 \quad \text{m}$$

B.2.2. Calculation of the heating time

The heating time of an object in a resistance furnace can be calculated according to the relation :

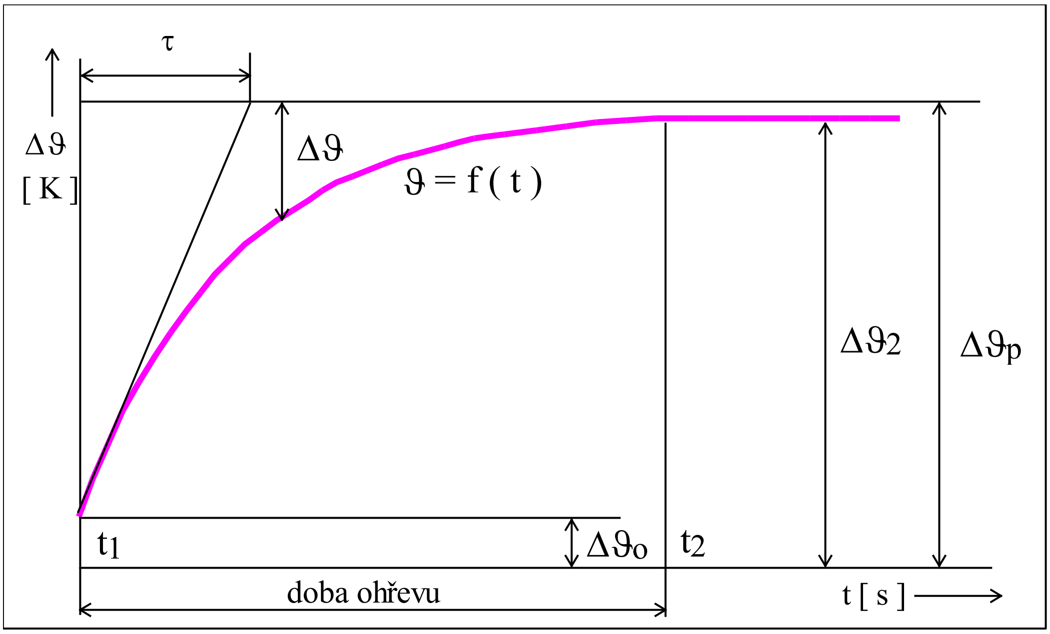
$$t = \frac{Q}{P_s + P_p} \qquad (\text{s} ; \text{J} , \text{W} , \text{W})$$

whereQ .... heat capacity (the heat that must be put into an object during heating not accumulate) of the heated object

P_s .... heat output transferred to an object by radiation

P_p .... heat output transferred to an object by a flow

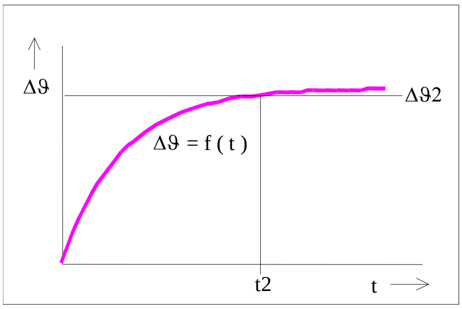
Heating curve of the heated object :



Valid $P_s = f(\Delta\vartheta)$ and $P_p = f(\Delta\vartheta)$,

where$\Delta\vartheta = f(t)$ is the difference between the temperature in the furnace and the temperature of the heated object.

To simplify the calculation, I replace the exponential by the parabola :



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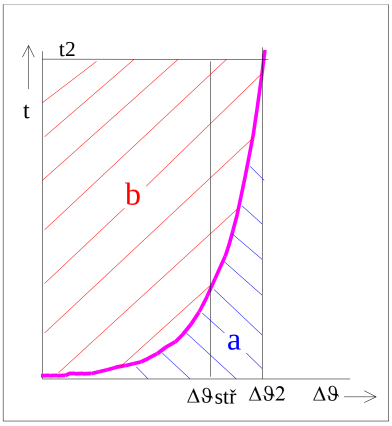


Find the mean difference in temperature (in the furnace and the heated object) over the heating time from the equality of the areas of the rectangle $\Delta\vartheta_{stř} \cdot t_2$ and the area labeled **b** in the figure:

$$b = t_2 \cdot \Delta\vartheta_2 - a = t_2 \cdot \Delta\vartheta_2 - \int_0^{\Delta\vartheta_2} p \cdot \Delta\vartheta^2 \cdot d\Delta\vartheta =$$

$$= t_2 \cdot \Delta\vartheta_2 - \frac{\Delta\vartheta_2{}^3}{3} \cdot p = t_2 \cdot \Delta\vartheta_2 - \frac{\Delta\vartheta_2{}^3}{3} \cdot \frac{t_2}{\Delta\vartheta_2{}^2} = \frac{2}{3} \cdot \Delta\vartheta_2 \cdot t_2$$

$$b = \Delta\vartheta_{stř} \cdot t_2 \quad => \quad \Delta\vartheta_{stř} = \frac{2}{3} \cdot \Delta\vartheta_2$$

In a simplified calculation, I include the heat output transferred by the radiant in the heat transfer coefficientα_{s+p} ($\alpha_{s+p} > \alpha_p$).

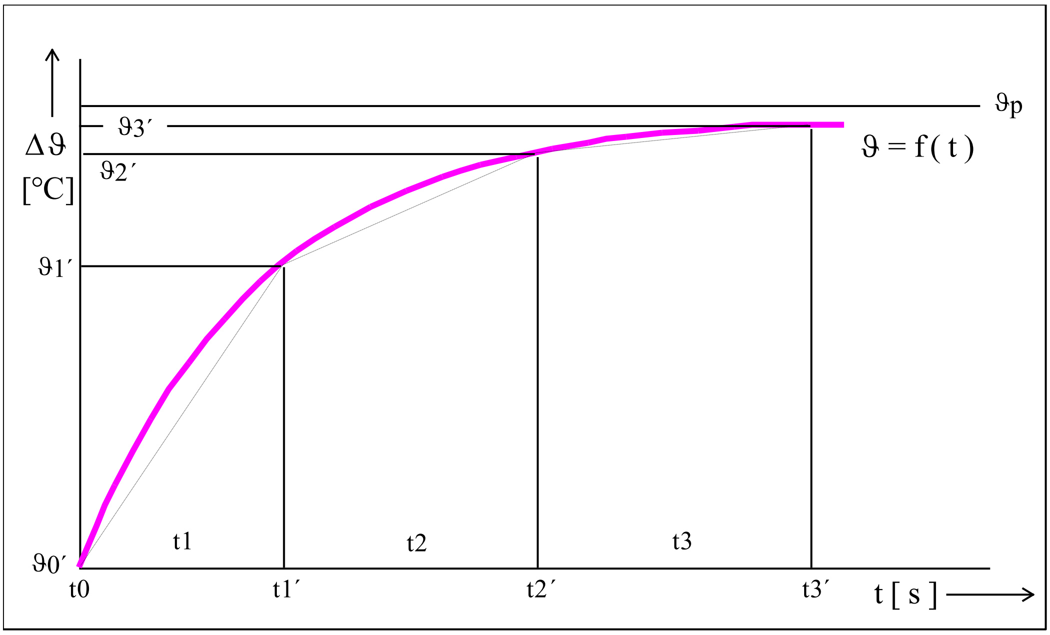
The heating time can then be calculated simplistically according to the relation :

$$t = \frac{Q}{\alpha_{s+p} \cdot \left(\vartheta_p - \vartheta_{stů} \right) \cdot S} \qquad (s ; J , W \cdot m^{-2} \cdot K^{-1} , K , m^2)$$

Where α_{s+p} .... heat transfer coefficient including both radiation and flow
ϑ_p .... oven temperature
$\vartheta_{stř}$ .... mean temperature of the heated object during the heating period
($\vartheta_{stř} = \vartheta_o + 2 / 3 \cdot (\vartheta_2 - \vartheta_o)$)
ϑ_o .... ambient temperature (of the object before the start of heating)
S .... the area through which heat is transferred to the heated object
Q .... heat accumulated in the object during heating

When calculating the exact heating time of an object in a resistance furnace, we consider the exponential temperature rise of the heated object. We divide the heating interval into sections within which we linearize the temperature rise.

We choose the division so that the time intervals are approximately the same length.



The duration of each interval is then calculated from the relation:

$$t_i = \frac{Q_i}{P_{s_i} + P_{p_i}} = \frac{m \cdot c \cdot \left(\vartheta_i{}' - \vartheta_{i-1}{}'\right)}{\frac{\sigma_{Ź}}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \cdot \left[\left(\frac{\Theta_p}{100}\right)^4 - \left(\frac{\Theta_{stů_i}}{100}\right)^4\right] \cdot S_s + \alpha_p \cdot \left(\vartheta_p - \vartheta_{stů_i}\right) \cdot S_p}$$

kdem .... weight of heated object

c .... specific heat of the heated object

$\vartheta_i{}'$,$\vartheta_{i-1}{}'$ .... boundary temperatures of the calculated interval

ε_1 ,ε_2 .... emissivities of the surface of the heated object and the inner surface of the furnace

ϑ_p ,Θ_p .... temperature and thermodynamic temperature in the furnace

($\Theta_p = \vartheta_p + 273.15$)

S_s .... the area over which heat is transferred to the heated object by radiation

S_p .... the area over which heat is transferred to the heated object by the flow of

α_p .... heat transfer coefficient for flow

$\Theta_{\text{stř i}}$,$\vartheta_{\text{stř i}}$ .... mean temperature and mean thermodynamic temperature of the heated of the object within the calculated interval

$$(\vartheta_{\text{stř i}} = (\vartheta_i{}' + \vartheta_{i-1}{}') / 2)$$

$$(\Theta_{\text{stř i}} = \vartheta_{\text{stř i}} + 273.15)$$

The total heating time of the object in the resistance furnace is then determined as the sum of the partial intervals calculated as above.

Example 1 :

Calculate the heating time of three prisms of dimensions 100 x 100 x 1000 mm in a chamber furnace. Three prisms are heated simultaneously in the furnace from an ambient temperature ofϑ_o = 10 °C to a temperature ofϑ_H = 800 °C. The temperature in the furnace during the heating period isϑ_p = 850 °C.

The prisms are steel :$\gamma_H = 7.8$ kg· dm^{-3} , c = 0.667 kJ· kg $^{-1}$· K^{-1} . The emissivity of the surface of the prisms is$\varepsilon_1 = 0.8$, the emissivity of the inner surface of the furnace is$\varepsilon_2 = 0.8$. The heat transfer coefficient for flow and radiation isα_{s+p} = 177.8 W· m $^{-2}$· K^{-1} , the heat transfer coefficient for flow isα_p = 14 W· m $^{-2}$· K^{-1} .

Make an approximate calculation of the heating time under the following conditions :

a, heat is transferred from above and below by radiation and convection

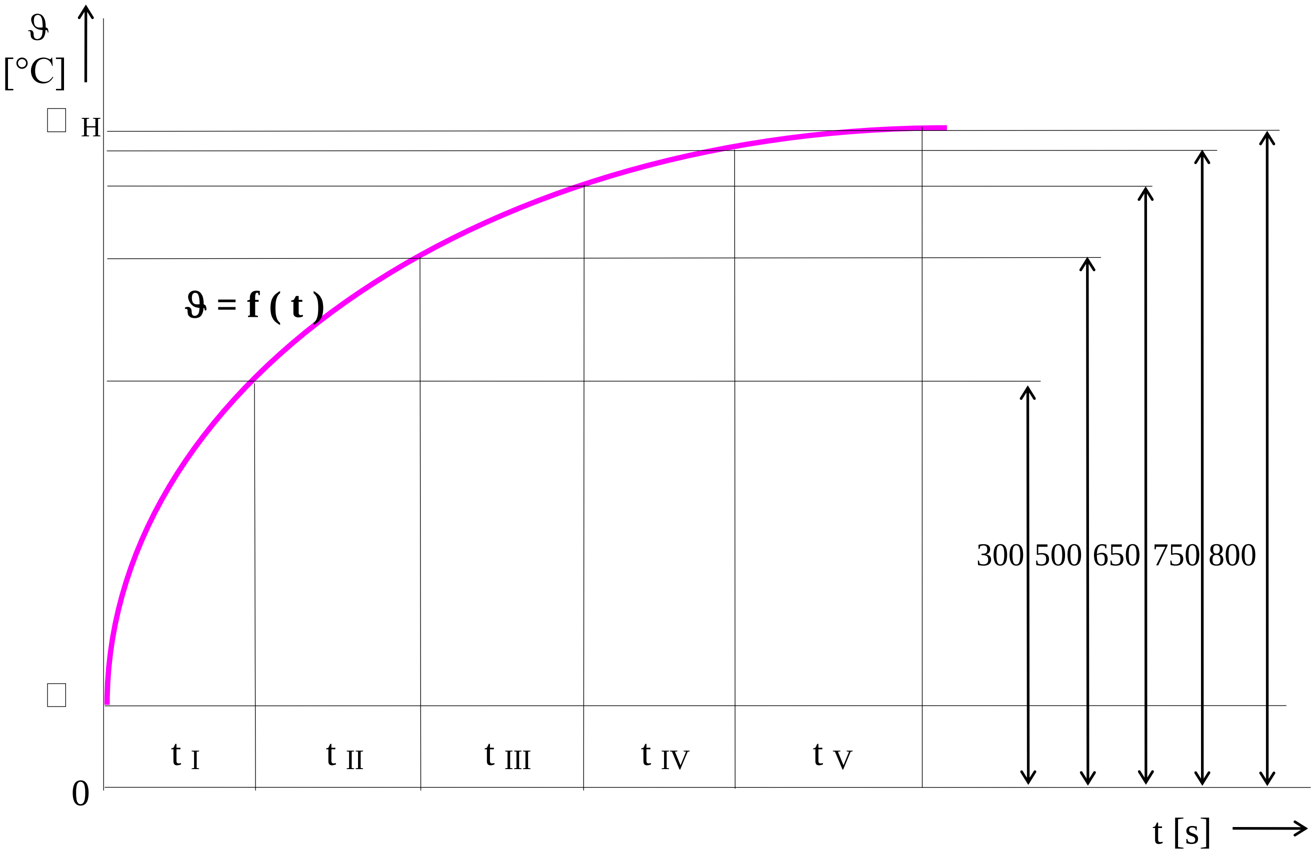
b, radiation is included in the heat transfer coefficient α_{s+p}

Next, perform an accurate calculation of the heating time of the prisms under the assumptions that

and, heat is transferred by radiation from above and below

b, heat is transferred by flow over the entire surface of the system.

Heating interval distribution for accurate calculation :



Solution :

Approximate calculation :

$$t = \frac{Q}{P_s + P_p} = \frac{m \cdot c \cdot \left(\vartheta_H - \vartheta_O\right)}{\alpha_{s+p} \cdot \left[\vartheta_p - \left(\vartheta_O + \frac{2}{3} \cdot \left(\vartheta_H - \vartheta_O\right)\right)\right] \cdot S}$$

$$t = \frac{3 \cdot 10 \cdot 7.8 \cdot 0.667 \cdot 10^3 \cdot (800 - 10)}{177.8 \cdot 850 - \left(10 + \frac{2}{3} \cdot (800 - 10)\right) \cdot 0.2 \cdot 3} = 3648 \quad \text{s}$$

Exact calculation:

$$t_I = \frac{m \cdot c \cdot \left(\vartheta_2 - \vartheta_1\right)}{\dfrac{\sigma_{\text{Ź}}}{\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1} \cdot \left[\left(\dfrac{\Theta_p}{100}\right)^4 - \left(\dfrac{\Theta_{\text{stů}}}{100}\right)^4\right] \cdot S_s + \alpha_p \cdot \left(\vartheta_p - \vartheta_{\text{stů}}\right) \cdot S_p}$$

$$t_I = \frac{3 \cdot 10 \cdot 7.8 \cdot 0.667 \cdot 10^3 \cdot (300 - 10)}{\dfrac{5.67}{\dfrac{1}{0.8} + \dfrac{1}{0.8} - 1} \cdot \left[\left(\dfrac{1123.15}{100}\right)^4 - \left(\dfrac{428.15}{100}\right)^4\right] \cdot 3 \cdot 2 \cdot 0.1 + 14 \cdot \left(850 - \dfrac{300 + 10}{2}\right) \cdot 0.86} = 1036 \quad \text{s}$$

Calculate the length of the remaining intervals in a similar way:

- second interval t_{II} = 839 s
- third interval t_{III} = 842 s
- fourth interval t_{IV} = 928 s
- fifth interval t_V = 831 s

$$\Rightarrow \quad t = \sum_{i=1}^{5} t_i = 4449 \quad s$$

The total heating time of the prisms is obtained as the sum of the partial time intervals.

Electric resistance furnace design

The following assignment is intended to enable you to practice the topics covered so far and to become aware of the interrelationships and practical application of computational procedures in the form of an independently developed program.

Assignment :

Design a crucible resistance furnace for melting aluminium and determine :

1. Furnace dimensions
2. Amount of heat required to melt aluminium
3. Temperature of the spirals
4. Heat losses including graphical representation of the temperature evolution in the composite cylindrical wall
5. Power input and power consumption
6. Accumulated heat
7. Time required to place the first bet
8. Dimensions of heating elements, electrical wiring diagram including regulation, sizing of inlets and protection.

Default values :

Default betting temperature . . . . $\vartheta_1 = 20$ °C
Melting temperature of aluminium . . . . $\vartheta_{tav} = 658$ °C
Casting temperature of the wedge . . . . $\vartheta_2 = 750$ °C
Weight of the bet . . . . m = 80 kg
Bet density . . . . $\gamma_{Al} = 2.7 \cdot 10^3$ kg· m^{-3}
Heating time . . . . t = 65 min
Heat transfer coefficient . . . . $.\alpha = 11.6$ W· m^{-2}· K^{-1}
Specific heat of aluminium . . . . $c_1 = 0.894$ kJ· kg^{-1}· K^{-1}
Specific heat of aluminium . . . . $c_2 = 397.1$ kJ· kg^{-1}
The specific resistance of the spiral material. . . . . $\rho = 1.1$ Ω· mm^{2}· m^{-1}
Density of resistive material. . . . . . . $\gamma = 8.27 \cdot 10^3$ kg· m^{-3}
Allowable surface load . . . . p = 10 700 W· m^{-2}
Emissivity of the surface of the spirals . . . . $\varepsilon_1 = 0.85$
Emissivity of the crucible surface . . . . $\varepsilon_2 = 0.85$
Specific thermal conductivity
    Tercalite . . . . $\lambda_t = 0.278$ W· m^{-1}· K^{-1}
    silocel . . . . $\lambda_s = 0.232$ W· m^{-1}· K^{-1}
    Steel . . . . $\lambda_o = 46.4$ W· m^{-1}· K^{-1}

Solution :

1. Furnace dimensions

Volume of aluminium after melting :

$$V^{/} = \frac{m}{\gamma_{Al}} = \frac{80}{2.7 \cdot 10^{3}} = 29.6 \cdot 10^{-3} \qquad m^{-3}$$

I will increase this volume by 50% :

$$V = 1.5 \cdot V^{/} = 1.5 \cdot 29.6 \cdot 10^{-3} = 44.4 \cdot 10^{-3} \qquad m^{-3}$$

From practical experience we choose the diameter of the cup :

$$d = 0.36 \qquad m$$

Height of the cup :

$$v^{/} = \frac{4 \cdot V}{\pi \cdot d^{2}} = \frac{4 \cdot 44.4 \cdot 10^{-3}}{\pi \cdot 0.36^{2}} = 0.436 \qquad m$$

Round the height of the cup to :

$$v = 450 \qquad mm$$

The dimensions of the crucible give me the other dimensions of the furnace - see drawing.

2. Amount of heat required to melt aluminium

Heat required for heating from 20 °C to 750 °C :

$$Q_1 = m \cdot c_1 \cdot \Delta\vartheta = 80 \cdot 0.894 \cdot (750 - 20) = 52209.6 \qquad kJ$$

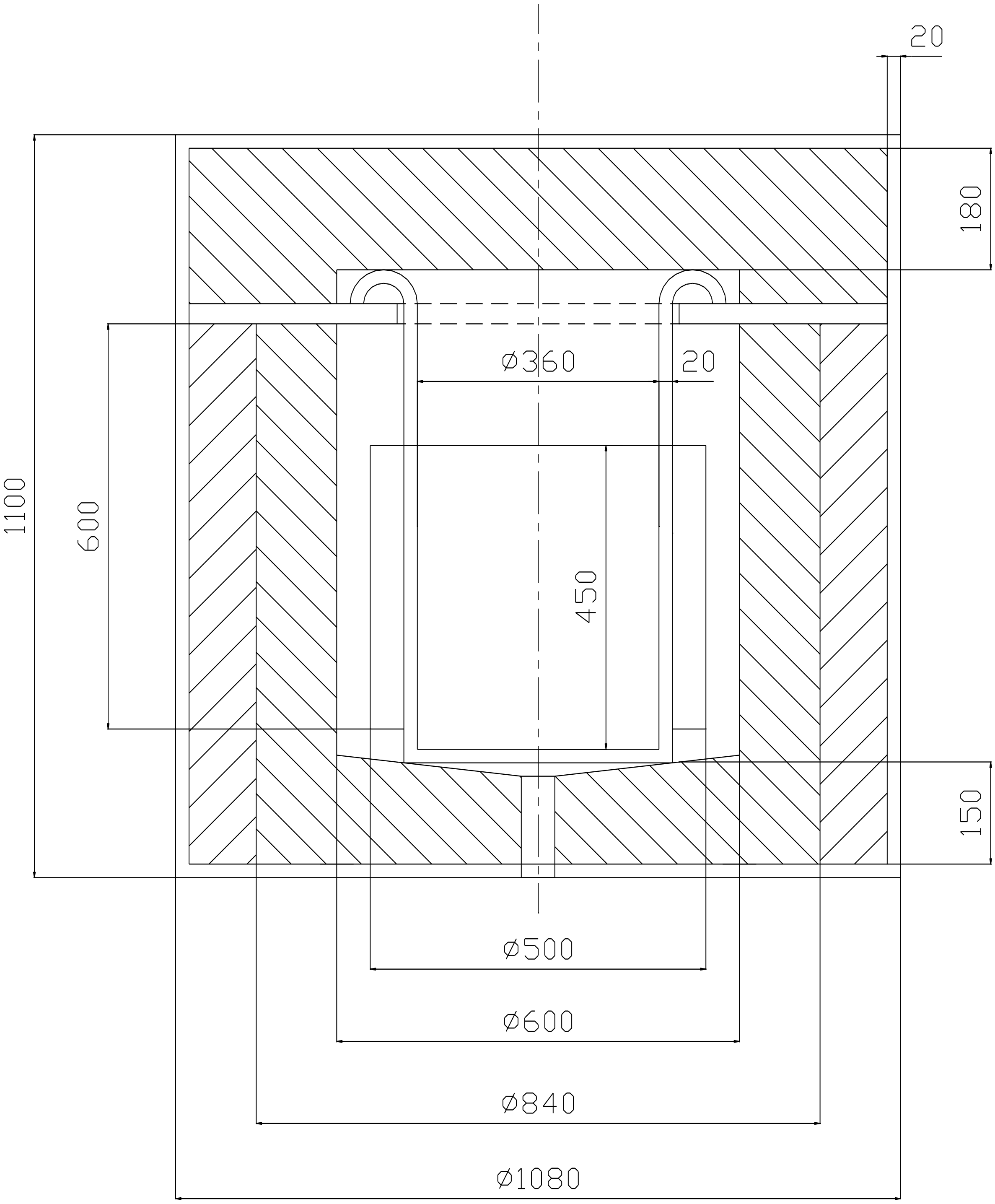
The heat required to change state :

$$Q_2 = m \cdot c_2 = 80 \cdot 397.1 = 31768 \qquad kJ$$

Total amount of heat to melt aluminium :

$$Q_{Al} = Q_1 + Q_2 = 52209.6 + 31768 = 83977.6 \qquad kJ$$

Schematic representation of a section of the proposed resistance crucible furnace :



3. Temperature of resistance spirals

The surface area of the spiral-irradiated and heat-transferring aluminium :

$$S_1 = \pi \cdot (d + 2 \cdot t_k) \cdot v = \pi \cdot (0.36 + 2 \cdot 0.02) \cdot 0.45 = 0.5655 \qquad m^2$$

Required heat flux :

$$P = \frac{Q_{Al}}{t} = \frac{83977.6}{65 \cdot 60} = 21.533 \qquad kJ \cdot s^{-1} = 21.533 \qquad kW$$

Temperature of the outer wall of the crucible :

$$\vartheta_k = \vartheta_2 + \frac{P}{2 \cdot \pi \cdot \lambda_o \cdot v} \cdot \ln \frac{d_2}{d_1} = 750 + \frac{21.533 \cdot 10^3}{2 \cdot \pi \cdot 46.4 \cdot 0.45} \cdot \ln \frac{0.4}{0.36} = 767.3 \qquad {}^oC$$

Required thermodynamic temperature of resistance spirals :

$$\Theta_2 = \sqrt[4]{\frac{P \cdot \left[\frac{1}{\varepsilon_1} + \frac{S_1}{S_2}\left(\frac{1}{\varepsilon_2} - 1\right)\right]}{\sigma_{\dot{Z}} \cdot S_1} + \left(\frac{\Theta_1}{100}\right)^4} \cdot 100$$

$$\Theta_2 = \sqrt[4]{\frac{21533 \cdot \left[\frac{1}{0.85} + \frac{0.4}{0.5} \cdot \left(\frac{1}{0.85} - 1\right)\right]}{5.67 \cdot \pi \cdot 0.4 \cdot 0.5} + \left(\frac{767.3 + 273.15}{100}\right)^4} \cdot 100$$

$$\Theta_2 = 1197 \qquad K$$

Required temperature of resistance spirals :

$$\vartheta_S = \Theta_2 - 273.15 = 1197 - 273.15 = 923.85 = 924 \qquad {}^oC$$

According to this temperature, we would choose the appropriate resistive material for the production of heating coils. This temperature will be maintained by the furnace control.

4. Heat loss

Calculate the heat loss under the simplifying assumption that the temperature on the inside of the refractory lining (tercalite) is equal to the temperature of the outer surface of the crucible on the furnace wall, lid and bottom.

To calculate the heat flux loss through the furnace wall, I consider a composite cylindrical wall with a height of l = 0.85 m :

$$P_s = \frac{(\vartheta_4 - \vartheta_o) \cdot \pi \cdot l}{\frac{1}{2 \cdot \lambda_t} \cdot \ln \frac{d_4}{d_3} + \frac{1}{2 \cdot \lambda_S} \cdot \ln \frac{d_3}{d_2} + \frac{1}{2 \cdot \lambda_o} \cdot \ln \frac{d_2}{d_1} + \frac{1}{\alpha \cdot d_1}}$$

$$P_s = \frac{(767.3 - 20) \cdot \pi \cdot 0.85}{\frac{1}{2 \cdot 0.278} \cdot \ln \frac{840}{600} + \frac{1}{2 \cdot 0.232} \cdot \ln \frac{1040}{840} + \frac{1}{2 \cdot 46.4} \cdot \ln \frac{1080}{1040} + \frac{1}{11.6 \cdot 1.08}}$$

$$P_s = 1742 \quad W$$

For the calculation of the heat flux loss through the furnace lid, we consider the mean area of the lid in the middle of its thickness S = 0.554 m^2 :

$$P_v = \frac{S \cdot (\vartheta_4 - \vartheta_o)}{\frac{s_t}{\lambda_t} + \frac{s_o}{\lambda_o} + \frac{1}{\alpha}} = \frac{0.554 \cdot (767.3 - 20)}{\frac{0.180}{0.278} + \frac{0.02}{46.4} + \frac{1}{11.6}} = 564 \quad W$$

For the calculation of the heat loss through the furnace bottom, we consider the same surface area as for the lid and the mean thickness of tercalite with$_t$ = 0.150 m :

$$P_d = \frac{S \cdot (\vartheta_4 - \vartheta_o)}{\frac{s_t}{\lambda_t} + \frac{s_o}{\lambda_o} + \frac{1}{\alpha}} = \frac{0.554 \cdot (767.3 - 20)}{\frac{0.150}{0.278} + \frac{0.02}{46.4} + \frac{1}{11.6}} = 661 \quad W$$

Total heat loss heat flux of the furnace :

$$P_z = P_s + P_v + P_d = 1742 + 564 + 661 = 2967 \quad W$$

Layer interface temperatures for a composite cylindrical furnace wall :

tercalite - silocel

$$\vartheta_3 = \vartheta_4 - \frac{P_s \cdot \ln \frac{d_3}{d_4}}{2 \cdot \pi \cdot \lambda_t \cdot l} = 767.3 - \frac{1742 \cdot \ln \frac{840}{600}}{2 \cdot \pi \cdot 0.278 \cdot 0.85} = 372.5 \qquad {}^{o}C$$

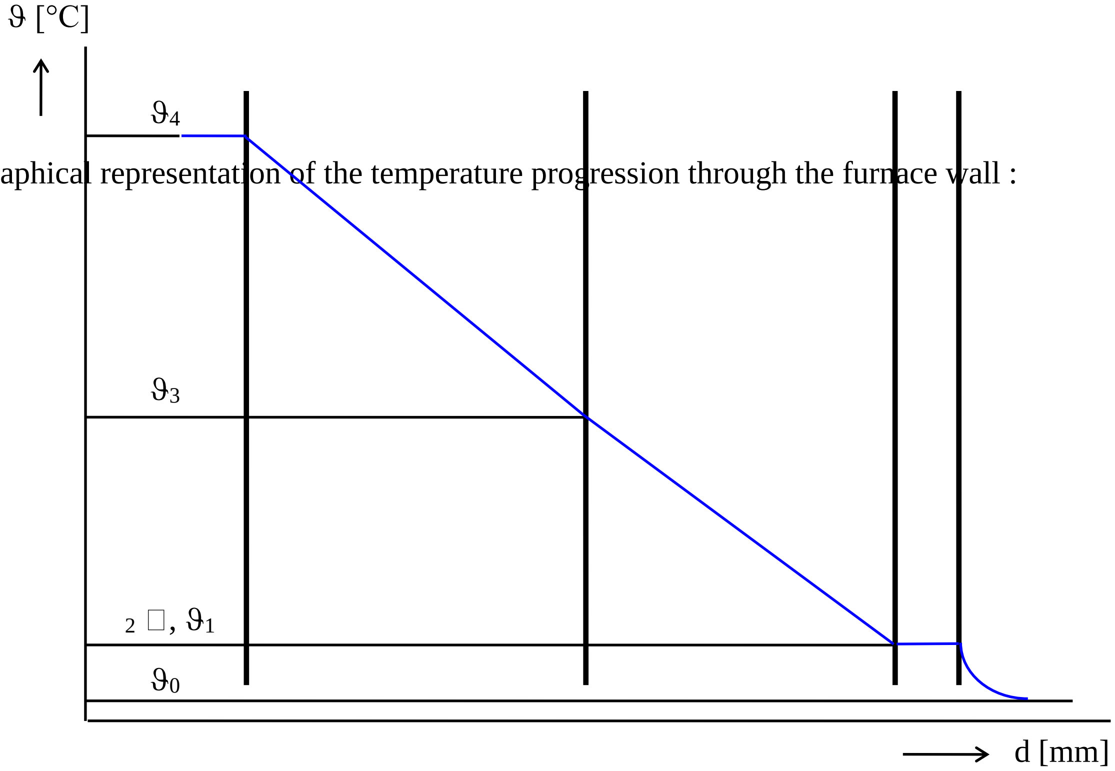
silocel - steel casing

$$\vartheta_2 = \vartheta_3 - \frac{P_s \cdot \ln \frac{d_2}{d_3}}{2 \cdot \pi \cdot \lambda_o \cdot l} = 372.5 - \frac{1742 \cdot \ln \frac{1040}{840}}{2 \cdot \pi \cdot 0.232 \cdot 0.85} = 72.2 \qquad {}^{o}C$$

surface of the steel furnace jacket

$$\vartheta_1 = \vartheta_0 + \frac{P_s}{\alpha \cdot \pi \cdot d_1 \cdot l} = 20 + \frac{1742}{11.6 \cdot \pi \cdot 1.08 \cdot 0.85} = 72.0 \qquad {}^{o}C$$

Graphical representation of the temperature progression through the furnace wall :



5. Power input and power consumption

The input power of the furnace is calculated as the sum of the required heat output going into the charge and the lost heat output of the furnace. This sum is increased by 15 % for power reserve reasons:

$$P_p = 1.15 \cdot (P_z + P) = 1.15 \cdot (2.967 + 21.533) = 28.2 \qquad \text{kW}$$

Power consumption per bet :

$$A = P_p \cdot t = 28.2 \cdot \frac{65}{60} = 30.6 \qquad \text{kWh}$$

6. Accumulated heat

The heat accumulated in the individual structural units of the furnace is calculated from the known relation :

$$Q = m \cdot c \cdot \Delta\vartheta$$

where$\Delta\vartheta$ is the mean temperature, i.e. the difference between the mean temperature of the material during furnace operation and the ambient temperature

Arrange the calculation in a table :

Name	Weight m	Specific heat c	Medium warming	Accumulated heat Qa
	[kg]	[$kJ \cdot kg^{-1} \cdot K^{-1}$]	[K]	[MJ]
The crucible	138	0.5	738.6	50.966
Lining	110	0.836	550.0	50.578
Insulation	80	0.670	202.4	10.848
Lid	30	0.836	400.0	10.032
Bottom	20	0.836	405.0	6.771
Cloak	836	0.5	52.0	21.736

Total accumulated heat : 150.931 MJ

7. Time required to place the first bet

This time consists of the time required to heat the furnace from ambient temperature to operating temperature - during this time the heat loss rises from zero to the value of the calculated loss heat flux P_Z - and the time required to melt the charge - during this time the loss heat flux has a value calculated as P_Z . During the time the furnace is heated to operating temperature we take the heat loss as half of the full loss heat flux P_Z .

Time required to heat the furnace :

$$t_1 = \frac{Q_a}{P_p - \frac{P_z}{2}} = \frac{150.931 \cdot 10^6}{28200 - 0.5 \cdot 2967} = 5981 \quad s$$

The melting time of the bet :

$$t_2 = \frac{Q_{Al}}{P_p - P_z} = \frac{83977.6 \cdot 10^3}{28200 - 2967} = 3328 \quad s = 56 \quad min$$

Heat loss during furnace heating :

$$Q_{z1} = \frac{P_z}{2} \cdot t_1 = \frac{2967}{2} \cdot 5981 = 8872.8 \quad kJ$$

Heat loss during melting of the charge :

$$Q_{z2} = P_z \cdot t_2 = 2967 \cdot 3328 = 9874.2 \quad kJ$$

Time required to place the first bet :

$$t_I = \frac{Q_{Al} + Q_a + Q_{z1} + Q_{z2}}{P_p} = \frac{83977.6 + 150931 + 8872.8 + 9874.2}{28.2} = 8994 \quad s = 1$$

Thanks to the power reserve of the furnace and the simplified calculation method, the time needed to melt the charge was shorter than the specified time.

8. Heating elements, supply, protection, regulation

We use a resistive material in the form of a heating wire of circular cross-section.

Cell conductor diameter :

$$d = \sqrt[3]{\frac{4 \cdot \rho \cdot P^2}{10 \cdot p \cdot \pi^2 \cdot U^2}} = \sqrt[3]{\frac{4 \cdot 1.1 \cdot \left(\frac{28200}{3}\right)^2}{10 \cdot 1.07 \cdot \pi^2 \cdot 380^2}} = 2.94 \qquad \text{mm}$$

We choose a resistance wire with a diameter of 3 mm, the basic connection of the heating elements will be in a triangle.

Resistance of a single phase cell :

$$R = \frac{U^2}{P} = \frac{380^2}{\frac{28200}{3}} = 15.36 \qquad \Omega$$

Length of resistance wire for one phase :

$$l = \frac{R \cdot S}{\rho} = \frac{15.36 \cdot \pi \cdot 0.003^2}{4 \cdot 1.1 \cdot 10^{-6}} = 98.7 \qquad \text{m}$$

Weight of resistance material required for the whole furnace :

$$m = 3 \cdot l \cdot S \cdot \gamma = 3 \cdot 98.7 \cdot \frac{\pi \cdot 0.003^2}{4} \cdot 8.27 \cdot 10^3 = 17.3 \qquad \text{kg}$$

Current through the heating element :

$$I_f = \frac{P_p}{3 \cdot U} = \frac{28200}{3 \cdot 380} = 24.73 \qquad \text{A}$$

Current in the supply wires :

$$I_s = \sqrt{3} \cdot I_f = \sqrt{3} \cdot 24.73 = 42.83 \qquad \text{A}$$

The design of the supply cable is carried out in accordance with the standard ČSN 34 1020 - Regulations for sizing and securing conductors and cables.

The supply cable should be of type AYKY according to ČSN 34 7656. We assume an ambient temperature of max. 40 °C, cable installation on the wall - six cables on a common

NIEDAX rail. This environment and cable installation corresponds to the correction coefficient given in Table 7 and 19 of ČSN 34 1020 with values of 0.84 and 0.67.

The rated load capacity of the proposed cable shall be at least :

$$I_n = \frac{I_s}{k_r} = \frac{42.83}{0.84 \cdot 0.67} = 76 \qquad A$$

According to Table 59 of the standard CSN 34 1020, we design AYKY 3 x 35 + 25 mm cable^2 . This cable has a rated load capacity of 93 A.

For the furnace protection we choose fuses with rated current I_{np} = 50 A. We have to check whether the fuse complies with the condition according to Article 173 of ČSN 34 1020 :

$$I \geq \frac{I_{Np}}{k_p}$$

whereI . . . . the permissible current of the respective conductor stored in an environment of temperature ϑ

k_p . . .the coefficient of attachment of the fuse to the conductor, which is stored in an environment of temperatureϑ - z Figure 11 in the standard, for I_{np} = 50 A and an ambient temperature of 40 °C , read the value k_p = 1.05

$$93 \cdot 0.67 \cdot 0.84 = 5234 \qquad \rangle \qquad 47.6 = \frac{50}{1.05}$$

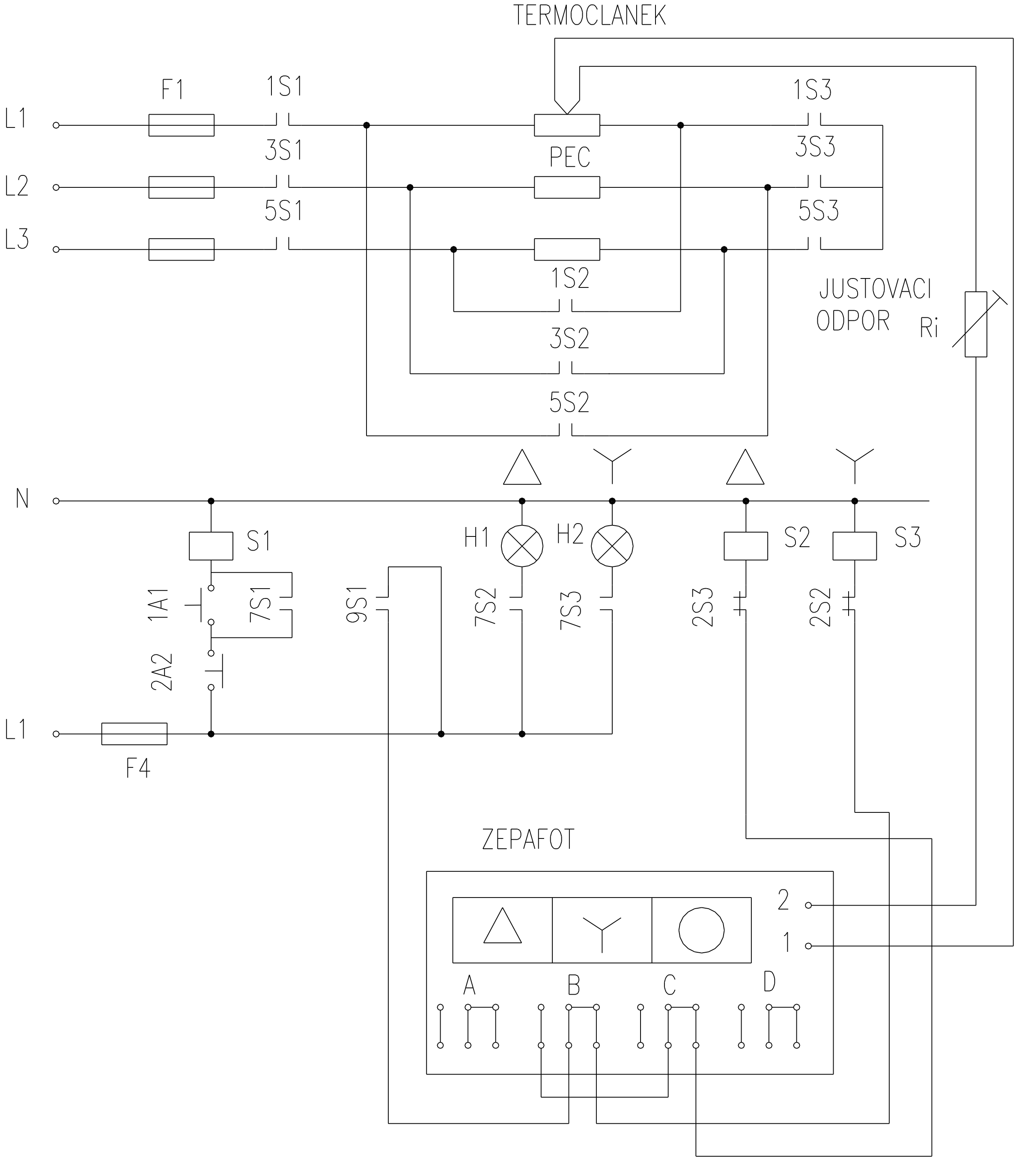
The condition is fulfilled, the 50 A fuse protects the cable against all overcurrents, overloads and short circuits.

Temperature control in the furnace will be implemented by the ZEPAFOT device, which switches the heating elements from a triangle to a star according to two set temperatures when the first set temperature is exceeded (thus reducing the furnace input to one third) or switches the furnace off when the second set temperature is exceeded.

The temperature is sensed by a thermocouple directly from the heating coil and this signal is fed to terminals 1 and 2 of the ZEPAFOT.

The A1 button switches on the furnace via the S1 contactor. When the heating elements are connected in a triangle, the S1 and S2 contactors are switched on, when they are connected in a star, the S1 and S3 contactors are switched on.

Circuit diagram of temperature control and regulation in resistance furnace :



B.3. ELECTRIC ARC HEATING EQUIPMENT

B.3.1. Circular diagram of an electric arc furnace

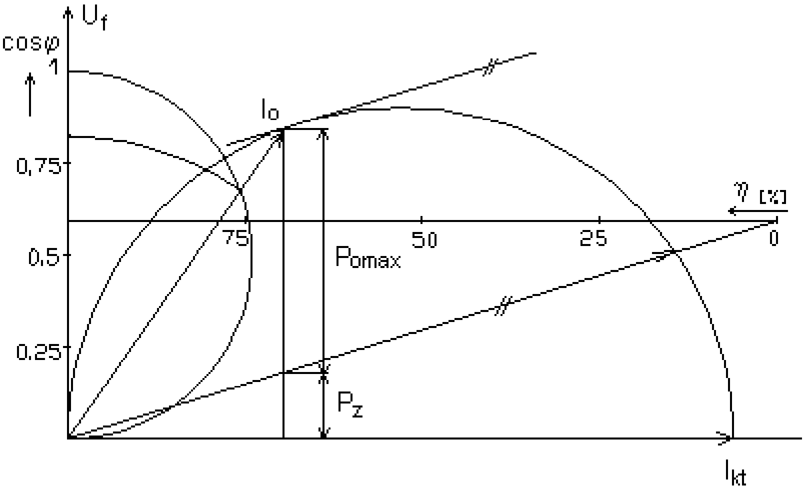
The circle diagram of a steel electric arc furnace is constructed from two basic current values :

1. theoretical short circuit current $$I_{kt} = \frac{U_f}{\sum X}$$

2. current shorted $$I_k = \frac{U_f}{\sum Z}$$

WhereU_f . . . . phase secondary voltage of furnace transformer
$\sum X$ . . . . short path single phase reactance
$\sum Z$ . . . . short path single phase impedance

and furthermore, the $\cos\varphi_k$ power factor, i.e. the phase shift between voltage and current when the electrodes short-circuit with the charge.
While I_{kt} is a theoretical value and can only be calculated, the current I_k can be calculated knowing the design of the short path. However, this value is difficult to calculate due to the geometric complexity of the short path and is therefore usually determined by a special measurement in a so-called short circuit soak test. In this test, electrodes are immersed in a molten bath at a suitably selected voltage level and the short-circuit current I_k and the power factor $\cos\phi_k$ are measured. Instead of the power factor it is possible to measure the active power of the supply circuit and the phase voltage (see chapter 5 for more details).



Circular diagram of an electric arc furnace.

The following data can be read from the circle diagram for a given current magnitude :

- power factor **cosφ**
- power supply circuit power **P**
- arc power **P_0**
- power losses on a short journey **P_Z**
- effectiveness **η**

The scales for reading power factor and efficiency can be constructed as shown. For power readings, it is necessary to know the power scale :

$$m_p = m_I \cdot U_f \ (kW\ m\cdot^{-1} ;\ kA\cdot m^{-1} ,\ V)$$

Where m_I . . . . current scale - usually selected according to the desired diameter of the circle diagram d : $m_I = I_{kt} / d$

AT_f . . . . phase secondary voltage of furnace transformer

Usually a good agreement of the readings with the actual values is not achieved due to the simplifications on the basis of which the circle diagram was constructed.

Example 1 :

The furnace transformer has a 6000/240V voltage ratio. With the electrodes meeting the load, the current on the primary side was measured to be I_{1k} = 1520 A. The power factor on the secondary side was equal to $\cos\phi_{1k} = \cos\phi_{2k} = 0.25$. Construct a circle diagram and determine the current for maximum arc power from it. The wiring of the furnace transformer is D/d.

Solution :

Secondary current shorted :

$$I_{2k} = I_{1k} \cdot p = 1520 \cdot 6000 / 240 = 38\ 000 \text{ A}$$

Impedance short :

$$Z_{2K} = \frac{U_2}{\sqrt{3}.I_{2K}} = \frac{240}{\sqrt{3}.38000} = 3{,}646.10^{-3}\,\Omega$$

Short path active resistance :

$$R_{2k} = Z_{2k} \cdot \cos\phi_{2k} = 3.646 \cdot 10^{-3} \cdot 0.25 = 9.116 \cdot 10^{-4}\Omega$$

Reactance of the short path :

$$X = {}_{2k}\sqrt{Z_{2k}^2 - R_{2k}^2} = \sqrt{0.003646^2 - 0.0009116^2} = 3.531 \cdot 10^{-3} \qquad \Omega$$

Theoretical current shorted :

$$I_{2kT} = \frac{U_{2f}}{X_{2k}} = \frac{240}{\sqrt{3}\cdot 3.351\cdot 10^{-3}} = 39246 \quad A$$

Scale of current :

$m_I = I_{2kT} / d = 39246 / 20 = 1962$ A· cm^{-1}

Power Scale :

$m_P = U_{2f}\cdot m_I = 240\cdot 1962 /\sqrt{3} = 271.862\cdot 10^3$ W· cm^{-1}

Values read from the diagram :

The current corresponding to the maximum arc power :

$I_{Pmax} = 24.7$ kA

Maximum arc power :

$P_{omax} = 2.175$ MW

Efficiency corresponding to the maximum arc power :

$\cos\phi_{Pmax} = 0.78$

B.3.2. Furnace transformer control, choke

The power supplied to the working area of an electric arc furnace is usually regulated in two ways:

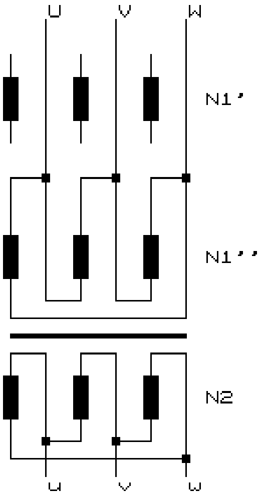
1. stepwise switching of taps on the primary side of the furnace transformer - in this way the voltage on the arc changes stepwise

2. by continuously changing the arc length - this is realized by automatic regulation of electrode movement. By increasing the arc length, the current through the furnace circuit decreases and vice versa.

Example 1 :

Design a four-stage furnace transformer voltage control for an electric arc furnace by switching two sections on the primary side of the furnace transformer winding. The secondary winding is connected in a triangle. The ratio of the number of turns of the primary winding sections is $N_1{}' = 0.262\ N\cdot_1{}''$. The primary voltage on the furnace transformer is 6000 V, the secondary voltage when the sections of $N_1{}''$ are connected in a triangle is 240 V. Calculate the secondary voltage for the other three stages of regulation.

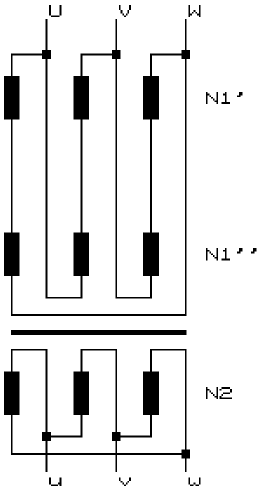
Solution : 1st stage of regulation



Secondary voltage of the 1st control stage :

$$U_{2I} = 240\ \text{V}$$

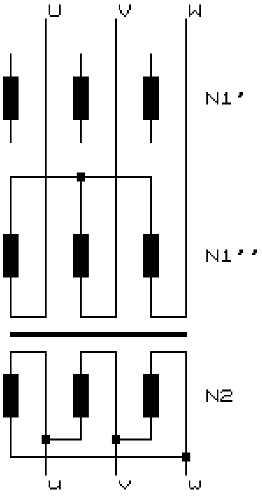
Level II regulation



Secondary voltage of the 2nd control stage :

$$U_{2II} = \frac{U_{2I}}{1.262} = \frac{240}{1.262} = 190 \quad \text{V}$$

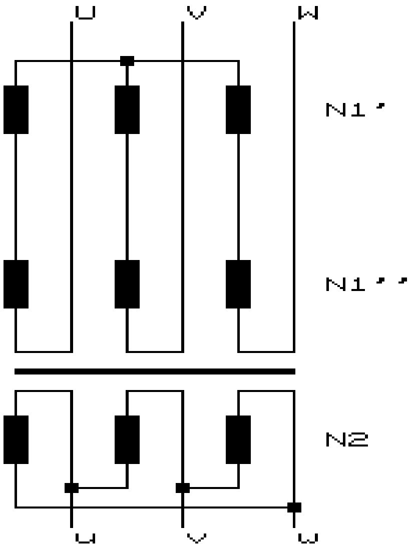
Level III regulation



Secondary voltage of control stage III :

$$U_{2III} = \frac{U_{2I}}{\sqrt{3}} = \frac{240}{\sqrt{3}} = 138 \quad \text{V}$$

Level IV regulation



Secondary voltage of the IV control stage :

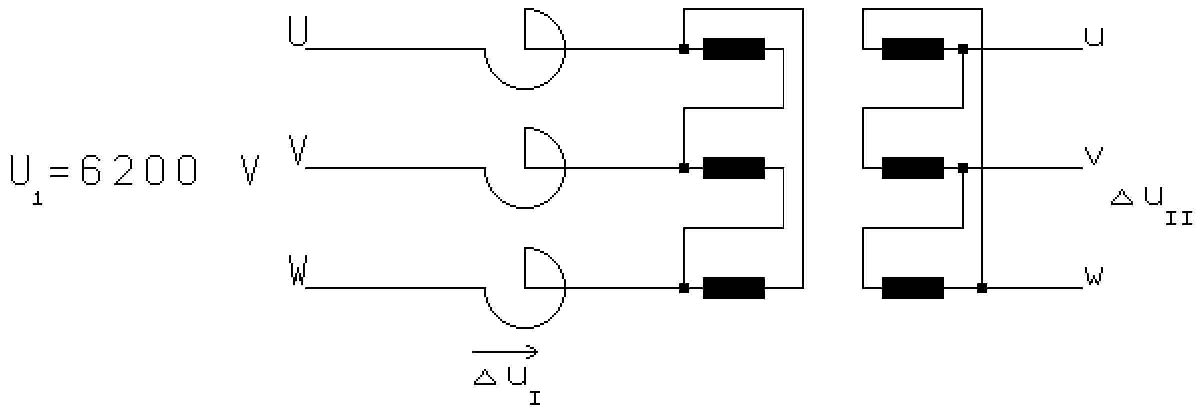
$$U_{2IV} = \frac{U_{2II}}{\sqrt{3}} = \frac{190}{\sqrt{3}} = 109.4 \qquad V$$

Example 2 :

Determine the voltage drop across the secondary side of the furnace transformer feeding the electric arc furnace caused by the reactor $U\Delta_n = 550$ V, $I_n = 400$ A included in the primary circuit if the combined primary voltage across the furnace transformer is equal to U_1 =6200V.

a, if $U_2 = 220$ V, $I_1 = I_n$
b, if $U_2 = 160$ V, $I_1 = I_n / 2$

Solution :



The cumulative voltage drop on the secondary side of the furnace transformer can be calculated from the relation :

$$\Delta u_{II} = u\Delta_I \cdot\ p \cdot \sqrt{}\ 3$$

wherep .... furnace transformer conversion
ΔAt_I .... voltage drop on the primary side of the furnace transformer

a, $u\Delta_{II} = U\Delta_n \cdot\ U_{II} / U_I \cdot \sqrt{}\ 3 = 550 \cdot 220 / 6200 \cdot \sqrt{}\ 3 = 33.8$ V

b, $u\Delta_{II} = 550 / 2 \cdot 160 / 6200 \cdot \sqrt{}\ 3 = 12.3$ V

Example 3 :

The electric arc furnace is powered by a furnace transformer with a rated apparent power of $S_n = 6$ MVA. The conversion is 6000/240 V when connected to Dd0. The short-circuit voltage of the furnace transformer is $at_k = 5\%$. The leads to the furnace have a reactance of 5% (neglect the active resistance).

Calculate the reactance of the inductor if the short-circuit current is to be equal to at most three times the rated current.

Solution :

The passage of rated current through the furnace circuit corresponds to the rated reactance, which is 100 % in percentage terms. Three times the rated current corresponds to a condition where the reactance of the circuit drops to 1/3, i.e. to 33.3 %. This reactance includes the reactances of the feeders (5 %) and the short-circuit voltage, i.e. the reactance of the furnace transformer (5 %).

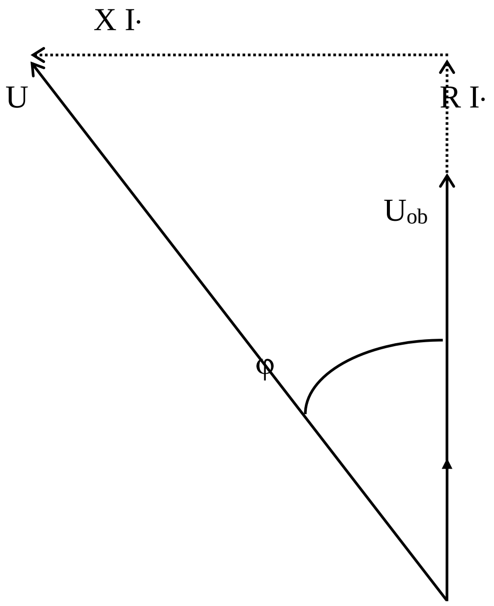
The reactance remains on the choke:
$x_{tl} = 33.3 - (5 + 5) = 23.3$ %
$X_{tl} = 0.233 \cdot X_n = 0.233 \cdot U_f / I_n = 0.233 \cdot U_s^2 / S_n = 0.233 \cdot 6000^2 / 6 \cdot 10^6 = 1.39\ \Omega$

Short Path Electric Arc Furnace

The short path of the electric arc furnace starts at the terminals of the secondary winding of the furnace transformer and ends with the electric arc burning between the electrode and the insert. The parameters of the short path (its active resistance and reactance) are important because they allow us to derive the parameters of the el. In the furnace working area, where the measurement of these parameters would be technically very difficult.

Diagnostic measurements on electric arc furnaces are usually carried out at the beginning of the short path and by correcting for the parameters of this short path, the adjustment of the furnace control is carried out on the basis of the values of the electrical quantities on the arcs.

The relationship between the electrical quantities at the beginning and end of the short path is shown in the following phase diagram:



In the diagram it indicates

U . . . . . . measured voltage at the beginning of the short path
R· I . . . . . . voltage drop across the short path active resistance
X· I . . . . . . voltage drop across the short path reactance

I . . . . the current through the furnace circuit
AT$_{ob}$ . . . . . voltage at the electric arc of the furnace

Example 1 :

A furnace transformer feeding a 15 ton steel arc furnace has a rated apparent power of S_n = 5 MVA and is connected to the grid at U_1 = 6000 V. The voltage on the secondary side is 220 V, the total short path voltage drop when the rated current is passed is $U\ \Delta_n$ = 100 V. Determine the following values (neglecting the short path active resistance) :

- rated primary current of furnace transformer I_{1n}
- rated secondary current of furnace transformer I_{2n}
- secondary short circuit current I_{2k}
- arc voltage U_{ob}
- active power on arcs P_{ob}
- effect at the beginning of a short journey $\cos\phi_n$

Solution :

Rated primary current of furnace transformer :

$$I_{1n} = \frac{S_n}{\sqrt{3}\cdot U_{1n}} = \frac{5\cdot 10^6}{\sqrt{3}\cdot 6\cdot 10^3} = 481 \qquad A$$

Rated secondary current of furnace transformer :

$$I_{2n} = \frac{S_n}{\sqrt{3}\cdot U_{20}} = \frac{5\cdot 10^6}{\sqrt{3}\cdot 220} = 13121 \qquad A$$

Voltage on the arc :

$$U_{ob} = \frac{1}{\sqrt{3}}\cdot\sqrt{U_{20}^2 - DU_n^2} = \frac{1}{\sqrt{3}}\cdot\sqrt{220^2 - 100^2} = 113 \qquad V$$

Active power on curves :

$$P_{ob} = 3\cdot U_{ob}\cdot I_{2n} = 3\cdot 113\cdot 13121 = 4.448 \qquad MW$$

Effect at the beginning of a short journey :

$$\cos\phi_n = \frac{P_{ob}}{S_n} = \frac{4.448\cdot 10^6}{5\cdot 10^6} = 0.89$$

Calculation of short path parameters of electric arc furnace

Assignment :

Calculate the active resistances and reactances of the short path of an electric arc furnace, determine the theoretical short circuit and short circuit current, construct a circle diagram of the furnace and determine from it :

- current corresponding to the maximum active power on the electric arc
- the value of the maximum active arc power
- power factor corresponding to the maximum active arc power
- furnace efficiency at maximum active arc power

Entered values :

voltage on the primary side of the furnace transformer	3 x 380 / 220 V
voltage on the secondary side of the furnace	transformerU_2 = 55 V
specific resistivity of the electrode material	$\rho_E = 10\ \Omega\cdot \text{mm}^{2}\cdot \text{m}^{-1}$
specific resistance of the material of the other parts	$\rho_{Cu} = 1/45\ \Omega\cdot \text{mm}^{2}\cdot \text{m}^{-1}$
sum of short path contact resistances	$\Sigma R_{ss} = 2\cdot 10^{-5}\Omega$
voltage drop across the contact resistance of the jaws	$U_{s\check{c}} = 0.5$ V
relative reactance of the inductor	x = 20 %
apparent power of the furnace transformer	$S_n = 150$ kVA
short-circuit losses of furnace transformer	$P_{2k} = 1.5$ kW
short-circuit voltage of furnace transformer	$e_k = 6\%$

The skin effect and the proximity effect are neglected, the arrangement and dimensions of the individual parts of the short path are clear from the figure.

Solution :

Silencer

$$X_{T1}^{/} = \frac{U_{1f}}{I_{1f}}\cdot x = \frac{220}{227.9}\cdot 0.2 = 0.193 \qquad \Omega$$

$$I_{1f} = \frac{S_n}{\sqrt{3}\cdot U_1} = \frac{150\cdot 10^3}{\sqrt{3}\cdot 380} = 227.9 \qquad A$$

Inductor reactance converted to the secondary side of the furnace transformer :

$$X_{T1} = X'_{T1} \cdot \left(\frac{I_{1f}}{I_{2S}}\right)^2 = 0.193 \cdot \left(\frac{227.9}{1574.6}\right)^2 = 4.043 \cdot 10^{-3} \qquad \Omega$$

$$I_{2S} = \frac{S_n}{\sqrt{3} \cdot U_2} = \frac{150 \cdot 10^3}{\sqrt{3} \cdot 55} = 1574.6 \qquad A$$

Neglect the active resistance of the inductor: $R_{T1} = 0$

Furnace transformer :

Determine the active resistance from the short-circuit losses :

$$R_{Tr} = \frac{P_{2k}}{3 \cdot I_{2S}^2} = \frac{1.5 \cdot 10^3}{3 \cdot 1574.6^2} = 2.017 \cdot 10^{-4} \qquad \Omega$$

Reactance :

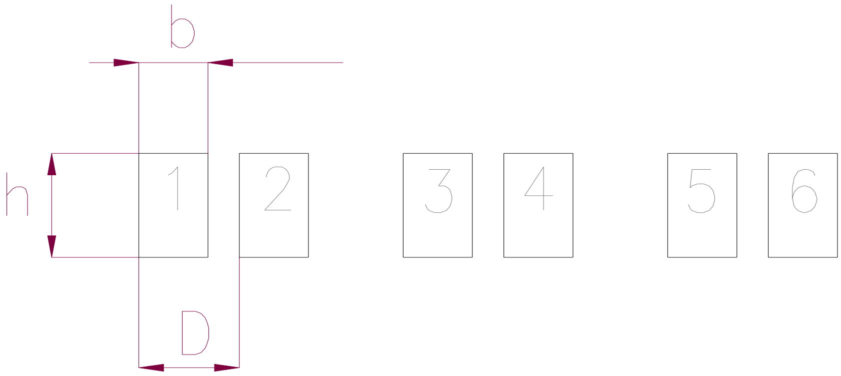
$$e_r = \frac{P_{2k}}{S_n} \cdot 100 = \frac{1.5 \cdot 10^3}{150 \cdot 10^3} \cdot 100 = 1 \qquad \%$$

$$e_x = \sqrt{e_k^2 - e_r^2} = \sqrt{6^2 - 1^2} = 5.916 \qquad \%$$

$$X_{Tr} = Z_n \cdot \frac{e_x}{100} \cdot \left(\frac{I_{1f}}{I_{2S}}\right)^2 = 0.965 \cdot \frac{5.916}{100} \cdot \left(\frac{227.9}{1574.6}\right)^2 = 1.196 \cdot 10^{-3} \qquad \Omega$$

$$Z_n = \frac{U_{1f}}{I_{1f}} = \frac{220}{227.9} = 0.965 \qquad \Omega$$

Bifilar lines :



Inductance of conductor 1 :

$$L_1 = L_{11} - M_{21} - \frac{1}{2} \cdot M_{31} + \frac{1}{2} \cdot M_{41} - \frac{1}{2} \cdot M_{51} + \frac{1}{2} \cdot M_{61} = L_{11} - M_{21}$$

$$L_{11} = 2 \cdot l \cdot \left(2.3 \cdot \log \frac{2 \cdot l}{r_e} - 1 \right) \cdot 10^{-9} \qquad [H; cm]$$

$$M_{21} = 2 \cdot l \cdot \left(2.3 \cdot \log \frac{2 \cdot l}{D_e} - 1 \right) \cdot 10^{-9} \qquad [H; cm]$$

$$r_e = m \cdot (h + b) = 0.2235 \cdot (55 + 2) = 12.73 \qquad mm$$

$r_e = 0.2235 \cdot (h + b)$ valid for rectangular cross-section

$r_e = 0.778 \cdot r$ valid for circular cross-section

D_e is read from the graph and is a function of $\frac{b}{h}$ a $\frac{D}{h}$ (see appendix)

for $\frac{b}{h} = \frac{2}{55} = 0.036$ and for

$\frac{D}{h} = \frac{20}{55} = 0.364$ $D_e = 1.35 \cdot D = 1.35 \cdot 20 = 27$ mm

After substituting l = 2530 mm, b = 2 mm, h = 55 mm, D = 20 mm the result is :

$$L_{11} = 2.519 \cdot 10^{-6} \quad H \qquad M_{21} = 2.139 \cdot 10^{-6} \quad H \qquad L_1 = 0.380 \cdot 10^{-6}$$

Reactance and active resistance :

$$X_{B1} = 2 \cdot \pi \cdot f \cdot L_1 = 2 \cdot \pi \cdot 50 \cdot 0.380 \cdot 10^{-6} = 0.119 \cdot 10^{-3} \quad \Omega$$

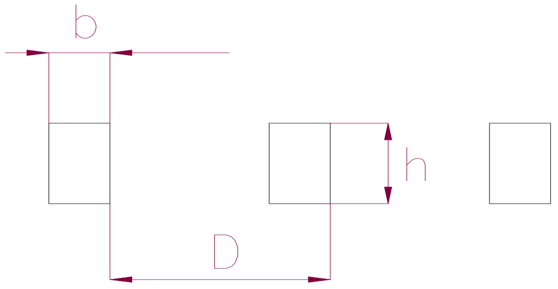
$$R_{B1} = \rho_{Cu} \cdot \frac{l}{S} = \frac{1}{45} \cdot \frac{2.53}{2 \cdot 55} = 0.511 \cdot 10^{-3} \quad \Omega$$

After converting from a triangle to a star :

$$X_B = \frac{2}{3} \cdot X_{B1} = \frac{2}{3} \cdot 0.119 \cdot 10^{-3} = 0.080 \cdot 10^{-3} \quad \Omega$$

$$R_B = \frac{2}{3} \cdot R_{B1} = \frac{2}{3} \cdot 0.511 \cdot 10^{-3} = 0.365 \cdot 10^{-3} \quad \Omega$$

Webbing :



Inductance of one phase :

$$L_1 = L_{11} - \frac{1}{2} \cdot M_{21} - \frac{1}{2} \cdot M_{31}$$

For the calculation of the self-inductance L_{11} and the mutual inductances M_{21} , M_{31} the same calculation relations as for bigilar lines apply.

After r_e , we'll put $r_e = m \cdot (h + b) = 0.2235 \cdot (50 + 10) = 13.41 \quad mm$.

After D_e we put $D_e = D = 300 \quad mm$ in case of M_{21} , and

$$D_e = 2 \cdot D = 2 \cdot 300 = 600 \quad mm$$

in the case of the M calculation$_{31}$.

After substituting l = 1150 mm, b = 10 mm, h = 50 mm, D = 300 mm the result is :

$$L_{11} = 0.951 \cdot 10^{-6} \quad H \qquad M_{21} = 0.237 \cdot 10^{-6} \quad H \qquad M_{31} = 0.078 \cdot 10^{-6} \quad H$$

$$L_1 = 0.636 \cdot 10^{-6} \quad H$$

Reactance and active resistance of one phase :

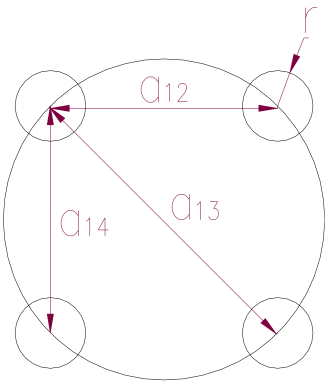
$$X_{p1} = 2 \cdot \pi \cdot f \cdot L_1 = 2 \cdot \pi \cdot 50 \cdot 0.636 \cdot 10^{-6} = 0.200 \cdot 10^{-3} \quad \Omega$$

$$R_{p1} = \rho_{Cu} \cdot \frac{l}{S} = \frac{1}{45} \cdot \frac{1.15}{10 \cdot 50} = 0.05 \cdot 10^{-3} \quad \Omega$$

Flexible cables :

$$r_e = \sqrt[4]{r \cdot a_{12} \cdot a_{13} \cdot a_{14}}$$

$$D_e = \sqrt[3]{d_{12} \cdot d_{13} \cdot d_{23}}$$



$$L_k = 2 \cdot l \cdot \left(2.3 \cdot \log \frac{D_e}{r_e} + 0.05 \right) \cdot 10^{-9} \qquad [H; cm]$$

After substituting l = 1300 mm, a = 200 mm, d = 19.55 mm, D = 300 mm the result is :

$$a_{12} = a_{14} = a \cdot \sin\left(45^{o}\right) = 200 \cdot 0.707 = 141.4 \quad mm$$

$$r = \frac{d}{2} = \frac{19.55}{2} = 9.775 \qquad \text{mm}$$

$$r_e = \sqrt[4]{9.775 \cdot 141.4 \cdot 200 \cdot 141.1} = 79.07 \qquad \text{mm}$$

$$D_e = \sqrt[3]{300 \cdot 600 \cdot 300} = 377.98 \qquad \text{mm}$$

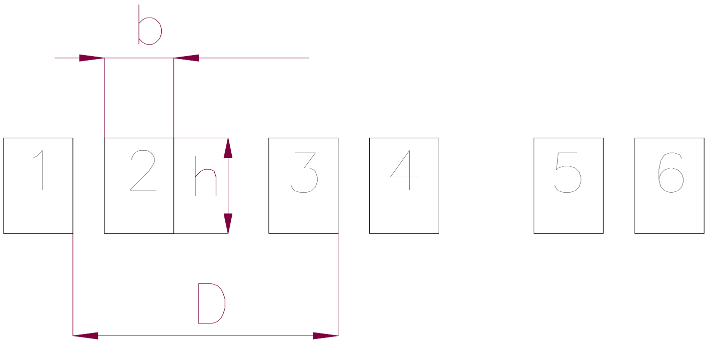
$$L_k = 0.419 \cdot 10^{-6} \qquad \text{H}$$

Reactance and active resistance of one phase :

$$X_{ok} = 2 \cdot \pi \cdot f \cdot L_k = 2 \cdot \pi \cdot 50 \cdot 0.419 \cdot 10^{-3} = 0.131 \cdot 10^{-3} \qquad \Omega$$

$$R_{ok} = \rho_{Cu} \cdot \frac{l}{S} = \frac{1}{45} \cdot \frac{1.3}{\frac{\pi \cdot 19.55^2}{4}} \cdot \frac{1}{4} = 0.05 \cdot 10^{-3} \qquad \Omega$$

Double webbing :



Inductance of conductor 1 :

$$L_1 = L_{11} + M_{21}$$

For the calculation of the self-inductance L_{11} and the mutual inductance M_{21} , the same relationships as for bifilar lines apply.

After r_e , we'll put $r_e = m \cdot (h + b) = 0.2235 \cdot (53 + 2) = 12.29 \qquad \text{mm}$

The dimension D_e is subtracted from the graph $D_e = f\left(\frac{b}{h}, \frac{D}{h}\right)$ for $\frac{b}{h} = \frac{2}{53} = 0.0377$ and $\frac{D_e}{D} = 1.06$ holds for $\frac{D}{h} = \frac{71}{53} = 1.3396$, so

$$D_e = 1.06 \cdot D = 1.06 \cdot 71 = 75.26 \quad \text{mm}$$

After substituting l = 710 mm, b = 2 mm, h = 53 mm, D = 71 mm the result is :

$$L_{11} = 0.531 \cdot 10^{-6} \quad \text{H} \qquad M_{21} = 0.274 \cdot 10^{-6} \quad \text{H} \qquad L_1 = 0.805 \cdot 10^{-6} \quad \text{H}$$

Reactance and active resistance of one phase :

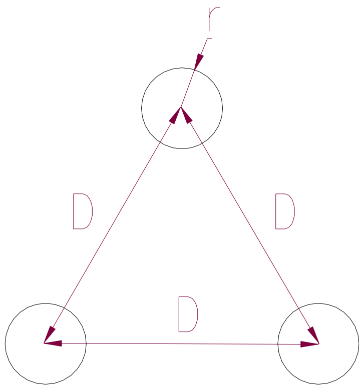
$$X_{p2} = 2 \cdot \pi \cdot f \cdot \frac{L_1}{2} = 2 \cdot \pi \cdot 50 \cdot \frac{0.805 \cdot 10^{-6}}{2} = 0.126 \cdot 10^{-3} \quad \Omega$$

$$R_{p2} = \rho_{Cu} \cdot \frac{l}{S} = \frac{1}{45} \cdot \frac{0.71}{2 \cdot 53 \cdot 2} = 0.074 \cdot 10^{-3} \quad \Omega$$

Electrodes :

$$r_e = r = 37.5 \quad \text{mm}$$

$$D_e = D = 167 \quad \text{mm}$$



$$L_E = 2 \cdot 1 \cdot \left(2.3 \cdot \log \frac{D_e}{r_e} + 0.05 \right) \cdot 10^{-9} \qquad [H; cm]$$

After inserting l = 1800 mm, r = 37.5 mm, D = 167 mm the result is :

$$L_E = 0.246 \cdot 10^{-6} \qquad H$$

Reactance and active resistance of one phase :

$$X_E = 2 \cdot \pi \cdot f \cdot L_E = 2 \cdot \pi \cdot 50 \cdot 0.246 \cdot 10^{-6} = 0.077 \cdot 10^{-3} \qquad \Omega$$

$$R_E = \rho_E \cdot \frac{l}{S} = 10 \cdot \frac{0.8 \cdot 4}{\pi \cdot 75^2} = 1.811 \cdot 10^{-3} \qquad \Omega$$

Short path contact resistors :

$$R_{ss} = 0.02 \cdot 10^{-3} \qquad \Omega$$

Contact resistance of the jaws :

$$R_{sŹ} = \frac{\Delta U}{I_{2S}} = \frac{0.5}{1574.6} = 0.318 \cdot 10^{-3} \qquad \Omega$$

Reactance, active resistance and short path impedance :

$$R = \Sigma R = 2.864 \cdot 10^{-3} \qquad \Omega$$

$$X = \Sigma X = 5.853 \cdot 10^{-3} \qquad \Omega$$

$$Z = \sqrt{R^2 + X^2} = 6.516 \cdot 10^{-3} \qquad \Omega$$

Short circuit current :

$$I_k = \frac{U_{2f}}{Z} = \frac{55}{\sqrt{3} \cdot 6.516 \cdot 10^{-3}} = 4.873 \cdot 10^{3} \qquad A$$

Theoretical short circuit current :

$$I_{kT} = \frac{U_{2f}}{X} = \frac{55}{\sqrt{3} \cdot 5.853 \cdot 10^{-3}} = 5.425 \cdot 10^{3} \qquad \text{A}$$

Scales for constructing a circle diagram :

Choose the diameter of the circle diagram d = 200 mm

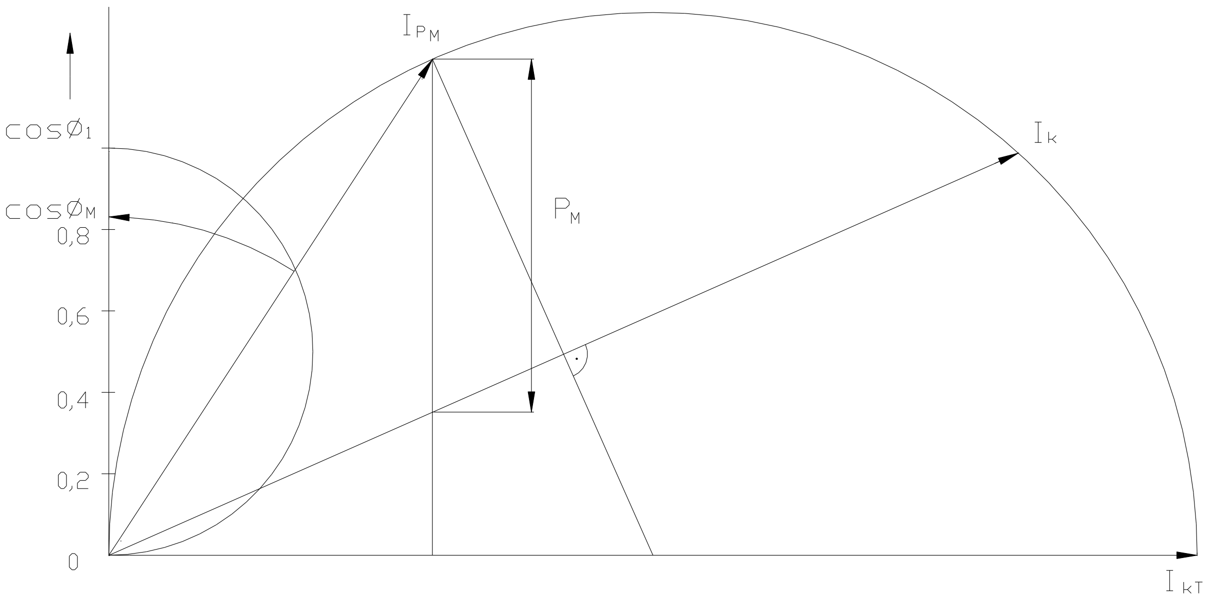
Scale of current :

$$m_{I} = \frac{I_{kT}}{d} = \frac{5425}{200} = 27.125 \qquad \text{A/mm}$$

Scale of active performance :

$$m_{P} = U_{2f} \cdot m_{I} = 55 \cdot \frac{27.125}{\sqrt{3}} = 861.3 \qquad \text{W/mm}$$

Circular diagram of electric arc furnace



I_{PM} = 2.9 kA $\cos\phi_M$ = 0.82 P_M = 53.4 kW

INDUCTION AND DIELECTRIC ELECTROTHERMAL DEVICES

B.4.1. Induction channel furnaces

Channel induction furnaces are built directly on the mains frequency. The molten charge in the circular channel forms a single secondary short thread. They have a better power factor because the magnetic flux passes through an iron core made of electrical sheets. The disadvantage is that molten metal must be poured into them during the first melting.

Example 1 :

Design an induction channel furnace with power P = 100 kW, voltage U_1 = 220 V, magnetic induction in the core B = 1.2 T, frequency f = 50 Hz.

Other initial data : $\cos\phi = 0.5$, coefficient c = 0.34,$\psi = 9.1$, current density in primary winding$\sigma = 3$ A· mm^{-2} , $N_2 = 1$.

Determine :

- iron core cross section S_{Fe} , if $S_{Fe} = c \cdot \sqrt{\frac{S \cdot \psi \cdot 10^5}{B \cdot \sigma \cdot f}}$ (cm)^2
- primary winding current I_1
- cross-section of the primary winding conductor S_{cu}
- number of primary threads N_1
- voltage on the secondary side U_2
- current on the secondary side I_2

Solution :

Cross section of iron core :

$$S_{Fe} = c \cdot \sqrt{\frac{S \cdot \psi \cdot 10^5}{B \cdot \sigma \cdot f}} = 0.34 \cdot \sqrt{\frac{200 \cdot 9.1 \cdot 10^5}{1.2 \cdot 3 \cdot 50}} = 342 \qquad cm^2$$

Current through the primary winding :

$$I_1 = \frac{S}{U_1} = \frac{P}{\cos\phi \cdot U_1} = \frac{100 \cdot 10^3}{0.5 \cdot 220} = 909 \qquad A$$

Primary winding conductor cross section :

$$S_{Cu} = \frac{I_1}{\sigma} = \frac{909}{3} = 303 \qquad mm^2$$

Number of primary threads :

$$N_1 = \frac{U_1}{4.44 \cdot B \cdot S_{Fe} \cdot f} = \frac{220}{4.44 \cdot 1.2 \cdot 0.0342 \cdot 50} = 24.1 = 25 \qquad \text{závit ů}$$

Voltage on the secondary side :

$$U_2 = \frac{U_1}{N_1} = \frac{220}{25} = 9.17 \qquad V$$

Current on the secondary side :

$$I_2 = N_1 \cdot I_1 = 25 \cdot 909 = 21816 \qquad A$$

Induction cup furnaces

A medium frequency crucible induction furnace consists of a crucible made of non-conductive material around which is a coil fed from a special frequency source of 500 - 10 000 Hz (tooth generator, power electronic oscillator). The coil is usually made of a copper tube through which cooling water flows. The efficiency of the furnace is very low (0.05 - 0.3) and is usually compensated by a capacitor bank connected in parallel, which is tuned to resonance with the inductance of the furnace during melting.

Example 1 :

The replacement induction crucible furnace scheme has the following parameters :

inductance of the oven	$L_I = 1.3 \cdot 10^{-4}$ H
active resistance of the furnace	$R_I = 4 \cdot 10^{-2} \Omega$
Capacitance of capacitor battery	$C = 2.2 \cdot 10^{-4}$ F
supply frequency	$f = 1000$ Hz
supply voltage	$U_G = 2500$ V

Determine :

- current drawn from the generator I_G
- Furnace current I_P
- capacitor bank current I_C
- resonant circuit quality factor Q

Draw a vector diagram.

Solution :

Replacement resistance :

$$R_Z = \frac{L_I}{C \cdot R_I} = \frac{1.3 \cdot 10^4}{2.2 \cdot 10^{-4} \cdot 4 \cdot 10^{-2}} = 14.75 \qquad \Omega$$

Current drawn from the generator :

$$I_G = \frac{U_G}{R_Z} = \frac{2500}{14.75} = 169.4 \qquad A$$

Resonant circuit quality factor :

$$Q = \frac{\omega \cdot L_I}{R_I} = \frac{2 \cdot \pi \cdot 10^3 \cdot 1.3 \cdot 10^{-4}}{4 \cdot 10^{-2}} = 20.4$$

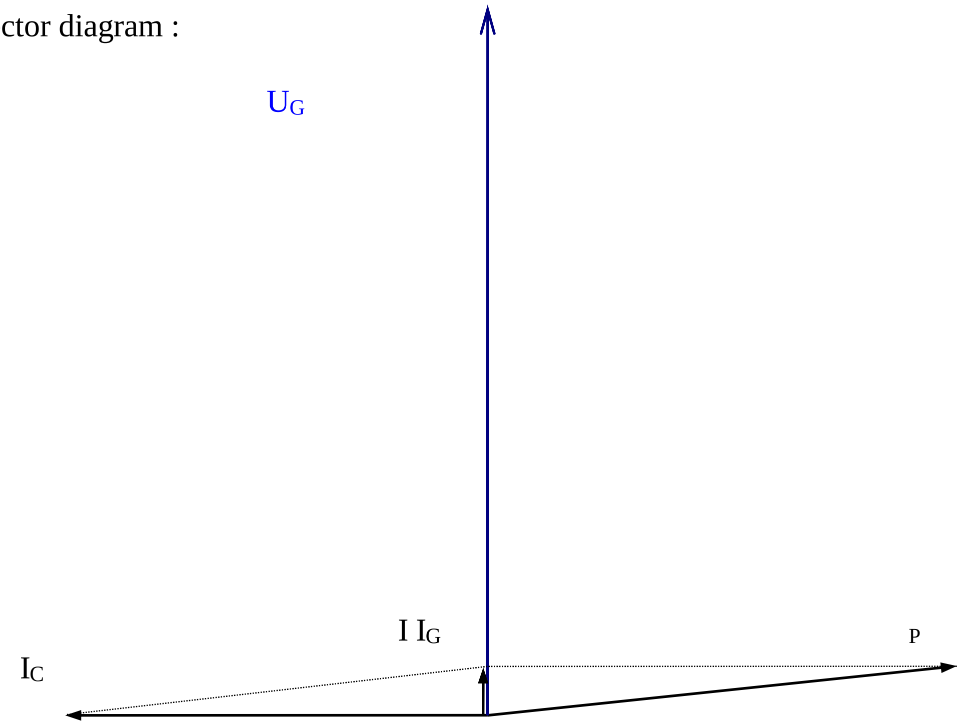
Capacitor battery current :

$$I_C = U_G \cdot \omega \cdot C = 2500 \cdot 2 \cdot \pi \cdot 10^3 \cdot 2.2 \cdot 10^{-4} = 3460 \qquad A$$

Furnace current :

$$I_P = \frac{I_C \cdot \sqrt{1+Q^2}}{Q} = \frac{3460 \cdot \sqrt{1+20.4^2}}{20.4} = 3465 \qquad A$$

Vector diagram :



Example 2 :

Determine the losses in the coil of an induction crucible furnace if the replacement coil diameter d_C = 1200 mm , the number of coil turns N_1 = 16, the specific resistance of the coil conductorρ = 0.0175 Ω· mm ^{2}· m^{-1} , penetration depth a = 0.284 cm, coil voltage U_G = 3000 V, coil active resistance R_I = 5.2· 10 $^{-2}$Ω ,
Inductance of coil L_I = 1.3· 10^{-4} H, frequency of generator f = 600 Hz, length of coil l_1 = 1.2 m, electric field strength in coil E = 150 V· cm^{-1} .

Solution :

Determine the active resistance of the coil from the relation :

$$R_1 = \rho \cdot \frac{l}{S} = \rho \cdot \frac{\pi \cdot d_C \cdot N_1}{a \cdot x}$$

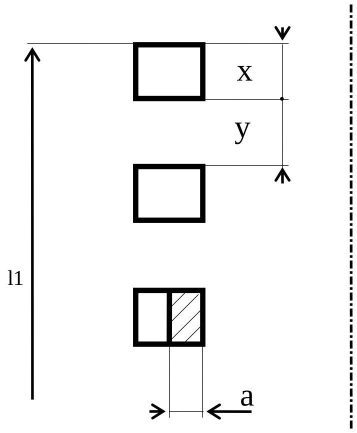
where x . . . .

The conductor height is derived from the relation for the electric field strength :

$$E = \frac{U_G}{(N_1 - 1) \cdot y}$$

$$y = \frac{U_G}{(N_1 - 1) \cdot E} = \frac{3000}{(16 - 1) \cdot 150} = 13.33 \qquad \text{mm}$$

$$x = \frac{l_1 - (N_1 - 1) \cdot y}{N_1} = \frac{1200 - (16 - 1) \cdot 13.33}{16} = 62.5 \qquad \text{mm}$$



Coil active resistance :

$$R_1 = \rho \cdot \frac{\pi \cdot d_C \cdot N_1}{a \cdot x} = 0.0175 \cdot \frac{\pi \cdot 1.22 \cdot 16}{2.84 \cdot 62.5} = 6 \cdot 10^{-3} \qquad \Omega$$

Current flowing through the coil :

$$I_p = \frac{U_G}{\sqrt{R_I^2 + (\omega \cdot L_I)^2}} = \frac{3000}{\sqrt{\left(5.2 \cdot 10^{-2}\right)^2 + \left(2 \cdot \pi \cdot 600 \cdot 1.3 \cdot 10^{-4}\right)^2}} = 6087 \qquad A$$

Induction furnace coil losses :

$$P_z = R_1 \cdot I_p^2 = 6 \cdot 10^{-3} \cdot 6087^2 = 222.3 \cdot 10^3 \qquad W$$

Dielectric heating

Dielectric heating occurs in electrically non-conductive materials placed in the electric field of a capacitor connected to a high frequency source. Frequencies on the order of 10^6 - 10^9 Hz are used for dielectric heating. The fastest heating is for a frequency with a period close to the relaxation time of the material.

Example 1 :

Determine the power and voltage of the generator for dielectric heating of the material from $\vartheta_1 = 20$ °C to $\vartheta_2 = 180$ °C. Specific heat capacity of the material c = 0.35 kcal/kg· K, relative permittivity $\varepsilon_r = 5$, loss factor tgδ = 0.035, specific gravity $\gamma = 900$ kg/m^3 .

The mass of the charge is m = 10 kg , the frequency of the source is f = 25 Mhz, the thickness of the heated material is d = 50 mm , the heating time is t = 20 min.

Solution :

We start from the relationship :

$$P = U \cdot I \cdot \cos\phi = U^2 \cdot \omega \cdot C \cdot tg\delta$$

Generator voltage :

$$U=\sqrt{\frac{P}{\omega\cdot C\cdot tg\delta}}=\sqrt{\frac{P}{\frac{2\cdot\pi\cdot f\cdot\varepsilon_o\cdot\varepsilon_r\cdot S}{d}\cdot tg\delta}}=\sqrt{\frac{\frac{m\cdot c\cdot\Delta\vartheta}{t}}{2\cdot\pi\cdot f\cdot\varepsilon_o\cdot\varepsilon_r\cdot tg\delta\cdot\frac{m}{\gamma\cdot d^2}}}$$

$$U=\sqrt{\frac{c\cdot\Delta\vartheta\cdot\gamma\cdot d^2}{2\cdot\pi\cdot f\cdot\varepsilon_o\cdot\varepsilon_r\cdot t\cdot tg\delta}}=\sqrt{\frac{0.35\cdot 4.186\cdot 10^3\cdot(180-20)\cdot 900\cdot\left(5\cdot 10^{-2}\right)^2}{2\cdot\pi\cdot 8.854\cdot 10^{-12}\cdot 5\cdot 25\cdot 10^6\cdot 20\cdot 60\cdot 0.035}}=1338 \qquad V$$

Generator output :

$$P_G=\frac{m\cdot c\cdot\Delta\vartheta}{t}=\frac{10\cdot 0.35\cdot 4.186\cdot 10^3\cdot(180-20)}{20\cdot 60}=1956 \qquad W$$

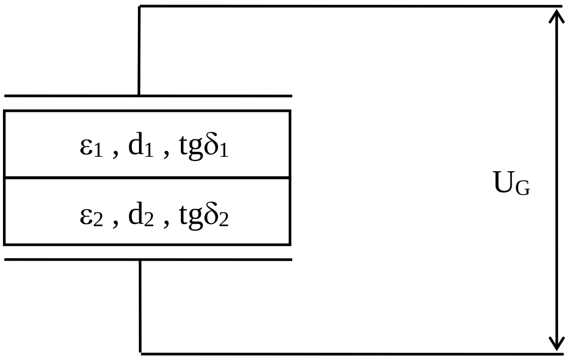
Example 2 :

Determine the gradients (voltage gradients) and volumetric heat outputs on the formulas loaded into the dielectric furnace if given :

Generator : U_G = 1200 V , f = 5 Mhz
the first sample : $\varepsilon_1 = 6$, $tg\delta_1 = 0.04$, d_1 = 30 mm
second sample : $\varepsilon_2 = 30$, $tg\delta_2 = 0.08$, d_2 = 50 mm

Solution :

Arrangement of samples :



Stress gradient in the first sample :

$$E_1 = \frac{U_1}{d_1} = \frac{U}{\varepsilon_1 \cdot \left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)} = \frac{1200}{6 \cdot \left(\frac{0.03}{6} + \frac{0.05}{30} \right)} = 38700 \qquad V \cdot m^{-1}$$

Stress gradient in the second sample :

$$E_2 = \frac{U_2}{d_2} = \frac{U}{\varepsilon_2 \cdot \left(\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} \right)} = \frac{1200}{30 \cdot \left(\frac{0.03}{6} + \frac{0.05}{30} \right)} = 7740 \qquad V \cdot m^{-1}$$

Volumetric heat output in the first sample :

$P_1 = E_1^{2} \cdot 2 \cdot \pi \cdot f \cdot \varepsilon_o \cdot \varepsilon_{r1} \cdot tg\, \delta_1$

$P_1 = 38700^{2} \cdot 2 \cdot \pi \cdot 5 \cdot 10^{6} \cdot 8.854 \cdot 10^{-12} \cdot 6 \cdot 0.04 = 99.98\ kW \cdot m^{-3}$

Volumetric heat output in the second sample :

$P_2 = E_2^{2} \cdot 2 \cdot \pi \cdot f \cdot \varepsilon_o \cdot \varepsilon_{r2} \cdot tg\, \delta_2$

$P_2 = 7740^{2} \cdot 2 \cdot \pi \cdot 5 \cdot 10^{6} \cdot 8.854 \cdot 10^{-12} \cdot 30 \cdot 0.08 = 40\ kW \cdot m^{-3}$

Symmetrization

Induction furnaces cause asymmetry in the three-phase network with their single-phase load. In order to remove this asymmetry, inductance and capacitance are inserted into the other phases, either in a star or triangle arrangement.

Example 1 :

Design a symmetrizing circuit for an induction cup furnace connected to a 3x380/220 V, 50 Hz star network. Generator input P_g = 200 kW. Determine the magnitudes of capacitance, inductance and currents and voltages in all branches.

Solution :

Replacement load resistor :

$$R_z = \frac{U_R^2}{P_g} = \frac{(3 \cdot U_f)^2}{P_g} = \frac{(3 \cdot 220)^2}{200 \cdot 10^3} = 2.17 \qquad \Omega$$

Required inductance :

$$L = \frac{R_z}{\sqrt{3} \cdot \omega} = \frac{2.17}{\sqrt{3} \cdot 2 \cdot \pi \cdot 50} = 3.98 \cdot 10^{-3} \qquad H$$

Required capacity :

$$C = \frac{\sqrt{3}}{\omega \cdot R_z} = \frac{\sqrt{3}}{2 \cdot \pi \cdot 50 \cdot 2.17} = 2.54 \cdot 10^{-3} \qquad F$$

Currents through individual branches :

$$I_R = I_L = I_C = \frac{U_R}{R_z} = \frac{3 \cdot U_f}{R_z} = \frac{3 \cdot 220}{2.17} = 304 \qquad A$$

Voltage on the furnace :

$U_R = 3 \cdot U_f = 3 \cdot 220 = 660$ V

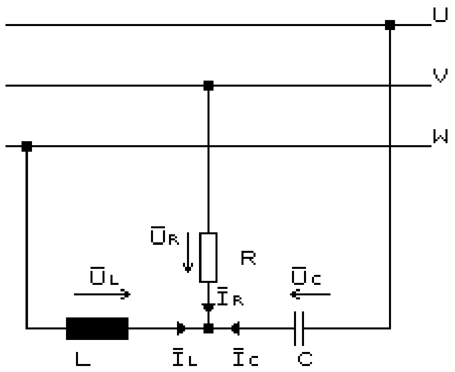
Voltage on inductance :

$U_L = 3\surd \cdot U_f = 3\surd \cdot 220 = 380$ V

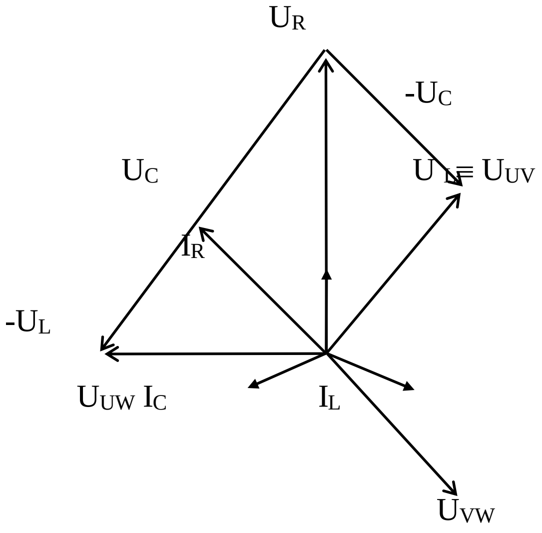
Voltage at capacity :

$U_C = 3\surd \cdot U_f = 3\surd \cdot 220 = 380$ V

Symmetrizing device connected to the star



Vector diagram



Example 2 :

Design a symmetrizing circuit for a 1000 kg steel induction crucible furnace connected directly to a 3x380 V, 50 Hz grid connected in a triangle. Determine the required capacitance and inductance and all currents. Power input P_g = 250 kW.

Solution :

Replacement load resistor :

$$R_z = \frac{U_R^2}{P_g} = \frac{380^2}{250 \cdot 10^3} = 0.575 \qquad \Omega$$

Searched symmetrization capacity :

$$C = \frac{1}{\sqrt{3} \cdot \omega \cdot R_z} = \frac{1}{\sqrt{3} \cdot 2 \cdot \pi \cdot 50 \cdot 0.575} = 3.2 \cdot 10^{-3} \qquad F$$

Search symmetrization inductance :

$$L = \frac{\sqrt{3} \cdot R_z}{\omega} = \frac{\sqrt{3} \cdot 0.575}{2 \cdot \pi \cdot 50} = 3.18 \cdot 10^{-3} \qquad H$$

Currents in feeders :

$$I_U = I_V = I_W = \frac{U_f}{R_z} = \frac{220}{0.575} = 382 \qquad A$$

Furnace current :

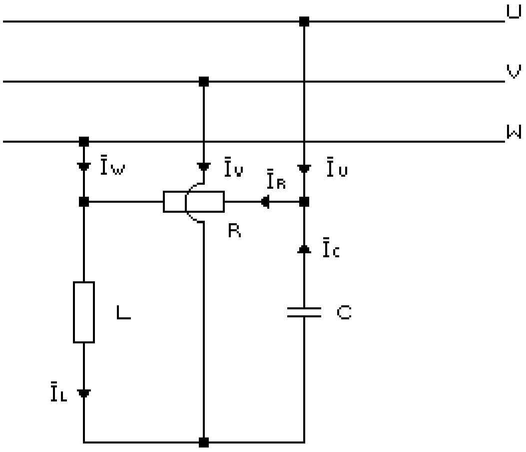
$$I_R = \frac{U_R}{R_z} = \frac{380}{0.575} = 660 \qquad A$$

Current Capacity :

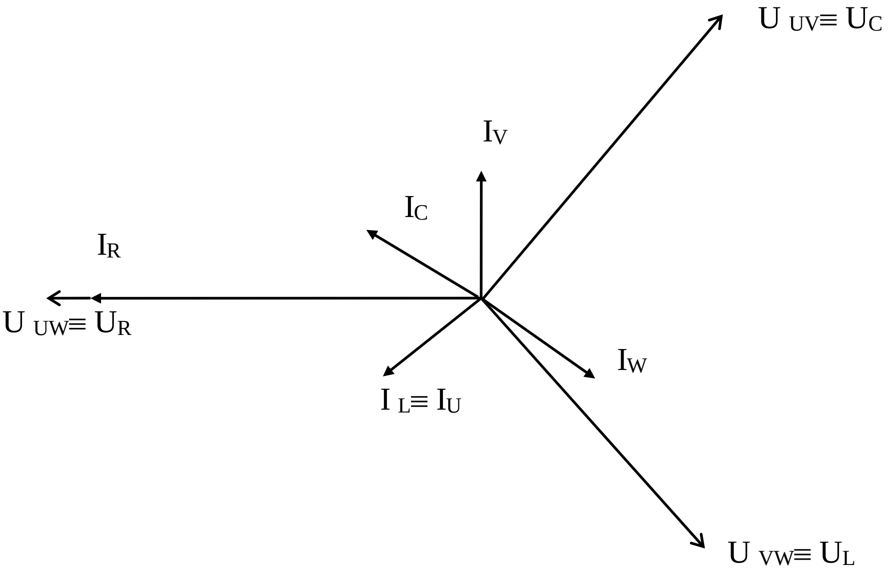
$I_C = I_R /\sqrt{3} = 660 /\sqrt{3} = 382$ A

Current inductance :

$I_L = I_R /\sqrt{3} = 660 /\sqrt{3} = 382$ A



Symmetrizing device connected in a triangle



Vector diagram